

Entropic Scalar EFT: From Entanglement Microstructure to Gravity and Cosmic Structure

Jacob Chinitz

August 29, 2026

Abstract

We propose that empty space is not a passive backdrop but a physical medium with a finite budget of quantum entanglement: the linking structure that allows parts of a quantum system to share state. Matter forms when some of that capacity becomes locked into stable, localized defects of the medium. A particle's mass measures how much entanglement is committed to such a defect. Gravity is the surrounding capacity-strain field: near matter, slightly less entanglement capacity is freely available, and in the weak-field limit the fractional shortfall gives the gravitational potential. The excess acceleration seen in galaxies, usually attributed to particle dark matter, is treated here as the large-scale continuation of the same capacity response rather than as a new unseen substance.

The central result is that this picture is not freely adjustable after the fact. Once one accepts the finite-capacity medium, the three founding postulates, and a specific minimal model for the smallest cell of space, finite counting fixes the cell entropy and the ordinary weak-field response. The resulting capacity action is the static scalar sector of the Einstein action written in the capacity variable, so it gives Newton's law and the leading no-slip metric without introducing another gravitational field. A separately identified transverse branch gives the galactic acceleration scale and the observed relation between galaxy rotation and ordinary matter, subject to the microscopic matching conditions stated in the paper.

The electron plays a double role. As the lightest clean charged defect, it fixes the exchange rate between committed entanglement and mass and calibrates the absolute cell scale. Many-Pasts supplies the history-space interpretation of that calibration while preserving ordinary Born-rule statistics and no-signaling. Applying the same faithful-resolution condition used for the cell ensemble makes the local renewal process memoryless. A reversible marked-transfer action then derives the finite charged response and routes it through the electron and the heavier charged-lepton shells. This adds no new founding premise and leaves the original tetrahedral construction intact.

We also test the cell model in a computer simulation of dynamical spacetime. Turning on the medium's weighting orders the microscopic cell states while the background geometry remains stable, and a scrambled control confirms that the ordering follows the closure structure itself. Inserted defects then strain the nearby capacity and measurably deform the local geometry. In a separate transport calculation, a conserved carrier responds to defects of different strength through one common rule, and the disturbance persists without detected screening across the measured range. A predicted shift of the host geometry likewise follows the cell model across a family of simulation settings, while the control follows its own distinct prediction. These tests are limited in scale and do not yet measure Newton's constant, but they connect the proposed medium to dynamical geometry through measured consequences rather than analogy alone.

Beyond ordinary weak gravity, the framework extends to time-dependent transport, clusters, cosmology, the early universe, dark energy, black holes, particle structure, and complexity growth at explicitly labeled levels of closure. At the microscopic level, an explicit group-field-theory action selects the finite capacity and record dynamics and, in its retained nine-state sector, the local geometry of quantum states. A selected geometric construction connects the same substrate to the familiar large-scale Einstein description. Other extensions retain the conditional or open grades stated in the paper.

Contents

Part I. Physical Idea and Foundations	5
1. Introduction: The Physical Claim	5
1.1 What Is Primitive, and What Is Closed	6
1.2 Physical Motivation for the Primitives	8
2. Canonical Field Content and Definitions	11
3. The Three Postulates	13
3.1 Information–Geometry Equivalence	13
3.2 Mass–Entropy Equivalence	13
3.3 Many-Pasts Hypothesis	14
4. Relativistic Continuum Structure	15
4.1 Capacity budget and continuum symmetry	15
4.2 Dependency Map of the Theory	16
Part II. UV Coefficient Chain	17
5. Why a Tetrahedral Boundary Ensemble	17
6. Admissibility Closure	19
6.1 Minimal isotropic kernel	19
6.2 Closure condition and uniqueness	19
6.3 Effective sharing entropy	20
7. Edge Kernel and Tree-Level Coupling	20
8. Finite-Loop Renormalization	21
9. Continuum Stiffness and SI Normalization	22
Part III. The Closed Static Branch: Einstein Gravity in the Capacity Variable	24
10. Einstein Parent Action and the Reduced Capacity Frame	24
11. Capacity Variable, Bridge Law, and Variational Status	26
12. Newtonian Gravity and the Point-Source Limit	27
13. Electron Anchor: One-Bit Mass Scale and Seven-Sector Length Scale	28
13.1 Why the electron is the anchor	28
13.2 One-bit mass anchor	28
13.3 Seven-sector length anchor	29
13.4 Decorated marked-transfer vertex	29
13.5 Consistency checks	31
13.6 Composite sectors	32
14. Baseline Metric Closure: No Slip and PPN	32
Part IV. The Carrier-Resolved Galactic Branch	32
15. Carrier-Resolved Galactic Dynamics	32

16. Galactic Metric, Lensing, and Local Tests	34
Part V. Transport, Clusters, and Cosmology	36
17. Causal Transport and Telegrapher Dynamics	36
18. Cluster Source Projection and the Diffuse–Decoupled Channel Split	37
19. Cosmology and the Hubble-Tension Sector	42
20. The Saturated Phase and the Cosmic Microwave Background	43
Part VI. Strong Fields, Many-Pasts, and Microstructure	48
21. Strong-Field Action: Spherical Closure and Its Boundary	48
22. Many-Pasts: The History-Space Ontology	50
23. Microstructure Hamiltonian and Underlying Dynamics	52
Part VII. The Substrate on a Dynamical Lattice	55
24. Lattice Tests: Compatibility, Defect Response, and Transport	55
24.1 The host geometry and the cell identification	55
24.2 The coupled ensemble and its controls	56
24.3 What the closure sector cannot supply: vacuum stiffness	56
24.4 The externally hosted vacuum and the role of conditioning	57
24.5 Compatibility: the weighting on dynamical geometry	58
24.6 The defect experiment: geometry responds to the theory’s mass	58
24.7 Reaching Newtonian range: the conservation requirement	58
24.8 What the finite-lattice results establish	61
24.9 The capacity-decorated continuum target	61
25. Equilibrium Vacuum and Cosmological Term	62
Part VIII. Closure Status, Falsifiability, and Comparisons	67
26. Closure-Status Table	67
27. Falsifiability and Observational Tests	75
27.1 Static weak-field falsifiers	75
27.2 Dynamical falsifiers	76
27.3 Cosmological falsifiers	76
27.4 Correlated-constant falsifiers	77
27.5 Many-Pasts status	78
28. What the Theory Would Have to Get Wrong to Fail	78
29. Comparison with Other Approaches	79
29.1 Relative to Λ CDM	79
29.2 Relative to MOND-like interpolation programs	79
29.3 Relative to Verlinde-style emergent gravity	79
29.4 Relative to TeVeS and other multi-field modified gravities	79
29.5 Relative to AeST	79
29.6 Relative to scalar-tensor gravity	80

29.7 Relative to quantum-mechanical interpretations	80
29.8 Relative to CDT, spin foams, and group field theory	80
29.9 Relative to algebraic and information-geometric gravity	81
30. Conclusion	82

Part I. Physical Idea and Foundations

1. Introduction: The Physical Claim

Space, in this proposal, is a finite medium of entanglement capacity. The particles we call matter are stable defects that lock away part of that capacity, and the surrounding medium responds to the commitment. Seen at large scales, that response is gravity:

- **Matter** is a localized capacity defect of the substrate.
- **Mass** is the entanglement that defect commits, read in mass units.
- **Gravity** is the extended capacity strain — the fractional capacity deficit — the medium carries around the defect.
- **Dark-matter phenomenology** comes from two further regimes of the same medium: the long-range capacity strain on galactic scales and the saturated phase in the early universe.
- **General relativity** is the low-energy geometry of this capacity medium.

Written as a continuum theory, this becomes a scalar EFT for a vacuum-relative entanglement field $S_{\text{ent}}(x)$ and its deficit δS relative to the background capacity. The defect sector is written at continuum scale in ordinary stress-energy variables, but its ontology is unchanged: inertial mass enters through the mass-per-entropy map κ_m , and the weak-field potential is the fractional deficit $\delta S/S_\infty$.

The gravitational response normally attributed to a dark halo is assigned here to the capacity structure of the vacuum itself. Ordinary weak gravity, the galactic excess, and the homogeneous cosmological mode are different regimes of one medium: general relativity supplies its low-energy geometry, and the capacity variable tracks how localized defects deplete and redistribute the available entanglement. Section 29.9 compares this proposal with recent information-theoretic constructions of gravity, after the paper’s own development is complete.

The inputs are the finite-capacity substrate, three postulates, and the tetrahedral cell ensemble. From these, one chain of finite calculations follows. Admissibility fixes the sharing entropy, edge transport fixes the tree stiffness, the decorated transfer vertex fixes the charged marked event, source projection fixes the ordinary coupling ratio, and the weak-field bridge relates fractional capacity deficit to gravitational potential. The microscopic result is an explicit capacity/record action within the group-field-theory framework. It splits a primitive shared face into a seven-state matched channel and a nine-state retained mismatch record. Restricting the same action to that record gives a real Lorentzian phase space whose transport and mismatch spectrum generate $\mathfrak{su}(9)$. The resulting unique positive complex structure, complete local response algebra, and phase quotient give $\mathbb{C}^9$ and $\mathbb{CP}^8$ rather than importing them for this sector (Appendix H.11). The action also prepares the renewed register, carries the persistent marked defect, and fixes the transfer gap used for scale setting. The same marked fiber supplies a coefficient-free dilation when a baryonic source clock loses its distinguishable carrier. Appendices O.2–O.14 complete the infrared bookkeeping of that event. The fixed-event history space has a unique volume singlet, so only the homogeneous projection of the marked drain is conjugate to the cosmological term; local marked-history fluctuations do not define a local $\Lambda(x)$. Carrier loss removes the source-label saturation constraint and reassigns the metric-visible remainder to the coarse retained carrier rather than creating a new material component. A first-entry excursion-set construction fixes the assembly history F_Z , while a three-dimensional compressible-flow passive-tracer surrogate supplies an explicit delayed mixing kernel K_{mix} . The resulting perturbation transfer has no independent dark-energy clustering variable: for $k \neq 0$ the release-hazard fluctuations cancel inside the total source-associated capacity stress, and $\delta\rho_\Lambda = 0$. The remaining registered astrophys-

ical input is a self-gravitating, cooling cosmological zoom with passive Lagrangian progenitor tags, whose measured $K_{\text{mix}}(\tau; M, z)$ is to be frozen before the final likelihood analysis (Appendices O.10–O.14 and Q.9a). The ordinary geometric branch is supplied by a selected fixed-facet GFT completion: maximal fusion gives the canonical $V_3^{\otimes N} \rightarrow V_{3N}$ blocking ray. On the selected metric-Regge branch, Appendix H.10 establishes the controlled massless two-helicity TT limit and an exact fixed- $j = 3$ primitive realization. The microscopic construction therefore supplies an explicit finite GFT-sector action and decorated gluing prescription. The remaining freedom lies in the geometric host: the finite-capacity premises do not select a unique native gluing tensor or measure, so a fully derived nonperturbative host requires an additional geometric principle. Several infrared branches remain conditional.

The most controlled branch is the ordinary static weak-field action and source map

$$\text{microstructure} \longrightarrow \text{coefficient chain} \longrightarrow \text{static capacity EFT} \longrightarrow \{G, \text{baseline metric}\}.$$

It recovers the Newtonian point-source limit, the leading no-slip metric, and the parametrized post-Newtonian values of general relativity through its Einstein parent. A specified carrier-resolved transverse branch produces the galactic acceleration scale a_0 , the radial-acceleration relation, its conservative nonspherical extension, and the leading lensing metric without per-system tuning, subject to the microscopic matching conditions stated in Sections 15–16 and Appendix N. The electron anchor, memoryless dressing, and marked vertex fix the substrate length inside the stated support-to-rate branch, and the resulting Newton normalization and corrected charged-lepton ratios agree with current measurements within one standard deviation. The comparison also runs backward: with the marked weight held at its action value, the measured Newton constant selects the vertex’s routing integer, and the unique survivor is the same seven fixed by the tetrahedral alphabet (Section 13.5). Appendix L separates this action-level closure from the historical fact that the residuals were already known.

Later parts treat time-dependent transport, galaxy clusters, cosmology, the saturated early phase, strong fields, and particle and gauge extensions. Their derivational status is listed in Part VIII.

Many-Pasts, the third postulate, already does work in this chain: faithful sector resolution selects the memoryless electron-dressing kernel, and Many-Pasts supplies the history space in which that kernel operates. Its consequences for quantum probability, branch realization, and the arrow of time are developed in Section 22 and Appendix G.

1.1 What Is Primitive, and What Is Closed

The word “closure” is used here in a specific sense. The paper does not derive the existence of a finite entanglement substrate or the tetrahedral boundary architecture from a deeper microscopic Hamiltonian; those are theory-defining inputs, and the closure claim begins only after they are fixed. There are five such inputs: finite local entanglement capacity; geometry–capacity equivalence; mass–entropy equivalence, including matter as localized defects of committed capacity; the Many-Pasts ontology of Postulate III, with its operational quantum measure imported as stated below; and the tetrahedral ultraviolet architecture, including positive oriented matching of the two primitive descriptions of a shared face. Appendix B writes that matching rule as a primitive pair operator and separates its exact consequences from the remaining incidence condition. This refines the existing ultraviolet input rather than adding a fourth postulate. Maximum caliber is likewise not a sixth input: it restates, for histories, the same faithful full-support condition already used to select the admissibility ensemble — every pass carries the largest path entropy compatible with the same fixed marginal. The equivalence has to be stated explicitly because finite capacity by itself does not imply renewal.

Given those inputs, the chain closes step by step. Finite counting fixes the admissibility weighting and the effective entropy. Faithful full-support resolution fixes the memoryless replacement kernel, and the decorated native-cell vertex realizes its reversible update. The state-weighted determinant supplies the baseline seven-channel recurrence, and the same closure amplitude, projected through two directed singlet returns and canonically dilated, supplies the marked correction and its 21-edge determinant. The electron anchor fixes the proper-time cadence, and the spatial and temporal readings of the same phase mode give $L_* = c\tau_*$. Edge transport and source projection then determine the ordinary static response. The longitudinal functional turns out to be the Einstein scalar-constraint sector in a different variable, so it yields the Newtonian limit, the baseline no-slip metric, and the PPN values of general relativity. Controlled Regge results and the exact primitive pullback establish the Einstein transverse-traceless limit on the selected metric branch. The empty-capacity lift is exact. The host-underdetermination theorem shows that the present capacity premises do not determine the geometric gluing tensor or measure. The galactic branch is closed as a specified leading EFT; its carrier projector, thermal cell Hamiltonian, and metric contact still require a microscopic derivation.

The decorated vertex adds no sixth foundational premise. It specifies the minimal marked field content and gluing that realize the faithful full-support rule, native tetrahedral update, and one-bit fermionic electron anchor already listed above. Those action-level choices are stronger than a numerical ansatz because they determine the response multiplicity, determinant power, and routing together; they are also falsifiable, since the nonminimal and differently routed vertices give the alternative values displayed in Appendix H.9.

In compressed form, the central claim is

$$\begin{aligned} & \boxed{\text{primitive UV capacity hypothesis}} \rightarrow \boxed{\text{finite counting} + \text{admissibility}} \rightarrow \boxed{L_*} \\ & \rightarrow \boxed{\gamma, \kappa/\gamma} \rightarrow \boxed{\delta S \leftrightarrow \Phi} \rightarrow \boxed{G, \text{baseline metric}}, \\ & \boxed{a_0, \text{RAR, galactic lensing}} \text{ in the carrier-resolved transverse EFT.} \end{aligned}$$

The microstructure is an input: a finite ultraviolet counting problem from which the weak-field sector is derived.

The absolute length calibration uses the electron, the lightest elementary charged defect, as the dimensional anchor. Faithful full-support resolution forces the replacement kernel to be memoryless: a dressing pass can carry the full admissibility entropy only if it retains no memory of the endpoint it replaces (the one-line proof appears in Section 1.2). The lightest-defect functional selects the fermionic ceiling $k = 7$ with $\Delta_7 = 0$, and the state-weighted determinant of the seven renewed clouds is $r = e^{-7g_{\text{share,eff}}}$. The decorated vertex fixes the residual marked-fiber factor and its electron routing Z_e . Positivity gives the raw survival energy $E_{\text{raw}} = -(\hbar/\tau_*) \ln(1 - r)$, and the dressed electron identification $m_e c^2 = (3/2)Z_e E_{\text{raw}}$ then fixes τ_* , and with it $L_* = c\tau_*$, without an independent clock or geometric diameter. Appendix H gives the finite action and both adversarial audits.

The reduced capacity functional can superficially resemble the scalar sector of a Brans–Dicke theory [56], but the resemblance is misleading. In the ordinary static branch it is the Einstein constraint action rewritten through $\delta S = -2S_\infty \Phi/c^2$, with no second scalar–tensor action. The open action questions are narrower: the microscopic origin and nonlinear covariant variation of the carrier-resolved transverse contact, the generic covariant capacity observable outside controlled reductions, and the saturation-boundary functional.

Several tasks remain: derive the ensemble from a deeper Hamiltonian, complete and audit the coupled geometry–capacity interaction beyond the factorized branch, audit the separate finite-loop stiffness return operator, derive the carrier projector and transverse metric contact

microscopically, and construct the strong-field boundary action. The selected metric-Regge GFT branch already has the massless two-helicity TT limit and an exact fixed- $j = 3$ primitive realization. The remaining geometric problem is to find a principle that selects a unique native gluing tensor and measure, then prove that its nonperturbative measure lies in the Einstein phase. The independent capacity-decorated CDT route must exhibit an extended four-dimensional phase with a massless transverse-traceless transfer sector; a CDT critical surface is the stronger optional cutoff-removal test. Each unresolved step is listed explicitly in the closure table.

The construction rests on five commitments.

First, the vacuum is a medium with a bounded local capacity for entanglement.

Second, spacetime geometry and that capacity structure are the same substrate seen at different scales, so that in the weak field gravity is the fractional deficit of locally available capacity.

Third, matter is localized committed capacity: a particle is a stable defect of the medium, and its inertial mass is the entanglement content of that defect read in mass units.

Fourth, a recorded present is supported by many compatible microscopic pasts. The operational branch assigns probabilities only to decoherent record histories through the standard quantum decoherence functional, then conditions them on the realized present. It preserves Born statistics and no-signaling. The reversible renewal dilation supplies a concrete microscopic role for the history degrees of freedom: they receive the previous local state while the present register is renewed.

Fifth, the ultraviolet cell has tetrahedral boundary architecture. Each shared face begins with two fermionic primitive slots and positive oriented matching. The matching operator transmits the maximal coupled multiplet. The marked present/history fiber has the representation content of the discarded complement only for $j_0 = \frac{3}{2}$; conditional on identifying it as that complement, the three-dimensional closure vector fixes the seven-state alphabet internally. Without that incidence identification, $j_0 = \frac{3}{2}$ remains the surviving branch of the discrete audit in Appendix B. Four relational ports, single-copy channel capacity, and the two orientations then give the 1680-state ensemble. The same faithful-resolution standard is applied to its histories: the local process carries the full available path entropy and retains no endpoint memory. “Maximum caliber” names this temporal application of the same standard; it adds no independent commitment to the five listed here.

The finite-capacity substrate is the ultraviolet premise; geometry-capacity equivalence, mass-entropy equivalence, and Many-Pasts are the three postulates; and the tetrahedral ensemble is the ultraviolet architecture. Faithful full-support resolution acts on both states and histories. From these inputs the paper derives the admissibility weighting, $g_{\text{share,eff}}$, the replacement kernel, the one-layer native-vertex update, the marked transfer, the tree edge factor, the weak-field bridge, and the Newtonian metric branch, all inside the displayed decorated action. The separate loop-dressed stiffness, the galactic acceleration scale, and the radial-acceleration relation carry the conditional grades recorded in the closure table.

The architecture therefore contains three postulates, one finite-capacity substrate premise, one minimal ultraviolet ensemble, and a small number of explicitly labeled conditional readings. Applied to the same ensemble, faithful resolution fixes both the state entropy and the maximum-caliber history process.

1.2 Physical Motivation for the Primitives

These commitments are premises, and the derivation begins only after they are fixed. They are nevertheless not arbitrary. Each is motivated by a place where established physics already strains against its own foundations — black-hole thermodynamics, quantum information, the

equality of inertial and gravitational mass, and the quantum-mechanical role of records — and each makes a known difficulty look less mysterious once it is adopted. That motivation does not prove the premises; the numerical and structural consequences derived from them provide the tests.

Mainstream gravitational physics has been converging on a finite-capacity substrate for decades. A black hole’s entropy scales with the area of its horizon rather than the volume it encloses, as though the contents of a region were written on its boundary; the Bekenstein bound limits the information a bounded region can hold; and entanglement-based reconstructions of geometry tie the shape of spacetime directly to patterns of entanglement. Each of these is usually treated as a deep clue without a mechanism. This paper takes the clue literally: the vacuum is a medium with a finite local budget of entanglement, and geometry is the large-scale description of that budget. Several long-standing puzzles then become the ordinary behavior of a medium that can fill up. A black hole is a region whose capacity is exhausted, so the only live bookkeeping sits at the boundary between exhausted and available capacity — which is why the entropy tracks the area and not the volume. The same finite state space supplies an ultraviolet cutoff: below the cell scale there is no continuum left to diverge, and a finite state space has nothing to renormalize away. The assumption that the empty state maximizes capacity agrees with the thermodynamic direction suggested by gravitational entropy, and three recent continuum results support the same reading from independent directions: the observer-dressed de Sitter algebra makes empty de Sitter the maximum-entropy gravitational state [24], a horizon modular calculation recovers Einstein curvature from the relative information carried by an excitation [23], and a geometric-relative-entropy action yields a local bulk information dynamics with an Einstein limit [25]. None of them supplies the finite cell or its coefficients; Section 29.9 states precisely what they do and do not establish for the present construction.

The mass–entropy identification addresses a coincidence that general relativity encodes but does not explain. General relativity builds in the equality of inertial and gravitational mass geometrically, through the equivalence principle, but gives no microphysical account of why the mass that resists acceleration and the mass that sources attraction should be one and the same. Here both are readings of a defect’s committed entanglement, so the equality follows from the construction instead of being imposed by hand. The same identification bears on why gravity is so weak. The induced gravitational scale contains the exact factor $Z_e^2 \ln^2(1 - e^{-7g_{\text{share,eff}}})$, whose dominant hierarchy is $e^{-14g_{\text{share,eff}}}$: a large sharing entropy makes gravity exponentially feeble, and the gap between gravity and the other forces becomes a matter of arithmetic rather than fine-tuning. The galactic extension carries a lower closure grade. In its carrier-resolved transverse branch, $a_0 = \epsilon c H_0$ with $\epsilon \equiv g_{\text{share,eff}}/(4\pi^2)$ ties the onset of anomalous rotation to the cosmic horizon. The thermal contact fixes the response of each retained carrier, and the auxiliary action relates that response to the resolved baryonic source. The phase-cell loading, carrier projector, and microscopic metric vertex are the tests of that connection.

The scale-setting proposal separates two questions that sound alike: how hard the medium works to keep a defect bound, and how often the defect fully re-forms. Because the medium never holds still — every instant it is re-drawn from all the ways it could be — a particle is a pattern the medium must continually re-form, and the entanglement it commits is a maintained quantity, re-established at every update. Keeping the knot bound means holding a grip on every strand at once, and separate holds add up, so the maintenance is large. Full re-completion, with every strand falling into alignment at the same instant, is a simultaneous coincidence, and coincidences multiply, so it is exponentially rare. The particle’s mass follows the second question, not the first: it is fixed by how rarely the binding fully re-closes. This is why a particle can be vastly lighter than the natural substrate scale with no small number inserted anywhere: the electron is deeply bound, and deep binding makes its full coherent recurrence exponentially rare. Quantitatively, seven channel entropies add, so their effective support multiplies; the state-weighted determinant

turns that support into a record-conditioned transfer rate; the positive survival operator gives the rest-energy gap; and the decorated marked vertex fixes the finite closure-response correction while the electron anchor fixes the clock. Section 23 and Appendix H give this its full form.

A memoryless update rule posits no hidden machinery carrying information forward from tick to tick, and in this construction it is also forced. For any stationary per-channel kernel with marginal p_{η_*} ,

$$H(B_{t+1} | B_t) = g_{\text{share,eff}} - I(B_t; B_{t+1}) \leq g_{\text{share,eff}},$$

so a dressing pass carries the full admissibility entropy if and only if the endpoint mutual information vanishes, which fixes the refresh kernel $K(b, b') = p_{\eta_*}(b')$. The allowed local single-label dynamics cannot realize this requirement: they freeze into disconnected sectors that never explore the full space (Appendix D.4), so the refresh must act nonlocally on the native cell. The decorated vertex prepares the diagonal fresh amplitude and realizes the charged recurrence; its geometric condensate embedding and durable history capacity remain open. A single universal refresh rate also lets the theory carry a smallest length without conflicting with the experiments that ended earlier discrete-spacetime proposals: the granularity lives in the capacity, not in a preferred spatial lattice, so gravitational waves and light travel at the same speed and there is no frame-dependent dispersion left to detect.

Many-Pasts holds that the present configuration of the entanglement network is supported not by one definite microscopic past but by a conditional ensemble of compatible decoherent histories. It addresses directly the quantum puzzles that interpretations of quantum mechanics were invented to handle. The interference in a double-slit experiment is the persistence of the unrecorded alternative histories in that ensemble; a durable which-path record conditions the ensemble, and the interference goes away. The correlations of an entangled pair come from weighting the histories of the whole joint system, which reproduces the nonclassical statistics with no signal passing between the two wings. Measurement adds no separate collapse law; it lays down a durable record, after which the relevant histories are the ones compatible with it. The operational construction is therefore ordinary quantum mechanics equipped with a history-space ontology, and it leaves laboratory predictions intact: Born-rule statistics and no-signaling both hold. Its arrow-of-time extension still needs a substrate typicality theorem. The postulate also has a concrete job beyond interpretation: faithful sector resolution independently selects the memoryless dressing, and Many-Pasts supplies the history-space setting in which that process operates and in which its exported registers live.

Tetrahedra are the standard building block in several approaches to quantum geometry, so the cell is a familiar object rather than one invented for this paper. What the framework adds is a precise primitive matching rule. After the two descriptions of a shared face are transported into one orientation, its positive mismatch operator has the maximal-spin sector as its unique null space. For the physical branch, the independently constructed nine-state marked register matches the complete nonmaximal fusion sector only at $j_0 = \frac{3}{2}$. Conditional on identifying those two objects dynamically, three-dimensional closure fixes seven face labels; four relational ports, single-copy channel capacity, and two orientations then give exactly 1680 boundary states. Without that identification the same result is the unique surviving point of a stated discrete branch audit, not a consequence of minimality. The closure structure also permits only three nondegenerate charged-lepton shells, a candidate answer to why the Standard Model contains three charged-lepton generations. Appendix L preserves the historical provenance of the construction. Once the branch is fixed, its entropy contains no adjustable continuous parameter.

Several mainstream results already point in the same direction: Einstein's equation derived as a thermodynamic relation of state, horizon entanglement entropy, and reconstructions of spatial connectivity from entanglement. The construction below makes a finite microscopic proposal within that program and derives quantities that can be checked.

Motivation alone carries little evidential weight, because a proposal of this scope can almost always assemble a list of mysteries explained after the fact. The relevant questions are how much freedom remained when the construction was chosen and whether one fixed construction survives measurements it did not anticipate. Appendix L audits the first question. The following sections derive the weak-field consequences and compare them with observation.

2. Canonical Field Content and Definitions

Before the symbols, four plain words recur throughout. *Capacity* is the entanglement support locally available in the medium. A *defect* is a stable, localized commitment of that capacity — what we coarse-grain into a particle. A *deficit* is capacity no longer freely available to the surrounding vacuum because a defect has committed it. *Strain* is the extended profile of that deficit reaching out into the medium, whose fractional size the weak-field potential tracks. The field variables below are the precise versions of these words.

We define the fundamental continuum variable as the vacuum-relative coarse-grained entanglement assigned to a UV probe cell of size L_* centered at x :

$$S_{\text{ent}}(x) \in \mathbb{R},$$

measured in nats and therefore dimensionless. This is not a literal microscopic entropy density at a mathematical point. It is the leading scalar order parameter associated with a vacuum-relative entanglement defect after coarse-graining over a UV cell.

This definition keeps the microscopic and continuum pictures tied together. At continuum level, $S_{\text{ent}}(x)$ is the field that appears in the action and field equations. At the microscopic level it is the coarse variable recording how much local entanglement capacity remains available in the underlying medium after averaging over a UV cell.

The asymptotic vacuum-capacity baseline is denoted S_∞ , and the deficit field is

$$\delta S(x) \equiv S_\infty - S_{\text{ent}}(x).$$

Positive δS denotes reduced available vacuum entanglement capacity in the neighborhood of a localized defect or defect distribution. It is the extended capacity-strain field sourced by the defect sector, not an independent medium acted on by matter from outside. For nonlinear work it is useful to define the bounded occupancy fraction

$$q(x) \equiv \frac{S_{\text{ent}}(x)}{S_\infty} = 1 - \frac{\delta S}{S_\infty} \in [0, 1].$$

The variables S_{ent} , δS , and q therefore describe the same local physics in three closely related ways: available capacity, missing capacity relative to vacuum, and surviving-capacity fraction. Each is used where it is most transparent: δS for the weak-field theory, because it maps directly onto the Newtonian potential; q for the nonlinear and strong-field completion, because boundedness is built in from the start; and S_{ent} itself for the covariant EFT, because it is the field that appears in the action. The operational meanings are:

- $q = 1$: vacuum capacity fully available in the absence of local defect-induced capacity strain;
- $0 < q < 1$: partial local capacity reduction around a defect configuration;
- $q = 0$: complete local exhaustion of available capacity on the physical branch.

Fixed-epoch normalization. The absolute normalization of S_{ent} and S_{∞} is a convention once an epoch and cell convention have been fixed. Under a constant rescaling

$$S_{\text{ent}} \mapsto K S_{\text{ent}}, \quad S_{\infty} \mapsto K S_{\infty}, \quad \delta S \mapsto K \delta S,$$

the observable bridge

$$\frac{\Phi}{c^2} = -\frac{\delta S}{2S_{\infty}}$$

is unchanged. The source equation is invariant in the same sense: rescaling the entropy field rescales the source coefficient with it, so the observable Newtonian normalization depends on the gauge-invariant combination $\kappa/(\gamma S_{\infty})$ rather than on S_{∞} alone. A cell-normalized description and a horizon-normalized description can therefore assign different numerical values to S_{∞} without changing Φ , G , or the PPN limit. This is not a time-dependent gauge symmetry; it is a fixed-epoch entropy-unit convention. Gravity sees fractional capacity depletion.

Substrate length scale. The canonical UV cell length is not taken to be the conventional Planck length as an input. Faithful full-support resolution fixes the renewal kernel and its history-space factorization. The state-weighted determinant gives the seven-channel recurrence

$$r = e^{-7g_{\text{share},\text{eff}}}, \quad L_*^{(0)} = -\frac{3}{2}\lambda_e \ln(1-r), \quad \lambda_e = \frac{\hbar}{m_e c}.$$

The decorated marked-transfer vertex derived in Appendix H fixes

$$\zeta_* = 9e^{-g_{\text{share},\text{eff}}} \left(1 - \frac{8\eta_*}{49}\right)^{21/2}, \quad Z_e = (1 + \zeta_*)(1 + 7\zeta_*^2).$$

The physical electron-anchored scale is

$$\boxed{L_* = Z_e L_*^{(0)}} = 1.6162537014 \times 10^{-35} \text{ m}.$$

The corresponding induced gravitational scale is

$$G_* := \frac{c^3 L_*^2}{\hbar} = \frac{9}{4} \frac{\hbar c}{m_e^2} Z_e^2 \ln^2(1 - e^{-7g_{\text{share},\text{eff}}}) = 6.6742890772 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}.$$

The lightest one-bit fermionic defect resolves the seven face sectors once and exports the transverse 2/3 share of that dressing block. The marked vertex accounts for its finite closure-response fiber and label-return loop. The conventional Planck length $L_P = \sqrt{\hbar G/c^3}$ remains useful for comparison and for standard black-hole thermodynamic notation, but it is not the primitive scale-setting input here.

The principal coefficients and derived quantities used throughout are:

$$\gamma : \text{entanglement-field stiffness}, \tag{1}$$

$$\kappa : \text{defect-entropy coupling}, \tag{2}$$

$$\kappa_m(\ell) : \text{mass-per-entropy map at scale } \ell, \tag{3}$$

$$L_* : \text{substrate cell length in the electron-anchored support-to-rate map}, \tag{4}$$

$$G_* : \text{gravitational scale induced by } L_*, \tag{5}$$

$$g_{\text{share},\text{max}} = \ln(1680), \tag{6}$$

$$g_{\text{share},\text{eff}} : \text{admissibility-weighted effective sharing entropy}, \tag{7}$$

$$J_{\text{bare}}, J_{\text{eff}}^{\text{tree}}, J_{\text{eff}}^{(\text{ren})} : \text{UV edge-kernel couplings}, \tag{8}$$

$$a_0 = \frac{c H_0 g_{\text{share},\text{eff}}}{4\pi^2} \quad \text{in the conditional compact two-phase normalization.} \tag{9}$$

The gravitational potentials are denoted Φ and Ψ , and the canonical weak-field bridge will be written as

$$\frac{\Phi}{c^2} = -\frac{\delta S}{2S_\infty}.$$

These same symbols reappear in the ultraviolet closure chain, the continuum action, and the phenomenology sections. From this point onward each one keeps the same meaning, so the later derivations build on a single notation rather than shifting between parallel conventions.

3. The Three Postulates

The framework rests on exactly three postulates because they answer three different questions. *What is spacetime geometry?* — Postulate I: geometry is the long-wavelength expression of the capacity substrate itself. *What is matter, and what is mass?* — Postulate II: matter is localized committed capacity, and mass is its inertial reading. *What supports the present, and what fixes the direction of history?* — Postulate III: the present is supported by a history-space ontology over the decoherent pasts compatible with one realized record. Operational probabilities come from the standard decoherence functional, while the proposed arrow of time requires an additional typicality result. Faithful sector resolution, applied to this history space, selects the memoryless dressing kernel used by the decorated scale-setting action.

3.1 Information–Geometry Equivalence

The first postulate states that spacetime geometry is the continuum expression of the capacity substrate. This is stronger than saying entanglement contributes an additional piece of stress-energy inside otherwise standard general relativity: the metric and the scalar capacity sector are two projections of one finite medium, and S_{ent} is not appended to an independent background geometry. In the weak field, gravitational potential is the fractional deficit of available capacity.

Two consequences of this reading should be kept distinct from the start. First, absolute S_{ent} is not itself “the gravitational potential”; the observable weak-field potential comes from the fractional deficit $\delta S/S_\infty$, which is why a fixed-epoch rescaling of entropy units leaves gravity unchanged (Section 2). Second, because geometry and capacity are two descriptions of one response, the deficit is not an extra force appended to an independently existing metric. Section 10 proves that the ordinary reduced capacity functional is the Einstein constraint action in the capacity coordinate. The remaining common-parent problem is the microscopic embedding of the carrier-resolved transverse contact and the saturation boundary.

3.2 Mass–Entropy Equivalence

The second postulate identifies mass as the inertial reading of localized capacity commitment. At scale ℓ ,

$$m(\ell) = \kappa_m(\ell) \Delta S.$$

A particle is already a localized defect of the entanglement substrate, so $m = \kappa_m \Delta S$ does not assert an analogy between two independent things; it asserts that the inertial content of the defect *is* its entanglement content, read in mass units.

For elementary fermionic sectors the canonical defect increment is

$$\Delta S_f = \ln 2.$$

The one bit here is not arbitrary. An elementary fermionic exclusion is binary — the face is occupied or unoccupied — and a binary distinction carries exactly $\ln 2$ of missing entanglement. This is the simplest possible defect increment, which is why the lightest such defect, the electron,

becomes the cleanest anchor for the mass–entropy map (Section 13). Composite sectors instead require their fully dressed bound-state entanglement budgets.

Two corollaries are used later. First, because mass and entanglement budget are two descriptions of the same defect, and the masses of separated defects add, capacity committed in service of one defect cannot simultaneously serve another: shared service would make the joint budget, and with it the joint mass, sub-additive. Commitment is therefore per-defect — each committed unit carries the label of the defect it serves. Second, the same bookkeeping makes the saturated early phase countable: its abundance follows from the number of committed units, with no double-counting across separated defects (Section 20).

3.3 Many-Pasts Hypothesis

The third postulate concerns the microscopic support of the present entanglement network. A recorded present can be compatible with many coarse-grained histories of the substrate. Many-Pasts takes those alternative pasts seriously while retaining one realized macroscopic present. Its probability theory must distinguish amplitudes for alternatives that still interfere, probabilities for recorded presents, and conditional probabilities for decoherent histories compatible with a given record.

The operational construction uses the decoherent-histories formalism [71]. A coarse history $h = (\alpha_1, \dots, \alpha_n)$ has class operator

$$C_h = \Pi_{\alpha_n}^{(n)} U_{n,n-1} \cdots \Pi_{\alpha_1}^{(1)} U_{1,0},$$

and decoherence functional

$$\mathcal{D}(h, h') = \text{Tr} \left(C_h \rho_0 C_{h'}^\dagger \right).$$

When a family decoheres, $\mathcal{D}(h, h') \simeq 0$ for $h \neq h'$, its diagonal entries obey the ordinary probability sum rules. If P denotes a final macroscopic record and $\mathcal{H}_P$ is a decoherent refinement of the histories ending in that record, then

$$p(P) = \sum_{h \in \mathcal{H}_P} \mathcal{D}(h, h) = \text{Tr}(\Pi_P \rho_{\text{now}}), \quad p(h \mid P) = \frac{\mathcal{D}(h, h)}{p(P)}.$$

The probability of the present is obtained from the common normalized measure over all records before the history distribution is conditioned on P . Unresolved alternatives remain combined at amplitude level; no positive probability is assigned to individual fine-grained paths that have not decohered.

This construction makes the operational branch standard quantum mechanics. Quantum instruments give the Born probabilities of laboratory records, and local trace-preserving instruments give no-signaling marginals. Many-Pasts changes their history-space interpretation without adding a collapse term or a signaling bias.

The ontological reading is record-retentive. The realized present includes both its current macroscopic configuration and the physical records that encode earlier events. Compatible pasts are distinguished to the resolution carried by those present records; they are not additional coexisting spacetimes. When a physical process erases a record distinction, the conditional history measure coarsens by summing the histories that the surviving record can no longer separate. Conditioning on a future record is defined only after that record belongs to a present configuration, so the construction supplies no future-to-past force or retrocausal update law.

For the retained one-mark sector, Appendix H.11 derives rather than assumes the local kinematics $\mathcal{H}_{\text{mark}} \simeq \mathbb{C}^9$ and the ray space $\mathbb{CP}^8$. Gleason’s theorem then constrains its probability

measure. On a Hilbert space of dimension greater than two, a normalized, noncontextual, additive measure on a sufficiently rich lattice of record projectors has the form

$$\mu(\Pi) = \text{Tr}(\rho\Pi)$$

by Gleason’s theorem [65]. In the local marked sector the Hilbert-space premise and its full rotated projector family are supplied by the derived $\mathfrak{su}(9)$ action; record completeness and noncontextual additivity remain hypotheses. A medium-decoherent family with a pure initial state admits orthogonal generalized records for its branch state vectors, so each history probability can be represented as the probability of a single-time record projector [66]. The Born form is therefore unique for record-defined decoherent families under those hypotheses. The full continuum/Fock-space kinematics, initial state, and substrate derivation of the global decoherence functional remain open; a mixed state requires the corresponding purification or generalized-record construction.

The postulate and the renewal theorem have separate logical roles. Many-Pasts supplies the ontology and the record-conditioned history space. The already-stated faithful full-support condition selects the memoryless dressing kernel when applied to paths at fixed admissibility marginal; “maximum caliber” names this history-space form and is independent of the Born measure on laboratory records. The lightest-defect functional selects the same absence of temporal memory and, within the support-to-length map, selects seven occupied channels with vanishing inter-channel correlation. A reversible dilation exports the old replaceable closure register to history. The decorated transfer vertex supplies the determinant recurrence, finite marked response, and Compton phase readout.

The renewal statement applies to the replaceable closure register, not to every degree of freedom carried by a charged defect. The marked internal fiber and its position degree of freedom belong to the retained system. Appendix H.11 proves the corresponding coherence criterion: after a dilation, a spatial off-diagonal is multiplied by the overlap of the discarded records produced by its two branches. Free marked transport must therefore export no cell address or other which-path label. This is a consistency condition on the microscopic realization of the existing operational quantum postulate, not an additional founding premise.

4. Relativistic Continuum Structure

4.1 Capacity budget and continuum symmetry

The continuum description is expected to be covariant because the substrate itself is finite-capacity, isotropic, and relational — covariance is read off from the substrate’s own properties rather than added as a geometric axiom at the outset.

The first ingredient is a finite maximal update rate, denoted by the same constant c that later appears in the transport relation $D/\tau_0 = c^2$. In the present interpretation, c measures the largest rate at which the substrate can propagate and reorganize information. A defect at rest spends that budget entirely on local temporal evolution. A defect in motion must spend part of the same budget on spatial transport within the surrounding network. Because the substrate is isotropic, the cost of motion depends only on the rotational scalar v^2 at leading order, with the temporal rate maximal at $v = 0$ and vanishing when the budget is exhausted at $v = c$. These endpoint conditions alone admit many interpolating functions and so do not fix the form of the time-dilation relation. The form is fixed once the finite update speed is treated as invariant across inertial coarse descriptions: homogeneity, isotropy, and the relativity principle then select the Lorentz group rather than the Galilean one, giving the invariant interval

$$c^2 d\tau^2 = c^2 dt^2 - d\mathbf{x}^2,$$

and hence

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{v^2}{c^2}}.$$

The capacity-budget picture supplies the substrate interpretation of this Lorentzian kinematics: motion allocates part of the finite update budget to spatial transport, leaving the remaining fraction as proper-time evolution.

The same capacity language also unifies motion-induced and gravity-induced clock slowing. In the nonlinear branch the surviving-capacity fraction is

$$q = \frac{S_{\text{ent}}}{S_{\infty}},$$

so smaller q means that less local update capacity remains available. Motion reduces the temporal share of the budget by consuming part of it in spatial transport; a nearby defect reduces the local budget by depleting available capacity. The two familiar time-dilation effects are therefore interpreted as two regimes of one mechanism.

The second ingredient is the relational character of the substrate. The network is not embedded in a prior physical manifold whose coordinate labels carry independent meaning; its physical content is the pattern of local capacities, defects, and neighborhood relations. Continuum coordinates are descriptive labels imposed on that relational structure, and smooth coordinate changes relabel the same underlying configuration. This is precisely why the low-energy theory must be written in generally covariant form.

The metric sector, then, is not introduced from outside. Lorentzian geometry is the natural coarse description of a finite-capacity, isotropic, relational substrate, and the Einstein sector is its lowest-order continuum gravitational expression, with the entanglement scalar tracking how localized defects redistribute the same capacity geometry. As with any discrete substrate, this continuum claim faces a sharp known obstacle: a discrete structure with a preferred rest frame feeds dimension-four Lorentz-violating operators into the infrared with order-unity coefficients through loops [32], against laboratory bounds many orders of magnitude below unity. The protection here is structural. The tetrahedral ensemble is combinatorial and pre-geometric: it lives in the state counting from which the continuum is constructed, defines no embedding lattice in the emergent spacetime, and imprints on the EFT only through the frame-independent scalars L_* , $g_{\text{share,eff}}$, and η_* . Discreteness of this class is compatible with exact low-energy Lorentz symmetry, as causal-set sprinkling demonstrates by construction [33, 34]. The cosmological bath does select a frame, but only in the environmental sense the CMB does: a state rather than an operator, while the laboratory bounds constrain operators. The supporting calculation this argument calls for — that substrate loops generate no dimension-four Lorentz-violating operators — is still required and is listed in the closure table.

4.2 Dependency Map of the Theory

The logical flow begins with the three foundational postulates — Information–Geometry, Mass–Entropy, and Many-Pasts — with faithful full-support resolution applied to both states and

histories, and runs through the static weak-field chain before reaching the conditional sectors:

{three postulates}

→ finite-capacity substrate ontology → tetrahedral boundary ensemble

→ faithful resolution of states and histories → local replacement / history export

→ edge transport / loop dressing / source map

→ capacity form of the Einstein constraint → Newton / baseline no-slip / GR PPN,

with carrier projection + thermal contact + horizon matching

→ $\{a_0, \text{RAR, nonspherical fields, lensing}\}$ in the leading EFT.

Only then come the conditional and frontier sectors — transport, clusters, cosmology, strong field, and the particle/gauge extensions — each developed as a consequence or completion of the same framework.

Two features of this map matter. First, it is a dependency graph, not an equality of closure status: the ordinary static branch is closed more tightly than the transverse, cosmological, or strong-field sectors, and Part VIII makes that difference explicit in a closure-status table. Second, Many-Pasts appears at the top of the map because the scale-setting chain uses it: faithful full-support resolution selects the local renewal process on the history space that Many-Pasts supplies. Their combination gives a reversible present/history exchange; neither the Born history measure nor record conditioning alone selects memorylessness.

Part II. UV Coefficient Chain

Part I fixed what the theory is about. The question now is whether the local capacity-sharing structure can actually be *counted*. If the substrate has finite local capacity, the coefficients that appear in the continuum weak-field theory should not be free continuum parameters; they should descend from a finite local boundary problem. The next five sections follow that problem through: the smallest boundary cell that can carry capacity and close isotropically, the weighting that selects well-closed configurations, the cost of neighboring cells disagreeing, the local returns that dress that cost, and the continuum coefficient they leave behind. The baseline calculation is internal to the microscopic construction. Appendix B separately propagates nearby discrete branches to the Newton scale, and Appendix L records that this comparison is postdictive rather than historically blind.

Several of the ultraviolet choices below may look at first like independent tunings: the tetrahedral cell, the seven labels, the injective assignment, the parity doubling, the admissibility kernel, the transverse export, and the electron anchor. None is a phenomenological knob, and none varies from galaxy to galaxy. Section 5 and Appendix B derive the finite ensemble and its entropy; Appendix C derives the edge projection; Appendices D and H derive the replacement process, the reversible history export, the factorized lightest branch, and the decorated marked-transfer action; Appendix L records the historical fork accounting. The remaining microscopic task is to embed that finite transfer vertex in a stable geometric continuum, through a specified GFT action or a capacity-decorated CDT transfer matrix on a critical trajectory.

5. Why a Tetrahedral Boundary Ensemble

The problem is to find a finite boundary cell that can carry channel entropy, close isotropically, and hand a scalar response to the continuum. In three spatial dimensions the minimal volumetric simplex is a tetrahedron. The construction uses five ingredients:

- a tetrahedral volumetric cell;

- half-integer primitive data on the two sides of each shared face;
- positive matching after the two sides are placed in one orientation;
- four distinguishable relational ports with single-copy channel capacity;
- binary cell orientation.

This package is not presented as the only possible ultraviolet completion. Postulate II assigns a half-integer primitive spin

$$j_0 = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$$

to each side of a shared face. Before gluing, the pair spans

$$V_{j_0} \otimes V_{j_0} = \bigoplus_{J=0}^{2j_0} V_J.$$

After orientation transport, positive coherent matching is generated by a nonnegative mismatch operator whose unique null space is the maximal coupled sector. Its sharp limit is therefore

$$\mathcal{G}_{\text{sharp}} = P_{2j_0},$$

and the transmitted face alphabet has

$$|M| = \dim V_{2j_0} = 4j_0 + 1$$

states. Appendix B derives this operator on the full tensor product, gives its finite-width spectrum, and states the Hessian tests that would falsify the primitive matching rule.

The marked sector provides a second, more selective relation. The independently constructed present/history response is a product of two spatial vectors,

$$\mathcal{H}_{\text{mark}} = V_1^P \otimes V_1^H = V_0 \oplus V_1 \oplus V_2.$$

The complete nonmaximal information left by maximal fusion is

$$Q(j) = (V_j \otimes V_j) \ominus V_{2j} = \bigoplus_{J=0}^{2j-1} V_J.$$

Because both sums are multiplicity-free,

$$\mathcal{H}_{\text{mark}}(s) \cong Q(j) \iff j = s + \frac{1}{2}.$$

The closure response is a three-component spatial vector, so in $d = 3$ it carries $s = 1$. Conditional on the physical incidence statement that the marked fiber is the faithful record of the nonmaximal fusion channels,

$$j_0 = \frac{3}{2}, \quad V_{2j_0} = V_3, \quad |M| = 7.$$

This is a conditional derivation from the already present marked and fusion structures, not a new minimality axiom. The representation equivalence and its complement isometry are exact. What remains to be shown by the complete microscopic vertex is that the two representation spaces are the same dynamical present/history record. If that incidence identification fails, $j_0 = \frac{3}{2}$ remains the branch selected by the discrete audit of Appendix B.5.

The four faces are relationally distinct ports. A displayed letter is the occupation of one mode in a single cell-level channel resource; the fermionic single-copy ceiling therefore forbids the same

channel from being routed through two ports at once. This gives an injective assignment without antisymmetrizing away the port labels. The two global orientations remain distinct microscopic states. The resulting count is

$$\Omega_{\text{tet}} = 2P(7, 4) = 1680,$$

and the combinatorial sharing ceiling is

$$g_{\text{share}, \text{max}} = \ln(1680) = 7.42654907240.$$

The exact equality

$$16 = 7 + 9$$

has a direct meaning: in the sharp primitive pair map, seven states form the geometric link and the other nine can be retained by the marked record. Within the physical three-dimensional rotation algebra the conditional chain is

$$d = 3 \implies V_1^P \otimes V_1^H \implies j_0 = \frac{3}{2} \implies V_3 \implies 7.$$

It is not asserted as a theorem under dimensional continuation to arbitrary $\text{Spin}(d)$.

The exact K^2 spectrum and branch audits are given in Appendix B. The j -labeled tetrahedron used here coincides with the quantum tetrahedron of simplicial spin networks [62, 63], whose discrete geometric spectra [64] arise from the same $SU(2)$ representation theory. The present construction differs in weighting these states by admissibility closure rather than by a spin-foam amplitude, and in routing them to a capacity entropy rather than to area and volume operators.

6. Admissibility Closure

6.1 Minimal isotropic kernel

Not every boundary configuration should count equally. The raw combinatorial ensemble is too permissive to be the complete ultraviolet input: some configurations sit close to the regular closure pattern expected of a smooth local cell, while others are badly distorted. Admissibility closure is the statement, in its mildest form, that more poorly closed configurations contribute less to the coarse ensemble. The minimal rotationally invariant measure of that distortion is a single quadratic closure-defect scalar K^2 , and the weighting it induces is

$$p_\eta(b) \propto e^{-\eta K^2(b)}.$$

Normalization, isotropy, and a fixed quadratic closure moment select this maximum-entropy kernel. Higher invariants such as K^4 carry additional ultraviolet information and enter as subleading refinements. One distinction is fixed here and holds throughout: this capacity (admissibility) closure is soft, and its invariant K^2 is strictly positive on the ensemble, while the exact Gauss closure of local gauge redundancy is a different operator on a different tensor factor. The two cannot be one operator — Appendix Q proves the impossibility and locates every quantity of the numerical spine on the gauge-inert factor.

6.2 Closure condition and uniqueness

The admissibility precision η is not chosen externally; it is fixed by maximizing the normalized closure evidence. Tetrahedral closure is the vanishing of the three-component oriented-face sum, so the closure-defect space is three-dimensional, and the quadratic family on it carries a determinant weight $\eta^{3/2}$. The closure-evidence functional is therefore

$$\mathcal{F}(\eta) = \ln Z(\eta) + \frac{3}{2} \ln \eta,$$

and its stationary point gives the closure condition

$$\langle K^2 \rangle_\eta = \frac{3}{2\eta},$$

in which the factor $3/2$ is the determinant weight of the three independent closure components, while the discreteness and multiplicities of the spectrum stay inside the exact sum $Z(\eta)$. This is the stationary normalized-evidence point of the exact closure spectrum, and it is a maximum rather than a bare root (Appendix B.2).

On that spectrum it is unique,

$$\eta_* = 0.0298668443935.$$

The closed branch is locally stiff: small fractional changes in η produce only small fractional changes in the downstream effective sharing entropy.

6.3 Effective sharing entropy

The admissibility-weighted effective sharing entropy is

$$g_{\text{share,eff}} = 7.41980002357.$$

The gap between $g_{\text{share,max}}$ and $g_{\text{share,eff}}$ is therefore not loss imposed by hand. It is the difference between the raw combinatorial ceiling and the admissibility-closed effective boundary entropy that actually propagates into observable couplings.

The continuum description does not inherit the naive channel-counting ceiling; it inherits the portion of the channel space that survives after closure is imposed. The downstream couplings should therefore be read as consequences of admissibility-closed sharing, not of raw combinatorics alone.

With η_* fixed, the effective sharing entropy carries no remaining freedom; the exact spectrum, multiplicities, and uniqueness proof are given in Appendix B.2.

7. Edge Kernel and Tree-Level Coupling

Admissibility determines the capacity of one cell. The edge kernel determines the cost when neighboring cells differ, and that cost becomes the continuum stiffness: stronger resistance to local disagreement makes capacity deficits spread less readily. The same ultraviolet closure data fix both quantities. The geometric bridge is the tetrahedral identity

$$\sum_{i=1}^4 \hat{n}_i \hat{n}_i^\top = \frac{4}{3} I_3,$$

which implies a channel-averaged transverse fraction of $2/3$ and gives the bare edge smoothness coupling

$$J_{\text{bare}} = \frac{2}{3} \eta_*.$$

If adjacent cells disagree strongly the edge pays a larger penalty; if they agree, the penalty is small. The factor $2/3$ is the geometric fraction that survives after averaging the four tetrahedral channel directions into the isotropic continuum limit — the part of the disagreement that the scalar sharing channel actually carries.

For a $z = 4$ regular coarse adjacency graph, the tree-to-lattice reduction then yields

$$J_{\text{eff}}^{\text{tree}} = \frac{J_{\text{bare}}}{3} = \frac{2\eta_*}{9}.$$

The division by 3 comes from the branching geometry of the rooted $z = 4$ graph. One neighboring link points back toward the source, while the remaining $z - 1 = 3$ links carry forward transport into the tree. The net long-range transport $J_{\text{eff}}^{\text{tree}}$ is therefore the portion of the microscopic edge penalty that survives this local branching.

Origin of the horizon target. The horizon target

$$\sigma_* = \frac{\pi}{g_{\text{share,eff}}}$$

is the closure-consistency value required by the horizon-normalized field convention. In the admissibility-closed boundary ensemble, one active microscopic sharing unit carries effective entropy $g_{\text{share,eff}}$. In the continuum normalization used for the weak-field scalar, the occupancy variable is normalized by

$$S = \pi Q_{\text{occ}},$$

so a coarse horizon-normalized channel with occupancy $Q_{\text{occ}} = 1$ carries entropy π in the S -field convention. If σ_* denotes the asymptotic conditional-independence weight seen by the rooted shell hierarchy (Appendix B.3), consistency between the boundary entropy count and the horizon-normalized continuum field requires

$$\sigma_* g_{\text{share,eff}} = \pi,$$

and therefore

$$\sigma_* = \frac{\pi}{g_{\text{share,eff}}} = 0.42340665 \dots$$

The factor σ_* matches the admissibility-closed microscopic entropy normalization to the horizon-normalized scalar-field convention. The rooted shell observable converges rapidly to this closure target, constraining the nonlocal correction at small shell depth. The four tetrahedral channel directions average to the isotropic tensor structure in the continuum limit, so the combinatorial data that fix admissibility also fix tree-level transport. Appendix C gives the shell hierarchy and phase-selection checks.

8. Finite-Loop Renormalization

Tree level is not the end of the ultraviolet chain. The full lattice admits local closed-return motifs that recycle part of the transmitted information before it contributes to net coarse transport. The leading correction is organized as a local Dyson self-energy dressing,

$$J_{\text{eff}}^{(\text{ren})} = \frac{J_{\text{eff}}^{\text{tree}}}{1 + J_{\text{eff}}^{\text{tree}} \Sigma_{\text{ret}}}.$$

A purely tree-like transmission rule would let the relevant amplitude move outward once and never locally return; a real coarse graph is not that simple. Some of the transmitted information cycles back through short closed motifs before contributing to long-distance transport. The renormalized coupling is therefore the true stiffness felt by the coarse field after these local returns have been resummed.

The structure of that self-energy is not a generic loop number. The returns split into seven sector-diagonal channels and one collective mode. The seven are the face-label channels, each returning independently without mixing. The one is the permutation-symmetric combination across channels, which returns as a shared closure-singlet rather than as a channel-specific loop,

and it is weighted by the same transverse projection and branch-dilution factors that define the tree edge map,

$$\binom{2}{3} \binom{1}{3} = \frac{2}{9}.$$

The leading local self-energy is therefore

$$\Sigma_{\text{ret}} = 7 + \frac{2}{9} = \frac{65}{9}.$$

Equivalently, on the seven-channel scalar return space,

$$R_{\text{ret}} = I_7 + \frac{2}{9}P_{\text{sing}}, \quad P_{\text{sing}} = |u\rangle\langle u|, \quad u = \frac{1}{\sqrt{7}}(1, \dots, 1),$$

with $\Sigma_{\text{ret}} = \text{Tr}(R_{\text{ret}})$. The orthogonal six-dimensional sum-zero sector carries no net scalar charge in the coarse branch and so adds no separate scalar return. The singlet weight is fixed by the tree map, not introduced here, so no new loop parameter appears. Permutation symmetry fixes the existence of the singlet but not the placement of the suppression factors on it alone; that placement is the minimal-return-operator reading whose graph-level derivation Appendix C.3 records as the outstanding audit. The induced uncertainty is bounded: replacing $\Sigma_{\text{ret}} = 65/9$ by 7, $7 + 2/3$, or 8 shifts $J_{\text{eff}}^{(\text{ren})}$ and γ by at most 0.5% and leaves G , a_0 , and κ/γ exactly unchanged, since $J_{\text{eff}}^{(\text{ren})}$ cancels in the source-to-stiffness ratio (Appendix C.5).

$$c_{\text{loop}}^{(\text{ren})} \equiv \frac{J_{\text{eff}}^{(\text{ren})}}{J_{\text{eff}}^{\text{tree}}} = \frac{1}{1 + J_{\text{eff}}^{\text{tree}}\Sigma_{\text{ret}}} \approx 0.95426,$$

and

$$J_{\text{eff}}^{(\text{ren})} \approx 0.00633348.$$

This reproduces the shell-target crossing near $J_{\text{bare,cross}} \sim 0.019$ at the 0.05% level.

The loop correction is no longer schematic: the finite renormalization is written as an explicit local self-energy. The remaining audit task is the independent graph-level derivation of the relative diagonal and singlet weights of the same scalar-return operator, not the introduction of any new loop parameter.

9. Continuum Stiffness and SI Normalization

The last UV step reads the lattice weighting as a quantum action rather than a thermal one: the lattice quadratic form is interpreted as a Euclidean action weight,

$$\frac{I_E}{\hbar} = \frac{J_{\text{eff}}^{(\text{ren})}}{2} \sum_{a,i} (Q_a - Q_{a+L*\hat{n}_i})^2,$$

where the sum runs over one sublattice representative a of each bipartite primitive cell and its four outgoing bonds $\hat{n}_i$, so each undirected nearest-neighbor edge is counted once (the convention of Appendix C.4). The microscopic four-cell is assigned the volume

$$\Delta V_4 = \frac{L_*^4}{c}$$

as a coarse-graining convention: the abstract tetrahedral cell complex has no space-filling regular-tetrahedron Euclidean embedding (Appendix C.5), so cell volumes and face areas enter as normalization conventions of the coarse map, not as geometry supplied by the graph. Up to this

point the derivation has determined a dimensionless lattice weighting. The continuum EFT, however, needs a dimensionful coefficient multiplying derivatives of a field in spacetime. The Euclidean-action interpretation upgrades the lattice closure data into a continuum action density with the right units and the right covariant target.

The same tetrahedral identity used in the edge-kernel reduction then yields the continuum coefficient for the occupancy field Q_{occ} ,

$$\gamma_Q = \frac{4\hbar c}{3L_*^2} J_{\text{eff}}^{(\text{ren})}.$$

Here L_* is the canonical tetrahedral spacing, with one coarse cell carrying volume L_*^3 up to the fixed cell-shape convention, and $J_{\text{eff}}^{(\text{ren})}$ is the loop-dressed edge coupling. The numerical factor $4/3$ is the isotropic projection

$$\sum_i \hat{n}_i \hat{n}_i^T = \frac{4}{3} I_3$$

that turns the tetrahedral edge directions into the continuum gradient tensor.

The field normalization is fixed by horizon capacity:

$$S = \pi Q_{\text{occ}}.$$

Therefore the canonical EFT coefficient in the $\frac{\gamma}{2}(\partial S)^2$ convention is

$$\gamma = \frac{4\hbar c}{3\pi^2 L_*^2} J_{\text{eff}}^{(\text{ren})}.$$

Physically, γ is the continuum stiffness of the entanglement-capacity field. A larger γ makes spatial gradients more costly and suppresses the capacity-deficit response to a given source; a smaller γ allows larger variations of the field. The faithful sector-resolution principle fixes L_* without using G . It is nevertheless useful to define the gravitational scale induced by this length,

$$G_* := \frac{c^3 L_*^2}{\hbar}.$$

Then the stiffness may be written in Einstein-normalized form as

$$\gamma = \frac{4J_{\text{eff}}^{(\text{ren})}}{3\pi^2} \frac{c^4}{G_*}.$$

This is the same algebra as the familiar Planck-cell rewrite, but read in the opposite direction: the substrate cell length induces the gravitational scale rather than being chosen by first inserting the measured value of G . Within the Euclidean-action and cell-volume conventions stated above, the SI-normalized stiffness coefficient is fixed; because the absolute normalization of an isolated scalar functional is conventional, the invariant content of this step is the ratio $\kappa/(\gamma S_\infty)$ that the weak-field matching of Section 11 consumes. In that ratio $J_{\text{eff}}^{(\text{ren})}$ cancels (Appendix C.5), so the loop-dressed coupling carries no content for the Newton normalization; its nontrivial input enters the stiffness itself and the dynamical and galactic sectors.

This completes the micro-to-continuum coefficient chain. The tetrahedral ensemble determines the effective sharing entropy; the edge kernel and loop dressing turn that entropy into a discrete stiffness; and the Euclidean matching turns the discrete stiffness into the continuum coefficient γ of the weak-field EFT.

Closed UV-to-IR chain. The UV coefficient chain can now be summarized as

$$\{\Omega_{\text{tet}}, K^2, \eta_*, g_{\text{share,eff}}, L_*, J_{\text{bare}}, J_{\text{eff}}^{\text{tree}}, \Sigma_{\text{ret}}, J_{\text{eff}}^{(\text{ren})}, \gamma\} \longrightarrow \{\kappa, G, a_0, g_{\text{obs}}(g_{\text{bar}})\}.$$

The first bracket is the micro-to-continuum closure chain; the second collects the weak-field observables it feeds. All later weak-field coefficients come from this chain.

The remaining microscopic question is independent confirmation of the same action-kernel interpretation from fuller inhomogeneous dynamics, not an unresolved normalization constant.

Part III. The Closed Static Branch: Einstein Gravity in the Capacity Variable

10. Einstein Parent Action and the Reduced Capacity Frame

The ordinary longitudinal capacity branch has a covariant parent action, and it is simply the ordinary metric action — no independently varied capacity scalar is added to it:

$$I_0[g, \psi] = \frac{c^3}{16\pi G} \int_{\mathcal{M}} d^4x \sqrt{-g} (R - 2\Lambda) + I_{\text{GHY}}[g] + I_{\text{matter}}[g, \psi],$$

where covariant coordinates use $x^0 = ct$. When the time integral is instead written in seconds, $dx^0 = c dt$ and the ADM prefactor is correspondingly $c^4/(16\pi G)$. Matter is coupled once, to one physical metric. The capacity functional is the static scalar-constraint reduction of I_0 , expressed in a different field coordinate. This statement can be proved without appealing to the final Poisson equation.

Static scalar reduction of Einstein–Hilbert gravity. Use Newtonian gauge,

$$ds^2 = - \left(1 + \frac{2\Phi}{c^2}\right) (dx^0)^2 + \left(1 - \frac{2\Psi}{c^2}\right) \delta_{ij} dx^i dx^j, \quad x^0 = ct,$$

and retain the static scalar sector through quadratic order. In ADM variables [8] the shift and extrinsic curvature vanish, so the Einstein–Hilbert plus Gibbons–Hawking–York action is

$$I_{\text{ADM}}^{\text{static}} = \frac{c^4}{16\pi G} \int dt d^3x N \sqrt{h} {}^{(3)}R + I_{\text{matter}}^{\text{static}} + I_{\infty}.$$

Expanding $N = 1 + \Phi/c^2$ and $h_{ij} = (1 - 2\Psi/c^2)\delta_{ij}$, cancelling the reference boundary term at infinity, and using $I_{\text{matter}}^{(1)} = - \int dt d^3x \rho \Phi$, gives

$$I_{0,\text{scal}}^{(2)}[\Phi, \Psi] = \int dt d^3x \left[\frac{1}{8\pi G} ((\nabla\Psi)^2 - 2\nabla\Phi \cdot \nabla\Psi) - \rho\Phi \right].$$

The lapse perturbation remains a constraint variable. Its variation and the spatial-scalar variation give, respectively,

$$\nabla^2\Psi = 4\pi G\rho, \quad \nabla^2(\Phi - \Psi) = 0.$$

Asymptotic flatness removes the harmonic difference, so $\Phi = \Psi$. Eliminating Ψ therefore produces

$$I_{\text{Newton}}[\Phi] = \int dt d^3x \left[-\frac{(\nabla\Phi)^2}{8\pi G} - \rho\Phi \right].$$

Thus no independent scalar stress tensor is needed to create the linear potential, and the equality $\Phi = \Psi$ in the baseline branch is a metric constraint equation rather than an anisotropic-stress assumption. Appendix N gives the expansion, boundary bookkeeping, and degree-of-freedom audit in full.

Exact reduced-action identity. On the renormalized static branch, write $S_{\text{ent}} = S_{\infty} - \delta S$. Source-independent extensive terms are removed by the vacuum normalization proved for the regulated joint measure in Section 25. The capacity functional is then

$$I_{\text{cap}}^{\text{static}}[\delta S; \rho] = \int dt d^3x \left[-\frac{\gamma}{2}(\nabla \delta S)^2 + \kappa \rho \delta S \right].$$

The field redefinition

$$\delta S = -\frac{2S_{\infty}}{c^2}\Phi$$

turns it into

$$I_{\text{cap}}^{\text{static}} = \int dt d^3x \left[-\frac{2\gamma S_{\infty}^2}{c^4}(\nabla \Phi)^2 - \frac{2\kappa S_{\infty}}{c^2}\rho \Phi \right].$$

Using

$$G = \frac{c^2 \kappa}{8\pi \gamma S_{\infty}}$$

gives the action-level equality

$$\boxed{I_{\text{cap}}^{\text{static}} = Z_S I_{\text{Newton}}, \quad Z_S \equiv \frac{2\kappa S_{\infty}}{c^2}.}$$

The field-independent factor Z_S cannot affect the classical reduced equations. It can matter when the UV construction is asked to normalize fluctuations or correlation functions, but it does not represent a second determination of G and it does not license adding I_{cap} to I_0 . The equality assumes the same asymptotically flat or Dirichlet boundary data on both sides. At a finite boundary the Newton surface term and its image under $\delta S = -2S_{\infty}\Phi/c^2$ must be included as well.

Scope of the background-covariant notation. For transport calculations the same reduced equation is packaged as

$$I_{\text{cap}}[S_{\text{ent}}; \chi | g_{\text{ref}}] = \int d^4x \sqrt{-g_{\text{ref}}} \left[-\frac{\gamma}{2} g_{\text{ref}}^{\mu\nu} \partial_{\mu} S_{\text{ent}} \partial_{\nu} S_{\text{ent}} - \lambda S_{\text{ent}} - \kappa \chi S_{\text{ent}} \right].$$

The vertical bar is essential: g_{ref} and the reduced source projection χ are held fixed while S_{ent} is varied. This notation is useful for extending the reduced response in time, but it is not a covariant scalar-tensor parent action. In the static nonrelativistic sector $\chi \simeq \rho$; covariantly the source is the full stress tensor through $I_{\text{matter}}[g, \psi]$.

Capacity coefficients and the source theorem. The UV calculation still fixes how the geometric constraint is coordinatized by the substrate variable. With

$$\sigma_{\text{def}} = \frac{\rho}{\kappa_m(L_*)},$$

the Green-matched projection is

$$\nabla^2 \delta S = -\frac{3L_*}{4G_{\text{tet}}(0)} \sigma_{\text{def}}, \quad \frac{\kappa}{\gamma} = \frac{3L_*}{4G_{\text{tet}}(0)\kappa_m(L_*)}.$$

This is the microscopic map between defect density and the capacity coordinate on the Einstein constraint surface. It is not an extra matter coupling in the covariant parent theory.

The length backbone of Newton’s constant. The weak-field normalization is

$$G = \frac{c^2 \kappa}{8\pi \gamma S_\infty}.$$

Faithful full-support resolution and the positive marked-transfer spectrum fix

$$L_* = -\frac{3}{2} Z_e \lambda_e \ln(1 - e^{-7g_{\text{share,eff}}}), \quad \lambda_e = \frac{\hbar}{m_e c},$$

and therefore induces

$$G_* = \frac{c^3 L_*^2}{\hbar} = \frac{9}{4} \frac{\hbar c}{m_e^2} Z_e^2 \ln^2(1 - e^{-7g_{\text{share,eff}}}).$$

Substituting the source-map identities

$$\frac{\kappa}{\gamma} = \frac{3L_*}{4G_{\text{tet}}(0)\kappa_m(L_*)}, \quad \kappa_m(L_*) = \frac{\hbar}{cL_* \ln 2}, \quad S_\infty^{\text{cell}} = \frac{3 \ln 2}{32\pi G_{\text{tet}}(0)},$$

into the weak-field expression gives identically

$$G = \frac{c^3 L_*^2}{\hbar} = G_*.$$

Thus there is one scale-setting route to G : electron recurrence fixes L_* , and L_* fixes the gravitational scale. The stiffness, source coefficient, and capacity normalization are a consistent static-EFT representation of that same length backbone, not a second determination that could have disagreed with it. The only numerical comparison in this sector is G_* against the measured Newton constant.

The decorated scale gives $G_* = 6.6742890772 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$, or -0.073σ relative to CO-DATA. Because both the early entropy construction and the later residual-closing vertex were developed with the discrepancy known, Appendix L treats this as a high-precision postdiction. The nontrivial content is the shared action that also fixes the two charged-lepton corrections.

11. Capacity Variable, Bridge Law, and Variational Status

Varying the reduced capacity-frame functional with respect to S_{ent} gives

$$\gamma \square S_{\text{ent}} = \lambda + \kappa \chi.$$

On the renormalized static, nonrelativistic branch this becomes

$$\nabla^2 \delta S = -\frac{\kappa}{\gamma} \rho.$$

Define the surviving fractional capacity

$$q(x) \equiv \frac{S_{\text{ent}}(x)}{S_\infty} = 1 - \frac{\delta S(x)}{S_\infty}.$$

The action reduction above fixes the weak-field bridge directly,

$$\frac{\Phi}{c^2} = -\frac{\delta S}{2S_\infty}.$$

Equivalently,

$$q = 1 + \frac{2\Phi}{c^2} + O(c^{-4}).$$

The bounded nonlinear rule

$$N^2 = q$$

is the continuous multiplicative completion selected by the capacity-composition rule. Its status must now be stated more precisely. In a static spherical exterior, q is the invariant geometric scalar

$$q = h^{ab} \partial_a R \partial_b R = 1 - \frac{2GM_{\text{MS}}}{c^2 R},$$

and in Schwarzschild coordinates it equals N^2 . In a generic spacetime the lapse is foliation dependent, so $N^2 = q$ by itself is not a covariant constraint. The metric-only parent therefore treats the weak-field δS and the spherical q as reduced or composite geometric variables; it does not promote either to an unconstrained second gravitational field. Appendix N.6 formulates the remaining target as a diffeomorphism-invariant, potentially quasilocal and state-dependent functional, rather than presuming that an additional fundamental scalar is needed.

Combining the static source equation with the weak-field bridge gives

$$G = \frac{c^2 \kappa}{8\pi \gamma S_\infty}.$$

This relation uses only the invariant combination $\kappa/(\gamma S_\infty)$: a fixed-epoch rescaling of the entropy units changes S_∞ and κ together and leaves the observable potential unchanged.

Two coordinates on one response. The metric parent solves the Hamiltonian and spatial constraints for Φ and Ψ ; the capacity frame uses δS as a field coordinate on that reduced solution. Around a constant background, a canonical scalar stress would begin as $(\partial \delta S)^2 = O(\rho^2)$ and could not be the source of the observed $O(\rho)$ potential. The action identity removes that mismatch: the linear capacity response is the reduced metric constraint itself.

Matter enters once. For the nonrelativistic static branch, the microscopic source theorem reduces the full metric source to the defect density ρ . Covariantly, matter enters only through $I_{\text{matter}}[g, \psi]$, so the full stress tensor gravitates, including trace-free radiation, and the Bianchi identity enforces the usual conservation law. The notation $\chi \simeq \rho$ belongs only to the reduced nonrelativistic source map; an explicit universal term $S_{\text{ent}} T^\mu{}_\mu$ is neither required nor adopted.

Parent-action decision. The preferred minimal construction for the ordinary branch is therefore metric-only:

$$I_{\text{parent}}^{\text{long}} = I_0[g, \psi], \quad \delta S = \delta S[g, \psi] \quad \text{after constraint reduction.}$$

It propagates the two tensor polarizations of general relativity and no extra scalar. Constrained-clock and scalar-tensor alternatives remain useful control cases, but both add structure and generically add a mode; Appendix N records why neither is selected. The galactic excess belongs to the separate carrier-resolved contact of Sections 15–16; its leading EFT is explicit, while its microscopic metric vertex remains conditional.

12. Newtonian Gravity and the Point-Source Limit

In the renormalized static weak-field sector the scalar equation reduces to

$$\nabla^2 \delta S = -\frac{\kappa}{\gamma} \rho.$$

After the background is renormalized away and the source is taken to be nonrelativistic, the deficit field obeys an ordinary Poisson equation. Its mathematical structure is the one used in standard weak-field gravity, with δS as the field variable.

For a point source M ,

$$\delta S(r) = \frac{\kappa M}{4\pi\gamma r}.$$

Using the bridge law,

$$\frac{\Phi}{c^2} = -\frac{\delta S}{2S_\infty},$$

the gravitational acceleration becomes

$$g(r) = \frac{c^2\kappa}{8\pi\gamma S_\infty} \frac{M}{r^2} = \frac{GM}{r^2}.$$

Thus Newtonian gravity is recovered as the weak-field response of the entanglement-capacity medium: the sourced scalar equation and the bridge law together imply the familiar point-mass force law, with nothing further assumed.

Interpretation. A point defect produces a $1/r$ capacity deficit, and the bridge maps its gradient to the Newtonian inverse-square force. Ordinary gravity is the small-deficit, weak-curvature limit of the extended capacity strain around localized defects. Section 24 exhibits the same structure on the discrete substrate: cell-by-cell re-equilibration screens at sub-cell range, whereas a conserved capacity current with maintenance sinks obeys the massless graph-Poisson equation and produces the $1/r$ deficit. The Newtonian form therefore diagnoses the conservation law behind it.

13. Electron Anchor: One-Bit Mass Scale and Seven-Sector Length Scale

The electron supplies the elementary mass anchor $m_e/\ln 2$. Its Compton scale also assigns the length associated with the seven-sector effective support. The decorated marked-transfer action closes the finite correction to that scale and routes the same correction through the heavier charged-lepton shells. These are two readings of one dimensionful datum, not independent measurements, so the weak-field normalization remains a single calibration.

13.1 Why the electron is the anchor

The mass-entropy map needs an elementary anchor because the elementary matter sector is the localized defect sector. The electron is the lightest simple charged fermionic defect, and its mass is not obscured by hadronic or QCD dressing. A single fermionic face-exclusion defect carries the canonical increment

$$\Delta S_f = \ln 2,$$

one bit of missing entanglement, because an excluded face is a binary occupied/unoccupied defect of the local network.

13.2 One-bit mass anchor

At the electron Compton scale $\ell = \lambda_e$ the mass-entropy map reads

$$\kappa_m(\lambda_e) = \frac{m_e}{\ln 2}.$$

Dividing the electron mass by the fixed one-bit increment fixes the mass-per-entropy conversion at the electron's own scale. Run back to the cutoff cell, the conversion is

$$\kappa_{m,\text{UV}} = \frac{\hbar}{cL_*} \frac{1}{\ln 2},$$

with canonical running law

$$\kappa_m(\ell) = \kappa_{m,\text{UV}} \left(\frac{L_*}{\ell} \right)^{1+\alpha_{\text{cl}}}, \quad \alpha_{\text{cl}} = 0$$

in the closed branch. One bit fixes the electron-scale conversion; the running law carries it to the UV scale; the same conversion then feeds the weak-field source map. This is the only point at which the mass anchor enters gravity.

13.3 Seven-sector length anchor

The same electron anchors the cell length through the marked support-to-rate map. Faithful full-support resolution uniquely selects the memoryless kernel. Within the stated recurrence mass functional, fermionic exclusion and electron lightness jointly select the maximum channel count $k = 7$ and vanishing inter-channel correlation $\Delta_7 = 0$. On the commutative renewed-state algebra, the state-weighted determinant of the likelihood multiplication operator is

$$\Delta_{\tau_p}(\mathbf{R}) = \exp[\tau_p(\ln \mathbf{R})] = e^{-g_{\text{share},\text{eff}}}.$$

Multiplicativity across the seven factorized clouds gives

$$r = \Delta_{\tau_p^{\otimes 7}}(\mathbf{R}^{\otimes 7}) = e^{-7g_{\text{share},\text{eff}}}.$$

This is the record-conditioned geometric transfer rate, not the collision probability of two independently sampled blocks. The positive survival spectrum gives the baseline scale

$$L_*^{(0)} = -\frac{3}{2}\lambda_e \ln(1 - r),$$

with 3/2 the transverse export factor of Appendix C.5.

Appendix H realizes this determinant transfer in a finite charged action. The construction separates the baseline recurrence from the defect-bound closure response, so the one-channel mixing projector, seven-channel survival operator, and marked-fiber determinant remain distinct.

This does not mean an electron is a single tetrahedral cell carrying seven simultaneous labels. The local ensemble supplies seven distinguishable dressing layers. Each layer has fermionic occupation at most one, and the recurrence mass minimum occupies all seven once. The electron is the lightest coherent one-bit defect on that selected branch and exports the transverse share of the resulting support. The Compton scale calibrates the substrate length hierarchy; the one-bit mass calibrates the source map. The two uses impose a nontrivial joint requirement on the electron's role in the gravitational normalization without duplicating a single input.

13.4 Decorated marked-transfer vertex

The original Newton normalization was low by about 1.05%. Because $G_* \propto L_*^2$, the missing amplitude in the substrate length was 0.5309%. The baseline muon ratio required a 0.5124% uplift. The tau ratio required 0.6562%, which separates into the same universal uplift and a smaller 0.1431% second-shell factor. Their common sign and scale motivated one small charged response with shell-dependent routing. Appendix L records that this was an action-level postdiction.

That response also had to be additive. The tetrahedral ensemble already fixed the entropy, closure spectrum, edge projection, source map, and the micro-to-macro coefficient chain. Changing its label count, admissibility weight, or transverse export to repair the residuals would move results that did not share the discrepancy. The admissible repair was consequently required to vanish in the unmarked vacuum, reuse the established closure incidence, and act only on the charged transfer graph. This criterion motivates the marked vertex below; it does not determine the answer numerically. The field content, determinant power, and routing still have to follow from the displayed action and survive the alternative-kernel audits.

The renewed closure amplitude has the exact Gaussian representation

$$e^{-\eta_* C^2/2} = \int \frac{d^3\xi}{(2\pi)^{3/2}} \exp\left[-\frac{1}{2}\xi^2 + i\sqrt{\eta_*}\xi_a C_a\right].$$

The amplitude-level closure incidence is therefore $\sqrt{\eta_*}$. For each occupied unordered channel pair $e = (m, m')$, the two directed scalar returns have row operator

$$R_e = \frac{2}{7}(\langle m \rightarrow m' | + \langle m' \rightarrow m |).$$

The contraction $B_e = \sqrt{\eta_*}R_e$ obeys

$$B_e B_e^\dagger = 2\eta_* \left(\frac{2}{7}\right)^2 = \frac{8\eta_*}{49} \equiv u.$$

Its canonical unitary dilation has no-event amplitude $\sqrt{1-u}$. The seven-channel lightest branch activates all $\binom{7}{2} = 21$ pair records, giving

$$Z_{\text{edge}} = \left(1 - \frac{8\eta_*}{49}\right)^{21/2}.$$

The present and history Gaussian strands each have three normalized first excitations. Their ordered products form nine orthonormal marked states, so the internal trace of one hard-core marked fiber gives

$$\boxed{\zeta_* = 9e^{-g_{\text{share,eff}}} \left(1 - \frac{8\eta_*}{49}\right)^{21/2}} = 0.005123584484947.$$

This internal trace computes a scalar vertex weight at fixed path branch; it is not a measurement that discards the defect's position label. Free transport applies the same internal contraction on every position branch, as required by Appendix H.11. The finite transfer graph has a universal marked alternative, one two-vertex label return, and one second-shell singlet passage. Its exact factors are

$$Z_\mu = 1 + \zeta_*, \quad Z_e = (1 + \zeta_*)(1 + 7\zeta_*^2), \quad Z_{\tau,2} = 1 + \frac{2}{7}\zeta_*.$$

The dressed scale is

$$\boxed{L_* = Z_e L_*^{(0)}}.$$

Appendix H writes the controlled vertex, decomposes its complete edge Hessian, and enumerates all 56,800 states of the one-step routing block.

The anchor is a gauge choice. Because counting fixes only dimensionless quantities, exactly one dimensionful measurement must be supplied, and which one is a convention. The physical content of the framework is carried by anchor-invariant statements: the lepton ratios m_μ/m_e and m_τ/m_e ; the hierarchy

$$\frac{m_e}{m_P} = -\frac{3}{2}Z_e \ln(1 - e^{-7g_{\text{share,eff}}});$$

the horizon normalization identity $1/4$ within the stated cell convention; the abundance ratio Ω_c/Ω_b ; and, within its branch assignment, a_0/cH_0 . The electron remains the anchor because its mass is measured most precisely. Appendix L records that the high-precision marked correction was constructed after the residuals were known, so anchor invariance does not turn the agreement into a blind prediction.

13.5 Consistency checks

The marked action uses no continuously fitted coefficient. Substitution gives

$$G_* = 6.6742890772 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \quad (-0.073\sigma),$$

$$\frac{m_\mu}{m_e} = 720 \frac{2}{7} Z_\mu = 206.768280237 \quad (-0.535\sigma),$$

$$\frac{m_\tau}{m_e} = 720^2 \left(\frac{2}{7}\right)^4 Z_\mu Z_{\tau,2} = 3477.343310 \quad (+0.481\sigma).$$

These three comparisons probe one marked-fiber weight through different graph polynomials. The scalar stiffness, source map, and weak-field bridge reproduce the same dressed G_* after substitution, so they remain one consistency chain rather than an independent determination.

Routing-integer audit. The displayed action makes the coefficient of ζ_*^2 in Z_e equal to the number of persistent charged labels: the second marked return sums over the alphabet, so the factor is $1 + 7\zeta_*^2$ (Appendix H.9). Holding ζ_* fixed and temporarily treating that integer as unknown, each unit step in n moves the induced G_* by 2.34 CODATA standard deviations: $n = 6$ gives -2.41σ , $n = 7$ gives -0.073σ , and $n = 8$ gives $+2.26\sigma$. Within $-200 \leq n \leq 200$, seven is the only integer within two standard deviations. Dropping the second-order term ($n = 0$) gives -16.4σ . Appendix B now supplies two readings of this result. Conditional on the marked-fusion incidence identification, the three-dimensional closure vector fixes the same seven internally and the Newton comparison checks its routing. Without that identification, the comparison selects the seven-state member of the stated discrete family. Neither reading is historically blind: the target and the residual were already known, as Appendix L records.

Two remarks keep this honest. The resolvability is partly fortunate: $\zeta_*^2 \simeq 2.6 \times 10^{-5}$ places the rung spacing just above the CODATA uncertainty, so the integer is measurable at all; a smaller ζ_* would bury the ladder inside the error bar, and a larger one would leave $n = 7$ one near-miss among many. And since the alphabet is symmetric about zero, one might expect the neutral label $m = 0$ to drop out of a return sum, giving coefficient six; $n = 6$ lies outside the band, so the neutral label demonstrably participates in the second marked return. At the tau the analogous test is weaker: among the forty-three simple rationals with denominator up to eleven, both the derived $2/7$ ($+0.49\sigma$) and $3/11$ (-0.49σ) survive, so the second-shell routing is supported rather than uniquely resolved.

The same comparison has a dimensionless form. Squaring the hierarchy gives the electron's gravitational coupling,

$$\frac{G_* m_e^2}{\hbar c} = \frac{9}{4} Z_e^2 \ln^2(1 - e^{-7g_{\text{share,eff}}}),$$

with $9/4$ the square of the transverse export factor and Z_e^2 the marked-transfer dressing. The relation also has an area reading. A horizon stores one bit in area $4 \ln 2 L_P^2$, while the electron spreads its single bit over the Compton area λ_e^2 . The marked factor changes the finite packing correction without altering the dominant hierarchy $e^{-14g_{\text{share,eff}}}$.

13.6 Composite sectors

For composite hadrons the claim is weaker and different in kind. The relevant quantity is the dressed, vacuum-subtracted bound-state entropy,

$$m_{\text{hadron}} = \kappa_m(\ell_H) S_{\text{ent},H}^{\text{dressed}},$$

with the dressed budget generated by confinement, gluonic structure, trace-anomaly dynamics, and chiral vacuum reorganization. A lattice derivation of that dressed entropy is not yet available. The present claim is limited to structural compatibility between the mass–entropy map and the standard QCD mass budget. The elementary-fermion anchor is settled in the simple sectors; the hadronic coefficients remain open.

14. Baseline Metric Closure: No Slip and PPN

For the branch that reproduces Newtonian gravity, the physical metric is already fixed by the Einstein parent action. The scalar constraint reduction of Section 10 gives

$$\nabla^2(\Phi - \Psi) = 0,$$

so asymptotic flatness implies

$$\Phi = \Psi.$$

This is an action-level result. It is not inferred from the canonical stress tensor of δS , because no independently gravitating capacity scalar is present. Since the parent action of this branch is exactly Einstein–Hilbert plus minimally coupled matter, its vacuum post-Newtonian solution has

$$\gamma_{\text{PPN}} = \beta_{\text{PPN}} = 1$$

and the remaining standard PPN parameters vanish, subject to the usual assumptions on the matter sector and boundary conditions. The capacity redefinition leaves these values unchanged. The carrier rule introduced below also removes localized Solar-System sources from the transverse influence functional. The leading Solar-System metric is therefore the Einstein solution in the specified carrier-resolved EFT. A nonlinear covariant audit of the record projector and the cosmic-state vector remains required for time-dependent preferred-frame terms.

Part IV. The Carrier-Resolved Galactic Branch

15. Carrier-Resolved Galactic Dynamics

The Einstein branch responds to the full stress tensor and supplies ordinary gravity at every scale. The galactic excess is a second, coarse-grained response associated with retained source records. Its leading effective completion consists of a carrier projector, a thermal contact law, and a conservative auxiliary-field action. These ingredients fix the static equations without introducing an independently coupled matter charge. Their microscopic origin in the GFT state and metric vertex remains conditional.

Carrier nesting. An active carrier P is a retained, dynamically coherent source record at the resolution relevant to the long-range response. Let $\mathcal{R}_P$ be its retained record algebra and χ_P its spatial support. The source projector is

$$\boxed{\Pi_P X = \chi_P \mathbb{E}[X \mid \mathcal{R}_P].}$$

The active set $\mathfrak{A}$ is an antichain: a carrier and one of its descendants cannot both supply transverse sources. Distinct active carriers have disjoint projected support, while nested record algebras obey the conditional-expectation tower rule,

$$\Pi_P^2 = \Pi_P, \quad \Pi_P \Pi_Q = 0 \quad (P \neq Q, P, Q \in \mathfrak{A}), \quad \Pi_P \Pi_A = \Pi_P \quad (A \subset P).$$

The transverse functional consequently has the carrier sum

$$\Gamma_{\perp}^{\text{nest}} = \sum_{P \in \mathfrak{A}} w_P \Gamma_{\perp, w_P}[g_{\pm}; \Pi_P T_{\pm}, \mathcal{R}_P].$$

The Einstein action continues to use the unprojected $T_{\mu\nu}$. A star contributes once to its galaxy's coarse transverse source and retains its resolved stellar field in the Einstein branch. It does not also carry an independent transverse halo. This source-space nesting removes parent–descendant cross-terms before the nonlinear response is evaluated.

Thermal contact and the acceleration scale. The transverse doublet has two compact phase angles, with Haar volume $(2\pi)^2$ for one phase cell. Loading one sharing entropy into that cell and coupling it reversibly to the apparent-horizon state gives

$$a_0 = \frac{g_{\text{share, eff}}}{4\pi^2} c H_0.$$

The phase-cell loading and reversible horizon coupling are physical matching premises. With the Planck value of H_0 , they give $a_0 = 1.231 \times 10^{-10} \text{ m s}^{-2}$, compared with the fitted RAR scale $1.20 \times 10^{-10} \text{ m s}^{-2}$ [1, 51]. The same premise gives $a_0(z) \propto H(z)$; that extrapolation is tested separately in Section 27.3.

For an active carrier P , the effective occupation cell is specified by

$$H_{\text{cell}, P} = k_B T_H x_P : N_P :, \quad x_P = \sqrt{\frac{|\nabla \Phi_{b, P}|}{a_0}}.$$

Its equilibrium free energy and response are

$$F_{\text{th}, P} = k_B T_H \ln(1 - e^{-x_P}), \quad \frac{\partial F_{\text{th}, P}}{\partial E_P} = n_B(x_P) = \frac{1}{e^{x_P} - 1}.$$

The dimensionless matching $E_P/(k_B T_H) = x_P$ is part of the leading EFT and awaits a microscopic derivation. The rank-one capacity Hessian of Appendix N remains a thermodynamic consistency condition for one capacity variable. Its massless tangent and massive eigenvalue $A_L + A_T$ do not supply the thermal energy or an additional propagating exchange mode.

Conservative static action. Define the carrier density and its Newtonian auxiliary potential by

$$\rho_P = \frac{u_{\mu} u_{\nu}}{c^2} \Pi_P T^{\mu\nu}, \quad \nabla^2 \Phi_{b, P} = 4\pi G \rho_P,$$

and set

$$z_P = \frac{|\nabla \Phi_{b, P}|^2}{a_0^2}, \quad x_P = z_P^{1/4}.$$

The required excess function is

$$\mathcal{Q}_\perp(z) = \int_0^z \frac{ds}{e^{s^{1/4}} - 1}, \quad \mathcal{Q}'_\perp(z) = n_B(z^{1/4}),$$

so the total auxiliary function $\mathcal{Q}(z) = z + \mathcal{Q}_\perp(z)$ satisfies

$$\mathcal{Q}'(z) = 1 + n_B(z^{1/4}) = \frac{1}{1 - e^{-z^{1/4}}}.$$

For each active carrier, introduce a total carrier potential Φ_P and the QUMOND-type action [19]

$$I_{Q,P} = - \int dt d^3x \left\{ \frac{1}{8\pi G} [2\nabla\Phi_P \cdot \nabla\Phi_{b,P} - a_0^2 \mathcal{Q}(z_P)] + \rho_P \Phi_P \right\}.$$

The functional added to the Einstein branch is the excess $I_{\perp,P} = I_{Q,P} - I_{N,P}$, where $I_{N,P}$ is the same auxiliary action with $\mathcal{Q}(z) = z$. Variation gives

$$\nabla^2 \Phi_P = \nabla \cdot [\mathcal{Q}'(z_P) \nabla \Phi_{b,P}].$$

Writing $\phi_{\perp,P} = \Phi_P - \Phi_{b,P}$ isolates the transverse excess:

$$\boxed{\nabla^2 \phi_{\perp,P} = \nabla \cdot [n_B(x_P) \nabla \Phi_{b,P}].}$$

This equation is conservative and applies to nonspherical sources. For spherical systems it reduces to

$$g_{\text{obs}} = g_{\text{bar}} \left[1 + n_B \left(\sqrt{g_{\text{bar}}/a_0} \right) \right] = \frac{g_{\text{bar}}}{1 - \exp[-\sqrt{g_{\text{bar}}/a_0}]}.$$

The high-acceleration limit is Newtonian, and the deep limit gives $g_{\text{obs}} \simeq \sqrt{a_0 g_{\text{bar}}}$ and $v^4 \simeq GM_b a_0$ [49, 50]. Disks and other nonspherical sources generally carry geometry-dependent curl corrections. Their predictions require direct Poisson solves using resolved baryonic maps; the pointwise spherical formula is not imposed on each pixel.

Empirical domain. The exponential law is specified for equilibrium active carriers in the regime established by rotationally supported galaxies and isolated-galaxy lensing. Dispersion-supported low-mass dwarfs now provide a distinct boundary test. A GravSphere analysis of 12 nearby dwarfs with $10^4 < M_{\text{bar}}/M_\odot < 10^{7.5}$ finds that most lie above the extrapolated RAR, trace multivalued loci, and show substantial galaxy-to-galaxy scatter; the reported tidal and nonequilibrium checks do not account for the result [39]. The carrier projector could address this evidence only through a derived phase-selection prediction. The present EFT therefore makes no universal RAR claim for ultrafaint dwarfs.

Status. The thermal contact free energy, auxiliary action, nonspherical field equation, and carrier nesting rule close the leading static EFT once its matching data are supplied. The one-entropy horizon loading, $E_P/(k_B T_H) = x_P$, and the active-carrier projector remain conditional. A microscopic GFT derivation must produce the cell Hamiltonian, projector, and metric vertex. Resolved baryonic-map calculations and the dwarf phase boundary remain empirical work.

16. Galactic Metric, Lensing, and Local Tests

No-slip metric contact. The leading stationary metric response of carrier P is

$$\boxed{h_{\mu\nu}^{\perp,P} = -\frac{2\phi_{\perp,P}}{c^2} (\bar{g}_{\mu\nu} + 2u_\mu u_\nu).}$$

In Newtonian gauge this gives $\Delta\Phi_{\perp,P} = \Delta\Psi_{\perp,P} = \phi_{\perp,P}$. A leading closed-time-path contact that produces this mean response is

$$\Gamma_{\perp}^{\text{QS}} = -\frac{1}{2} \sum_{P \in \mathfrak{A}} w_P \langle h_{\Delta}, \mathcal{E} h_{\perp,P} [\Pi_P T_c] \rangle,$$

where $\mathcal{E}$ is the linearized Einstein operator and the bracket includes the invariant spacetime integral. This contact satisfies $\Gamma_{\perp}[g, g] = 0$ and is real in the static mean. Its induced stationary source has $T_{\perp,P}^{00} = \rho_{\perp,P} c^2$ and $T_{\perp,P}^{ij} = 0$, so it carries no scalar anisotropic stress. The quasistatic contact introduces no independent scalar Cauchy data and adds no pole to the vacuum graviton propagator. A canonical local scalar-gradient stress would produce anisotropic stress and is not the metric realization used here.

The linear Ward identity follows from the Bianchi identity for $\mathcal{E}$. Nonlinear conservation requires the carrier and record variables to be varied with the metric. In covariant form the projector must obey

$$\Pi_P[\varphi^* g, \varphi^* \psi, \varphi^* \mathcal{R}] = \varphi^* \Pi_P[g, \psi, \mathcal{R}].$$

Freezing Π_P during metric variation would leave the nonlinear Ward identity unproved. The full covariant completion retains a conditional grade for this reason.

Lensing response. No additional lensing coefficient appears. The effective density inferred by dynamics and lensing is

$$\rho_{\text{lens},P} = \rho_P + \frac{1}{4\pi G} \nabla \cdot [n_B(x_P) \nabla \Phi_{b,P}].$$

For a deep-regime point mass, define $A = \sqrt{GMa_0}$. Then

$$g = \frac{A}{r}, \quad \rho_{\perp}(r) = \frac{A}{4\pi G r^2},$$

and the projected profiles and asymptotic deflection are

$$\Sigma_{\perp}(R) = \Delta\Sigma_{\perp}(R) = \frac{A}{4GR}, \quad \alpha_{\infty} = \frac{2\pi\sqrt{GMa_0}}{c^2} = \frac{2\pi v_{\infty}^2}{c^2}.$$

Joint kinematic and weak-lensing data extend the same RAR by about 2.5 decades in acceleration. With strict isolation and consistent stellar and gas masses, early- and late-type galaxies follow the same relation [46, 47]. This supports the leading no-slip choice. Direct solutions for resolved nonspherical lenses remain necessary.

Solar-System quadrupole and wide binaries. In the Solar System the Milky Way is the active carrier. The Sun and planets remain resolved descendants in the Einstein branch and do not re-enter $\Gamma_{\perp}$ as separate transverse sources. The Sun-generated anomalous quadrupole is therefore

$$Q_2^{\odot,\perp} = 0$$

at leading order. The remaining smooth Galactic contribution is a tide of order

$$\frac{g_{\text{MW}}}{R_0} \simeq 9.2 \times 10^{-31} \text{ s}^{-2},$$

about two thousand times smaller than the current Cassini 1σ uncertainty. Cassini tracking gives $Q_2 = (1.6 \pm 1.8) \times 10^{-27} \text{ s}^{-2}$ and places universal nonlinear functions of the total local field in 3–15 σ tension with galaxy fits; it also limits their Milky-Way boost at the Sun to two percent at 95% confidence [38]. Carrier nesting avoids that total-field cross-term.

Wide binaries inside the Milky-Way carrier are likewise Newtonian apart from the smooth Galactic tide. For a $1.5M_\odot$ pair the Hill radius is about 1.95 pc, and the estimated fractional speed corrections from the tide satisfy

$$\frac{\Delta v}{v_N} \lesssim 8 \times 10^{-6}, 6 \times 10^{-5}, 2 \times 10^{-4}$$

at separations of 10, 20, and 30 kAU, respectively. Current Gaia analyses remain divided: one recent treatment favors Newtonian dynamics after modeling unresolved triples, while two analyses report a low-acceleration anomaly [35, 36, 37]. The carrier-resolved branch predicts the Newtonian outcome.

The background vector u^μ enters the parent carrier response, but its local spatial variation is limited by the Galactic tide. Leading Solar-System PPN values and vacuum gravitational-wave propagation remain those of the Einstein branch. Time-dependent vector contractions and nonlinear preferred-frame terms remain part of the covariant projector audit.

Fluctuations. For N_{eff} independent occupation cells,

$$\text{Var}(\bar{n}) = \frac{n_B(1+n_B)}{N_{\text{eff}}}, \quad \frac{\sigma_g}{g} = \sqrt{\frac{n_B}{N_{\text{eff}}(1+n_B)}} \leq \frac{1}{\sqrt{N_{\text{eff}}}}.$$

A stochastic contribution below ten percent requires $N_{\text{eff}} \gtrsim 100$; three percent requires $N_{\text{eff}} \gtrsim 10^3$. The symmetrized occupation covariance gives a positive noise kernel. A dynamical completion must also satisfy the KMS fluctuation–dissipation relation. The bath coupling, equilibration rate, and correlation volume have not yet been derived.

Part V. Transport, Clusters, and Cosmology

The microstructure-to-static-weak-field chain is the most directly constrained part of the theory. The sectors of this part ask how the same finite-capacity substrate behaves when sources move. Transport asks how the capacity-strain field propagates after a source changes; the cluster sector asks how it couples to matter in distinct dynamical phases; cosmology treats the homogeneous mode; and the saturated phase treats the proposed committed carrier. These sectors share the static weak-field ontology but not its evidential status: the Einstein/capacity action equivalence, ordinary source map, and finite marked-transfer scale are the controlled baseline inside their displayed actions, while the transport, cluster, and cosmological additions carry their stated conditional or open grades.

17. Causal Transport and Telegrapher Dynamics

The static equations give settled fields. Moving sources require a causal response with the same zero-frequency limit. Relative to the substrate four-velocity u^μ , the ordinary capacity deficit retains the covariant telegrapher completion

$$\tau_0(u^\mu \nabla_\mu)^2 \delta S + u^\mu \nabla_\mu \delta S = D h^{\mu\nu} \nabla_\mu \nabla_\nu \delta S + A \chi, \quad h^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu,$$

with $A/D = \kappa/\gamma$. The carrier-resolved transverse excess has the corresponding local-frame equation

$$\boxed{\tau_0 \ddot{\phi}_{\perp,P} + \dot{\phi}_{\perp,P} - D \nabla^2 \phi_{\perp,P} = -4\pi G D \rho_{\perp,P} [\Pi_P T],}$$

where

$$\rho_{\perp,P} = \frac{1}{4\pi G} \nabla \cdot [n_B(x_P) \nabla \Phi_{b,P}].$$

Its static limit is exactly the Poisson equation of Section 15. No separate static form factor is introduced.

Causal propagation and the minimal infrared scale choice give

$$\frac{D}{\tau_0} = c^2, \quad \tau_0^{-1} = H_0, \quad D = \frac{c^2}{H_0}.$$

For a Fourier mode, the response relative to the static solution is

$$F(\omega, k) = \frac{Dk^2}{Dk^2 - \tau_0\omega^2 - i\omega}, \quad F(0, k) = 1.$$

The Laplace poles are

$$s_{\pm} = \frac{-1 \pm \sqrt{1 - 4\tau_0 Dk^2}}{2\tau_0}.$$

Both have negative real part, and the characteristic propagation speed is $\sqrt{D/\tau_0} = c$. For galactic source frequencies $\omega \sim vk$, departures from the static branch begin at $O(v^2/c^2) \sim 10^{-6}$; the linear damping correction is smaller on kiloparsec scales. The quasistatic RAR and lensing equations therefore survive without spectral suppression.

This transport law governs propagation delay, relaxation, and merger-era lag. Higher-derivative corrections at the block scale remain ultraviolet data. The cluster and merger phenomenology, which supplies the source weights evolved by this equation, is developed in Section 18.

18. Cluster Source Projection and the Diffuse–Decoupled Channel Split

Clusters require one account of both the hook-shaped residuals of relaxed systems and the collisionless-galaxy lensing peaks of mergers, without hidden baryonic galaxy mass or a modified galaxy law.

The proposed long-range entropic-excess channel couples differently to the three baryonic phases. A diffuse, phase-averaged medium carries the transverse projection ϵ inherited from the conditional galactic normalization; a dynamically decoupled collisionless component recovers the full projection; and a virialized coherent bath can be lifted above it. The projection value is fixed once that transverse branch is adopted. The phase-selection rule, lift profile, and resolved-map test remain open.

The cluster residual varies with acceleration. Lensing and kinematic analyses of relaxed clusters [4, 16, 17] find the ratio of observed to galaxy-RAR-predicted acceleration near unity in stellar-dominated centers, rising to roughly 3–5 at intermediate accelerations ($g_{\text{bar}} \sim 10^{-11}$ – $10^{-10} \text{ m s}^{-2}$), and apparently returning toward the galaxy relation at the lowest probed accelerations, subject to gas-extrapolation caveats [17]. The older integrated value of 1.5–2 from higher-acceleration hydrostatic analyses samples the high- g_{bar} edge of this hook-shaped profile. Merging clusters add a spatial constraint. In Bullet-type systems, ram pressure displaces the intracluster plasma from the collisionless galaxies while the lensing peaks remain with the outgoing collisionless components. A viable cluster sector must explain both observations without hidden baryonic mass in galaxies or a change to the galactic mass anchor.

Transport lag and extra galaxy mass do not supply the required offset. In the canonical $\tau_0^{-1} = H_0$ branch, disturbances propagate at c and cross a megaparsec-scale configuration in roughly 3 Myr, about two and a half orders below a gigayear merger timescale. The field therefore tracks the moving source; an underdamped configuration retains memory of the pre-merger centroid rather than the outgoing collisionless component. Extra galaxy mass is excluded because the galactic acceleration scale and gas-dominated-dwarf RAR fix the deficit per unit baryonic mass.

The projection coefficient as the galactic reduction factor. Section 15 fixes the galactic acceleration scale as the transverse reduction of the horizon thermal scale,

$$a_0 = \frac{g_{\text{share,eff}}}{4\pi^2} a_H, \quad a_H = cH_0,$$

where $(2\pi)^2$ is the angular Haar volume of one compact transverse phase cell and $g_{\text{share,eff}}$ is the admissibility-sharing content conditionally loaded into it. The same loading and horizon-coupling construction gives

$$\epsilon \equiv \frac{a_0}{a_H} = \frac{g_{\text{share,eff}}}{4\pi^2} \simeq 0.188.$$

This introduces no cluster-specific coefficient. It imports the reduction factor of the galactic branch and reads it as a source projection: a source restricted to the transverse static sector couples at strength ϵ relative to a source accessing the full horizon projection.

The diffuse and decoupled source classes. The assignment follows from coherence under coarse-graining. Diffuse matter is a continuum of locally uncorrelated, thermalized source elements; under coarse-graining its off-diagonal source cross-terms average away and only the diagonal transverse static projection survives. It therefore couples at ϵ . This includes shocked or unvirialized intracluster plasma, the warm-hot intergalactic medium, and cold but *diffuse* galactic H I when the galaxy is treated as a single smooth source—which is why gas-dominated dwarfs and low-surface-brightness galaxies remain on the galaxy RAR. The suppressed projection is thus a coherence effect, not a temperature effect; a hot phase that has *virialized* into a coherent bath is the exception, taken up below.

The unsuppressed projection is accessed by matter that is *not* part of the phase-averaged continuum: a collisionless overdensity that is spatially separated from, and dynamically decoupled from, a surrounding diffuse medium. The criterion is relational, not intrinsic compactness. Compactness alone would misclassify: stars in ordinary galaxies, isolated ellipticals, and globular clusters are compact and bound yet must remain on the galaxy RAR, and they do, because none is a collisionless node decoupled from a distinct diffuse continuum. A galaxy in a cluster is different only because it is embedded in, and decoupled from, the intracluster medium. The suppression is the property of participating in the continuum; matter that has decoupled from the continuum escapes it. So decoupled collisionless matter sits at the baseline weight, and incoherent diffuse gas is suppressed to ϵ , with $\epsilon = g_{\text{share,eff}}/4\pi^2$.

A *virialized* bath is the third state. Once the diffuse atmosphere relaxes into a coherent, extended phase it may open a collective response above the decoupled baseline. The source weight includes the ordinary unit response plus the maximally recruited excess. The cap reading is therefore

$$W_{\text{bath}}^{\text{max}} = 1 + \frac{1}{\epsilon} \simeq 6.32,$$

the same total-to-baryon factor that appears in the pinned abundance, $\Omega_m/\Omega_b = 1 + 1/\epsilon$. The earlier $1/\epsilon$ value counts only the excess/committed component and is not the ceiling on the *total* bath source weight. The lift profile between $W_{\text{bath}} = 1$ and $1 + 1/\epsilon$ is evaluated against cluster data below.

The effective entropic-channel source χ_{ent} is therefore regime-dependent. In a relaxed cluster the gas is a virialized bath,

$$\chi_{\text{rel}} = \rho_{\text{dec}} + W_{\text{bath}} \rho_{\text{bath}}$$

with ρ_{dec} the decoupled collisionless substructure and ρ_{bath} the virialized continuum; in a non-equilibrium merger the central gas is shocked and incoherent while only a residual atmosphere stays virialized,

$$\chi_{\text{merge}} = \rho_{\text{dec}} + \epsilon \rho_{\text{shock}} + W_{\text{bath}} \rho_{\text{vir}}$$

which in the Bullet limit, where the displaced gas is shocked and little virialized bath remains on the cores, reduces to $\chi_{\text{Bullet}} \simeq \rho_{\text{dec}} + \epsilon \rho_{\text{shock}}$. Ordinary matter continues to gravitate through the usual metric coupling; the projection rule concerns only the long-range entropic-excess channel.

The measured hot-atmosphere factor. A component is decoupled only relative to a surrounding medium, so the relevant factor is tied to a measured property of that medium: the fraction of the halo’s cosmic baryon allotment that has become an extended virialized hot phase,

$$\boxed{B_{\text{bath}} = \text{clip}_{[0,1]} \left[\frac{M_{\text{hot,vir}}(< r_{500})}{f_{b,\text{cos}} M_{500}} \right]} \quad f_{b,\text{cos}} = \frac{\Omega_b}{\Omega_m} \simeq 0.156,$$

with $f_{b,\text{cos}}$ fixed by Planck values [51] and $M_{\text{hot,vir}}$, M_{500} read from X-ray/SZ and total-mass estimates. Once the transverse branch is adopted, ϵ is shared across all systems, the baryon fraction is fixed cosmologically, and the per-system quantities are measured. The undetermined object is the coherence-growth profile $W_{\text{bath}}(B_{\text{bath}})$. In merging systems $M_{\text{hot,vir}}$ refers to the pre-merger virialized atmosphere.

Relaxed-cluster residual. For a *relaxed* cluster the diffuse gas is a virialized continuum, and the residual is carried by that continuum, not by the decoupled stellar component. Let

$$f_{\text{cont}} = \frac{M_{\text{hot,vir}}}{M_{\text{baryon}}}$$

be the fraction of observed baryons in the virialized diffuse continuum. The residual relative to a galaxy-RAR extrapolation is then

$$\boxed{\mathcal{R}_{\text{rel}} = 1 + (W_{\text{bath}} - 1) f_{\text{cont}}}$$

and the minimal linear candidate for the lift,

$$W_{\text{bath}} = 1 + (1 - \epsilon) B_{\text{bath}}, \quad 1 - \epsilon = 1 - \frac{g_{\text{share,eff}}}{4\pi^2} \simeq 0.812,$$

uses the amplitude $(1 - \epsilon)$, the part of the full horizon channel that the suppressed transverse branch is missing—not a new coefficient. The physical reading is that as a virialized diffuse bath forms, the continuum itself opens a collective cluster response whose amplitude scales with how complete the bath is (B_{bath}) and how much of the baryon budget sits in it (f_{cont}).

This linear form has the qualitatively correct mass trend: both B_{bath} and f_{cont} *increase* with halo mass—hot-gas fractions rise toward clusters [15, 14] and the atmosphere becomes more fully virialized—so their product rises monotonically from groups to massive clusters, reproducing the observed direction with no fitted parameter. Its amplitude, however, is excluded. For CLASH-scale clusters the measured factor gives $B_{\text{bath}} \simeq 0.83$, $f_{\text{cont}} \simeq 0.9$, hence $\mathcal{R}_{\text{rel}} \simeq 1.6$; but evaluating the source weight required to reproduce the published CLASH relation [4] across its data-supported acceleration window gives $\mathcal{R} \simeq 3.7$ at $g_{\text{bar}} = 10^{-10} \text{ m s}^{-2}$ rising to $\mathcal{R} \simeq 7.8$ at $10^{-11} \text{ m s}^{-2}$. The linear lift $(1 - \epsilon) B_{\text{bath}} f_{\text{cont}}$ is short of the observed peak residual by a factor of roughly 2.5–4, and no escape through missing baryons (a multiple of the X-ray gas mass would be required), hydrostatic bias (the masses are lensing-based), or sample heterogeneity is available at that magnitude.

The linear candidate is excluded on amplitude. Because $B_{\text{bath}} \leq 1$ and $f_{\text{cont}} \leq 1$, the linear candidate bounds the relaxed residual by $\mathcal{R}_{\text{rel}} < 1 + (1 - \epsilon) \simeq 1.81$. The measured peak residual of relaxed clusters is 3–5 [4, 16], well above this bound, so the linear lift is excluded. The exclusion is specific to the lift function: the channel-split ontology itself makes a structural prediction about the residual’s shape that the data support, taken up next.

The hook morphology. Independently of the lift amplitude, the channel split predicts the radial shape of the cluster residual. The decoupled BCG stellar component dominates cluster centers and carries weight 1, so the local residual starts near unity. Farther out, the virialized gas continuum dominates and raises the residual toward the bath-weighted value. At the lowest accelerations, the deep branch compresses a bounded source weight W toward $\sqrt{W}$ in acceleration terms and lowers the residual again. The resulting profile is a *hook*: near unity in the stellar-dominated center, maximal where the bath dominates at intermediate acceleration, and closer to the galaxy relation in the deep outskirts. Current measurements report this morphology [16, 17]; neither a total-baryon modified-gravity law with no relaxed-cluster excess nor a constant offset gives the same shape.

The linear lift nevertheless underestimates the amplitude: its predicted peak is $\simeq 1.5$ for CLASH-like parameters against the observed 3–5. The lift function must grow faster with bath development, reach 3–5 in developed clusters, and remain small enough for X-ray-faint groups to stay near the galaxy relation.

At the cap $W_{\text{bath}} = 1 + 1/\epsilon$, developed-bath parameters give

$$\mathcal{R}_{\text{cap}} = f_{\text{dec}} + \left(1 + \frac{1}{\epsilon}\right) f_{\text{cont}} = 1 + \frac{f_{\text{cont}}}{\epsilon} \simeq 5.7\text{--}5.9.$$

This is an upper bound, not a prediction that relaxed clusters sit at saturation. The ceiling 5.7–5.9 lies above the measured 3–5 mass-residual band, while the acceleration-resolved hook peak $S \simeq 3.7\text{--}4.9$ remains inside that band. Developed relaxed clusters therefore reach roughly three quarters of the available cap rather than saturating it. Applying the cap uniformly at group scale would overshoot strongly, so the attained fraction must depend on bath development; X-ray-faint groups remain far below it, with the group-scale data requiring $W_{\text{bath}} \lesssim 1.4$ there.

One caveat accompanies the bounded profile: the predicted central residual depends on the stellar/gas decomposition and on excluding multiphase cool-core gas from the coherent bath. The deep-outskirt question—power-law continuation [4] versus convergence toward the galaxy relation [17]—is adjudicated directly below.

The ceiling against cluster data. Inverting the X-COP hydrostatic measurements [72] gives twenty-four source-weight tests. For each point, S solves

$$\frac{Sg_{\text{bar}}}{1 - \exp[-\sqrt{Sg_{\text{bar}}/a_0}]} = g_{\text{obs}}.$$

Every point respects $S \leq 1 + 1/\epsilon$; the maximum is $S = 3.96$, and the relaxed systems span $S \simeq 1.8\text{--}2.7$ at R_{500} and $1.4\text{--}2.2$ at R_{200} .

The same inversion adjudicates the deep end: the power-law continuation requires $S \simeq 6\text{--}8$ at $g_{\text{bar}} \simeq 1\text{--}2 \times 10^{-11} \text{ ms}^{-2}$, precisely the accelerations of the $R_{500}\text{--}R_{200}$ points, which sit at $S \simeq 1.5\text{--}2.7$; within this sample the deep end converges rather than continuing, and the strongest published challenge to the bound is not borne out.

The methodological caveat is that the continuation was fit to lensing-based masses of higher-redshift systems while the inversion here uses local hydrostatic masses; breaching the bound at R_{500} would require the non-thermal-corrected masses to be low by a factor of $\simeq 2.3$, well beyond any claimed hydrostatic bias. The measured radial run of the source weight— $S \simeq 3.7\text{--}4.9$ in the hook-peak window, where the observed peak band is reached, declining to $\simeq 2.2$ at R_{500} and $\simeq 1.5$ at R_{200} —is the quantitative target the coherence-growth profile must reproduce.

Direct and fluctuation-based turbulence measurements find low non-thermal support in relaxed systems at all probed radii [73, 74], so the decoherence agent gating the lift cannot be the

cluster-to-cluster turbulence level: it must grow with radius even in fully relaxed atmospheres. The coherence-growth profile between the fixed endpoints, so constrained, is the object the channel-selection theorem must deliver, with the group end requiring $W_{\text{bath}} \lesssim 1.4$.

The decoupled-fraction form is excluded by the mass trend. A second candidate class assigns the residual to the decoupled stellar fraction, $\mathcal{R} = 1 + 4.32 B_{\text{bath}} f_{\text{dec}}$ with $f_{\text{dec}} = f_{\star}/(f_{\star} + f_{\text{gas}})$. Its amplitude can cross the observed band, but its mass trend is wrong. Because f_{dec} *falls* with mass while B_{bath} rises, their product peaks at the group or poor-cluster scale and declines toward massive clusters. The observed residual rises from groups to massive clusters. The trend therefore assigns the residual to the continuum, although its lift amplitude remains underived. The relaxed-cluster amplitude and Bullet morphology are distinct observables and require separate tests.

Bullet-type mergers. The relaxed residual and Bullet morphology use the same branch coefficient ϵ with different source expressions because the gas occupies different states. A relaxed atmosphere enters through the bath-lift term; shocked displaced gas carries the suppressed weight while collisionless cores remain decoupled. The two regimes share one conditional coefficient, not one universal scalar law.

In the merger, then, the decoupled galaxies and subcluster cores retain the unsuppressed projection and the shocked diffuse gas couples at ϵ . With a gas/galaxy baryon ratio near 5.7, the gas contributes $\epsilon \times 5.7 \simeq 1.07$ in the entropic channel against the galaxy contribution of 1.0: the projection brings the two components to near-parity, removing the factor ~ 5.7 by which the gas would otherwise dominate, but it does not by itself invert them. The inversion is completed by projected compactness. For two roughly symmetric outgoing components the ratio of one edge peak to the central gas contribution scales as

$$\frac{\Sigma_{\text{edge}}}{\Sigma_{\text{gas}}} \sim \frac{f_{\text{dec}}}{2\epsilon f_{\text{gas}}} \frac{A_{\text{gas}}}{A_{\text{edge}}}, \quad \frac{f_{\text{dec}}}{2\epsilon f_{\text{gas}}} \simeq 0.47,$$

so an edge peak dominates the projected map once the shocked gas is spread over more than about twice the projected area of a compact outgoing core—a condition the observed morphology satisfies by a wide margin. The projection rule and this geometry therefore produce the observed gas/lensing inversion—by projection and geometry together, not by projection alone and not by transport lag—as a spatial surface-density prediction rather than an integrated-mass argument. This is consistency, not yet a test of the coefficient. In standard flexible lens reconstructions the gas weight is degenerate with free halo and substructure components, so a model that fits comparably well with or without the fixed X-ray gas map constrains ϵ only weakly; Bullet-type mergers are thus consistent with the projection rule but do not yet measure ϵ . Peak location alone is in any case insensitive to the coefficient, since sufficiently broad shocked gas yields clump-centered peaks across a wide range of gas weights; the coefficient is tested only by the resolved amplitude fit below.

Relation to the transport sector. The telegrapher sector of Section 17 is not the cluster mechanism; it governs how the field propagates and relaxes once the source weights are set. The source-projection rule supplies the static weights χ_{ent} ; the transport sector then evolves them. This division avoids the failure mode of a transport-only account, in which a field sourced equally by all baryons cannot hold a lensing peak on the outgoing collisionless component. The full merger observable is obtained by evolving $\chi_{\text{ent}}(x, t) = \rho_{\text{dec}}(x, t) + [\dots]$ through the causal equation with the observed geometry as input.

Falsifiers and open status. The rule makes quantitative predictions beyond the relaxed normalization. (i) The linear candidate’s ceiling $\mathcal{R}_{\text{rel}} < 1.81$ lies below the measured peak

residual of relaxed clusters [4, 16], which excludes that form of the lift. The channel split predicts a hook: a residual near unity in BCG-dominated centers, one peak where the virialized bath dominates at intermediate acceleration, and convergence toward the galaxy relation in the deep outskirts. A profile monotonic in acceleration, or one that peaks in the stellar-dominated center, would falsify the channel split. The total-source cap supplies the ceiling $\mathcal{R}_{\text{rel}} \leq 1 + f_{\text{cont}}/\epsilon \simeq 5.7\text{--}5.9$ for developed-bath parameters. All twenty-four X-COP source-weight inversions respect it. A confirmed relaxed-cluster residual above this bound would falsify the cap reading.

(ii) At fixed mass, X-ray-bright bath-developed systems (larger B_{bath} , larger f_{cont}) should deviate more from the galaxy RAR than X-ray-faint systems; the residual turns on with the developed diffuse atmosphere, not with mass alone, so two systems of equal mass but different bath development should separate.

(iii) Resolved lensing maps test the three source components directly. Because the relaxed and merger regimes use different source expressions, the convergence is a *three-component* channel-weighted map—a relaxed virialized-bath component, a shocked non-equilibrium continuum at the suppressed weight, and a decoupled collisionless component,

$$\kappa_{\text{obs}}(x, y) = A \left[W_{\text{bath}} \Sigma_{\text{bath}}(x, y) + \epsilon \Sigma_{\text{shock}}(x, y) + \Sigma_{\text{dec}}(x, y) \right] + b$$

with the branch value $\epsilon = g_{\text{share,eff}}/4\pi^2 \simeq 0.188$ or with ϵ floated as a test, and W_{bath} fit within $[1, 1 + 1/\epsilon]$. Recovering $\epsilon \simeq 0.19$ across relaxed and merging systems would support both the cluster source rule and the inherited transverse normalization. A best fit near 1 or 0 would falsify the cluster branch. The fit must use the channel-weighted baryonic maps without free dark haloes, which would otherwise absorb the gas weight.

Three open items remain at the theory level. First, the relaxed residual and Bullet morphology use one inherited coefficient with two regime-specific source expressions. The boundary between the virialized and shocked regimes is not derived.

Second, suppression of a phase-averaged continuum and collective lift of a virialized bath are independent premises. Neither follows from the current microscopic source map. Their derivation must also reproduce the measured radial decline and the proposed branch endpoints.

Third, identifying the decoupled component with stellar or galaxy mass and the continuum with gas is a coarse split; intracluster light and tidally stripped stars blur it at a level the resolved-map fit would expose. Abell 520, whose reported gas-coincident dark core is disputed, is a phase-state stress case rather than a direct test: a re-cohering or quasi-bound central component would raise its effective B_{bath} and return lensing toward the gas. A confirmed *young* merger with a statistically secure gas-centered, galaxy-free lensing peak would leave no time for that re-coherence and would challenge the model.

This sector is a structured, falsifiable proposal. The branch value of ϵ is inherited rather than re-fitted, B_{bath} is measured from the hot-atmosphere fraction, and current data support the trend and hook morphology while excluding the linear lift candidate. The microscopic transverse normalization, coherence-growth profile, resolved-map test, and channel-selection theorem remain open.

19. Cosmology and the Hubble-Tension Sector

The scalar capacity field has two cosmological roles, and the sector works only if they stay separate. Its homogeneous mode $\bar{S}(t)$ affects the background expansion and the sound horizon; its inhomogeneous fluctuations $s(x, t)$ still govern local weak-field gravity. The cosmological sector is the homogeneous continuation of the same medium, not an unrelated dark-energy

component appended to the weak-field theory: what changes is the kinematic regime, not the ontology, as the background mode becomes dynamically relevant on horizon scales while the local branch stays encoded in the fluctuations.

The cosmological sector uses the same field split,

$$S(x, t) = \bar{S}(t) + s(x, t),$$

where $\bar{S}(t)$ is the homogeneous mode and $s(x, t)$ the inhomogeneous sector responsible for local weak-field dynamics. The vacuum baseline is fixed by apparent-horizon capacity,

$$S_\infty(t) = \pi \frac{R_A(t)^2}{L_*^2}.$$

This is the horizon-normalized representation of the same entropy field used locally. It is compatible with the cell-normalized source theorem because local observables depend on $\delta S/S_\infty$ and $\kappa/(\gamma S_\infty)$ rather than on an absolute entropy unit.

Two independent results clarify what this baseline means. First, the type II₁ static-patch construction makes empty de Sitter the maximum-entropy gravitational state and places excited semiclassical states below it [24]. Second, Bianconi obtains the same de Sitter area scaling from a bulk rather than a horizon entropy. In the $c = 1$ convention of her low-curvature Friedmann approximation, the local geometric-relative-entropy density and the causal-diamond four-volume scale as

$$\frac{\delta s}{\delta v} \simeq \bar{\omega}_{[1]} H^2, \quad V_{\text{dS}} \sim H^{-4},$$

so their product gives

$$S_{\text{GIE}} \simeq \frac{\bar{\omega}_{[1]}}{\ell_P^4 H^2} \propto H^{-2} \propto A_{\text{dS}}.$$

Integrating Bianconi’s local volumetric information density over the observer’s causal diamond gives an area-sized finite capacity [25]. Her result does not fix the coefficient used here, replace ℓ_P by the independently calibrated L_* , or derive the homogeneous evolution. Her intrinsic geometric k -temperatures scale as H^2 , whereas the Gibbons–Hawking temperature used in Section 15 scales as H .

Because the entanglement field couples to the trace of the stress-energy tensor, the homogeneous mode is largely dormant during radiation domination but can become active near matter–radiation equality. The conditional proposal would reduce the sound horizon and shift the CMB-inferred Hubble constant upward: the mechanism has the required sign and turns on at the required epoch. Its joint evolution with the committed component has not been computed, and a full Boltzmann calculation and likelihood analysis remain open.

The local weak-field predictions are protected by the separation between $\bar{S}(t)$ and $s(x, t)$. This is the role of the shear-lock logic: changing the homogeneous background mode does not rewrite the local static Poisson branch that governs galactic dynamics and lensing. The two regimes therefore use one scalar medium without changing the galactic coefficients, and a full perturbation calculation must still test that separation dynamically.

20. The Saturated Phase and the Cosmic Microwave Background

The conditional transverse normalization of Section 15 is now applied to the early universe. Its galactic response remains below the capacity ceiling, but the estimated recombination-era demand exceeds it. The resulting saturated phase contains a conserved count of committed channels that redshifts as a^{-3} . The zero-pressure constraint theorem below is conditional on the

pinned realization of that phase, and the numerical abundance inherits the transverse coefficient ϵ .

Saturation requirement at recombination. Use $y \equiv g_{\text{pert}}/a_0(z)$ for the acceleration ratio and $x \equiv \sqrt{y}$ for the thermal argument of Section 15. The linear-domain estimate $y \lesssim 2 \times 10^{-3}$ today and $y \propto \sqrt{a}$ gives $y \lesssim 6 \times 10^{-5}$ at $z \simeq 1100$, hence $x \lesssim 8 \times 10^{-3}$ and $\nu(x) \gtrsim 10^2$. This is well above the total-response cap $1 + 1/\epsilon = 6.32$, so the adopted response branch requires saturation if the same estimate applies to the relevant perturbation modes. The relation $a_0(z) = \epsilon cH(z)$ keeps the excess ratio $cH/a_0 = 1/\epsilon$ fixed at every redshift; adding the ordinary unit response gives the total cap.

Absolute capacity, source-normalized recruitment, and coherence. Three quantities that answer different questions must be kept separate. The availability field q measures the absolute local capacity remaining for ordinary transactions; in the static normalization it obeys $q = 1 + 2\Phi/c^2 + O(c^{-4})$. It is not a universal identity with $-g_{00}$ in arbitrary FLRW coordinates. Recruitment saturation is instead defined relative to each source's cap. For a Lagrangian source element A of baryonic mass M_A , let $M_{c,A}$ be the mass-equivalent committed allocation assigned to that source. Per-defect bookkeeping gives the cap and its normalized occupancy,

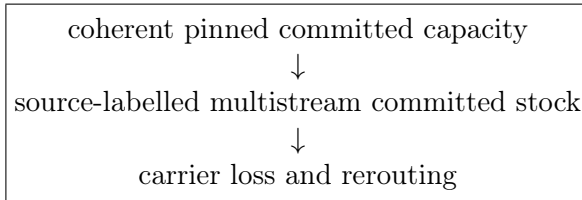
$$M_{c,A}^{\text{max}} = \frac{M_A}{\epsilon}, \quad \sigma_A = \frac{M_{c,A}}{M_{c,A}^{\text{max}}} = \epsilon \frac{M_{c,A}}{M_A} \in [0, 1].$$

The label A is physical bookkeeping: allocations belonging to distinct defects may overlap spatially, but they cannot be counted twice. In the continuum A becomes a Lagrangian source label, so σ_A is source-supported rather than a local projector attached independently to every cell.

Coherence is likewise a property of the committed flow, not a microscopic Boolean register. Write $\mathcal{C} = 1$ when the source-labelled currents combine into one hypersurface-orthogonal, single-stream flow with a single-valued renewal phase; locally this requires vanishing curl of the normalized flow, and globally it requires vanishing phase holonomy. At first shell crossing the Lagrangian map loses invertibility and several velocities occupy one Eulerian point, so $\mathcal{C}$ falls to zero. Loss of single-stream coherence ends the single-potential pinned description. It does not erase the Lagrangian source label or reroute the committed allocation: source-labelled multistream stock remains conserved until the separate carrier-loss condition $\mathbf{E}_A = 1$ is met. The physical pinned branch is therefore

$$\sigma_A = 1 \text{ for every recruited source,} \quad \mathcal{C} = 1,$$

and the subsequent sequence is



generally with the absolute availability still close to its vacuum value. Terminal strong-field saturation is the different condition $q_{\text{geo}} = 0$. Cosmological pinning saturates a small active subsector; it does not exhaust the vacuum budget.

The pinned reading and the zero-pressure constraint theorem. The *pinned reading* is the statement that the saturated bath holds exactly at the source-normalized ceiling and stays

there. The transfer law quantizes channel transport at one bit per tick as an upper bound; saturation is the attainment of that bound, so on the pinned reading every committed channel advances by exactly one unit per tick, and while the phase remains pinned no channel can decommit, since there is no slack for it to relax into. The committed count is then conserved and dilutes only with the expanding volume.

The unit-timelike normalization is not a further independent condition once saturation and coherence are granted. Let θ be the coherent renewal phase, increasing by one on each electron-calibrated interval τ_* . The dimensionful clock $\phi = \tau_* \theta$ therefore satisfies $d\phi/d\tau = 1$ along a committed worldline. Coherence identifies the flow covector as $u_\mu = -\nabla_\mu \phi$. Hence

$$1 = \frac{d\phi}{d\tau} = u^\mu \nabla_\mu \phi = -u^\mu u_\mu, \quad \boxed{X = -\frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi = \frac{1}{2}.}$$

Thus the spectral tick fixes the normalization of the coherent clock; no cosmological coefficient is inserted. This proves the constraint within the pinned coherent branch. It does not derive the transition that makes $\sigma_A \rightarrow 1$ and $\mathcal{C} \rightarrow 1$.

The coarse-grained field ϕ consequently carries a constraint rather than a generic kinetic term. The leading action consistent with it is

$$S_{\text{comm}} = \int d^4x \sqrt{-g} \lambda \left(X - \frac{1}{2} \right),$$

whose variation gives $\nabla_\mu (\lambda \partial^\mu \phi) = 0$ and, on the constraint surface,

$$T_{\mu\nu} = \rho u_\mu u_\nu, \quad \rho = E_0 n, \quad p = 0, \quad c_s^2 = 0, \quad \rho \propto a^{-3},$$

with $u_\mu = -\partial_\mu \phi$ and n the conserved spatial density of committed cells. This belongs to the constrained-scalar class [75]. Conditional on the pinned reading, this is the zero-pressure stress form of saturated committed capacity. It is an effective metric statement about the pinned capacity phase, not a particle ontology. The construction inherits caustic formation and the need for a small-scale completion. The manuscript does not derive the transition from the unconstrained weak-field branch onto $X = \frac{1}{2}$ or the commit/decommit energy accounting. Related relativistic Milgromian dynamics provides a useful comparison [31].

Uniqueness of the carrier. The constraint coupling is the unique survivor of the dynamics classes examined (Appendix M). Relaxational response kernels are excluded twice over: a growth clock calibrated on the cluster radial decline misses cosmological development by a factor of order thirty, and the precision of the measured acoustic peaks bounds any oscillatory leakage of a lagged response below one part in five hundred, a rejection no causal filter achieves over the few oscillation periods available before recombination. Bound-type rail readings sit at the sound-speed pole and carry no perturbation; plateau approaches have $c_s^2 = -1/(2n+1) < 0$ and are gradient-unstable; generic unconstrained scalar continuations have nonzero sound speed and are not adopted as post-release degrees of freedom. A general exclusion of relaxation-plus-cap carriers is recorded in Appendix M; the conserved committed density is precisely the additional integrating variable that the exclusion requires and the constraint reading supplies. Carrier loss does not continue this constrained field into a new scalar branch: it removes the source-label saturation condition and returns the metric-visible remainder to the ordinary source-associated capacity bookkeeping described in Appendix O.

Abundance. At full saturation the committed weight is the ceiling, giving

$$\frac{\Omega_c}{\Omega_b} = \frac{1}{\epsilon} = \frac{4\pi^2}{g_{\text{share,eff}}} = 5.321 \quad \text{against the measured } 5.364 \pm 0.065,$$

a 0.7σ agreement with the abundance derived from the commitment count [51]; equivalently $\Omega_m/\Omega_b = 1 + 1/\epsilon = 6.32$. A standard Einstein–Boltzmann computation [76] with the cold component tied to this value and the acoustic scale θ_* held fixed reproduces the quoted Planck best-fit temperature spectrum with a root-mean-square residual of 0.15% over $\ell = 2\text{--}2500$ and 0.08% in the third-peak region. At the likelihood level, the tied model carries one fewer free parameter than Λ CDM and sits at $\Delta\chi^2 = +2.15$ against the six-parameter best fit on the compressed Planck TT,TE,EE likelihood [41]. This fixed- θ_* comparison tests the inherited committed-capacity abundance only. It does not demonstrate the separate Section 19 claim that the homogeneous mode reduces the sound horizon. A joint Boltzmann calculation must evolve the homogeneous mode, commitment transition, and tied committed-capacity component together; that calculation remains open.

The acceleration and abundance relations contain a parameter-free structural test that does not depend on the numerical value of the phase-cell coefficient. Since $a_0 = \epsilon c H_0$ and $\Omega_c/\Omega_b = 1/\epsilon$,

$$a_0 \frac{\Omega_c}{\Omega_b} = c H_0.$$

Using the RAR scale $(1.20 \pm 0.02) \times 10^{-10} \text{ m s}^{-2}$, the Planck abundance ratio 5.364 ± 0.065 , and $H_0 = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ gives

$$\frac{a_0(\Omega_c/\Omega_b)}{c H_0} = 0.983 \pm 0.022$$

[1, 51]. This two-percent agreement tests the reciprocal structure of the two sectors; the quoted band uses the random error on the RAR scale, and its mass-to-light systematic widens it accordingly. It does not derive either conditional premise: the transverse loading controls a_0 , while the pinned recruitment reading controls the abundance.

That the committed density attains the ceiling exactly, rather than a development-dependent fraction of it, follows only within the recruitment assumptions of this branch. In the notation above, the assumption is $\sigma_A \rightarrow 1$ for every recruited Lagrangian source element. Commitment is source-driven: cells receiving no demand commit nothing, and each source’s allocation is capped at $1/\epsilon$ per unit source mass. In the recombination estimate above the relevant modes demand $\nu \gtrsim 10^2$, far above the cap, while the homogeneous background remains below it. With abundant supply and irreversible commitment in the pinned regime, each source recruits to its cap and allocations are assumed to add. The committed density is then

$$\rho_c = \frac{1}{\epsilon} \rho_b$$

pointwise at a commitment epoch. The manuscript has not derived that epoch or shown that commitment completes before the modes used in the Boltzmann calculation begin their relevant acoustic evolution. The initial condition $\delta_c = \delta_b$ is therefore an explicit assumption of the current numerical check, not an output of the recruitment argument. The abundance remains conditional on the pinned reading and per-defect bookkeeping; the transition history and energy accounting are open. Appendix O.6 registers a first-crossing computation that falsifies the late pointwise-tracking reading outright — the Silk-damped committed field never reaches a caustic — and, jointly with the Einstein–Boltzmann check above, selects pre-acoustic commitment; the recombination-era saturation estimate must be re-derived at that earlier epoch.

Conditional conservation and collective source-clock release. The saturated committed-capacity phase remains source-associated while its baryonic Lagrangian carrier is distinguishable. For each source A , the phase θ_A is the source/worldline renewal clock and $c_{A,c}$ is the identity-free committed allocation. Shell crossing can create several Eulerian streams, but it cannot

create a which-stream record in the one-register capacity action: exchanging identical stream assignments leaves the occupancy history unchanged, so

$$|e_i\rangle = |e_j\rangle, \quad \gamma_{ij} = 1, \quad C_A^{\text{rec}} = 1.$$

Carrier loss occurs only when matter itself ceases to preserve the source clock: gas from several progenitors becomes irreversibly mixed in one retained common well and baryonic records no longer resolve progenitor membership. Faithful renewal then fixes a complete fresh draw,

$$\boxed{E_A = \mathbf{1}[\text{the baryonic carrier of } A \text{ has lost progenitor identity}], \quad f_{\text{reroute}} = 1.}$$

The normalized nine-state dilation of Appendix O.13 is isometric; its $1/3$ marked amplitude normalizes nine outputs and is not a routing fraction.

The enabling well depth remains a collective matter condition rather than a microscopic capacity threshold. Atomic-cooling assembly supplies $V_{\text{thr}} \sim 17 \text{ km s}^{-1}$ before reionization, while photo-heated gas after reionization requires a scale nearer 30 km s^{-1} . The primitive action contains no local switch at either value. The transverse scale $a_0(z) = \epsilon c H(z)$ and the source-clock release gate therefore have different roles.

Appendix O measures carrier loss by

$$\epsilon_{\text{mix}}(t) = 1 - \frac{I(A; Y_t)}{H(A)},$$

with A the Lagrangian progenitor label and Y_t the retained coarse baryonic carrier. First eligible assembly starts the mixing clock rather than causing instantaneous release. The cosmological history is therefore

$$\boxed{F_{\text{rel}}(t) = \int^t K_{\text{mix}}(t - t_f; M, z) dF_Z(t_f).}$$

Appendix O.12 now fixes F_Z by an absorbing first-entry excursion-set construction and directly evaluates K_{mix} in a three-dimensional compressible-flow passive-tracer surrogate. The former Poisson number 0.8529 is retained only as a coverage statistic; it is not the Shannon information loss. For progenitors distributed through the full virial volume the surrogate gives $\epsilon_{\text{mix}}(70 \text{ Myr}) \simeq 0.20$, whereas centrally concentrated progenitors mix much faster. The decisive astrophysical calculation remains a cosmological self-gravitating zoom with passive progenitor tags; its kernel must be fixed independently of the cosmological likelihood.

Carrier loss does not create a new material component. The replaceable closure register returns to the universal renewed state, while the metric-visible remainder stays associated with the retained coarse carrier. The marked history output has no independent spacetime current. The unique volume-singlet projection of its universal drain changes the homogeneous cosmological term; every nonzero spatial mode remains internal to the source-associated capacity bookkeeping. Thus the background is described by one metric-visible capacity density,

$$\boxed{\rho_C(a) = \rho_{c,0}^{\text{wb}} a^{-3} [1 - \eta_* F_{\text{rel}}(a)],}$$

together with

$$\boxed{\rho_\Lambda(a) = \eta_* \rho_{c,0}^{\text{wb}} \int^a a'^{-3} dF_{\text{rel}}(a').}$$

There is no independent post-release scalar or local $\Lambda(x)$ branch. Appendix O.2–O.3 gives the volume-projection theorem and the resulting perturbation Ward identity.

Two consequences of the gate for other sectors. The release condition is collective and gated by retention, so a halo that never crosses the gate never undergoes source-clock release. A source whose gas is expelled before irreversible progenitor mixing preserves its source-associated saturated allocation at the ceiling ρ_b/ϵ , that is, 5.3 times its baryonic mass, in addition to the response-branch enhancement; the radial-acceleration relation of the faintest dwarfs is the corresponding test. Section 15 assumes that collapsed systems follow the baryonic response law with no additional collisionless matter species; under the present release law the detailed low-mass behavior therefore depends on whether the source remains in the saturated source-associated phase or exits it through the collective carrier-loss channel. The faintest retained systems are a cross-sector test of that bookkeeping, not evidence for a second material component.

Distinction from horizon saturation. The committed cosmological phase — channels advancing at capacity, a running clock — is distinct from the terminal saturation of the strong-field sector (Section 21), where capacity is exhausted and transactions cease. Whether the two termini are stages of one process is open and carries a stated consistency burden: the gravitating energy of committed capacity must coincide with the mass already accounted at infinity, with no double counting. This is recorded as an open question shared by the strong-field and cosmological sectors.

Part VI. Strong Fields, Many-Pasts, and Microstructure

The strong-field branch treats the zero-capacity boundary, and Many-Pasts supplies the record-conditioned history ontology in which the substrate's renewal process operates. The separate refresh theorem selects the memoryless dressing used in scale setting, while Section 23 asks for its microscopic dynamics. The same grading applies here: the exact spherical reduction and the operational quantum measure are controlled results, while the boundary microphysics, the arrow of time, and the condensate embedding carry their stated conditional or open grades.

21. Strong-Field Action: Spherical Closure and Its Boundary

The bounded variable remains the natural strong-field order parameter,

$$q = \frac{S_{\text{ent}}}{S_{\infty}} \in [0, 1].$$

Serial composition and weak-field matching select $N^2 = q$ in a static exterior. That relation is meaningful after the asymptotic Killing time has fixed the normalization of the lapse; it is not, by itself, a covariant field equation in a general foliation.

Why the former multiplier action is not retained. An exploratory ADM term $\sqrt{h} \lambda(N^2 - q)$ makes the problem look variational, but if q has no independent bulk dynamics or matter coupling its variation gives $\lambda = 0$. The remaining constraint merely renames the lapse, which is gauge dependent, and supplies neither the capacity Poisson equation nor a new covariant relation. Giving q its own kinetic term would instead add a scalar degree of freedom and reopen the fifth-force, stability, and double-counting problems. The multiplier construction is therefore a diagnostic control case, not the parent action.

Exact spherical reduction. Spherical symmetry supplies the invariant completion that the generic lapse constraint lacks. Write

$$ds^2 = h_{ab}(x)dx^a dx^b + R^2(x)d\Omega^2, \quad a, b \in \{0, 1\}, \quad x^0 = ct.$$

After integrating the Einstein–Hilbert plus GHY action over the two-sphere and removing a two-dimensional total derivative, the bulk action is

$$I_{\text{sph}} = \frac{c^3}{4G} \int d^2x \sqrt{-h} \left[R^2 {}^{(2)}R + 2h^{ab} \partial_a R \partial_b R + 2 \right] + I_{\text{matter}}^{(2)} + I_{\partial}^{(2)}.$$

Its vacuum equations imply conservation of the Misner–Sharp mass

$$M_{\text{MS}} = \frac{c^2 R}{2G} \left(1 - h^{ab} \partial_a R \partial_b R \right), \quad \nabla_a M_{\text{MS}} = 0.$$

The capacity-adapted invariant is therefore

$$q_{\text{geo}} \equiv h^{ab} \partial_a R \partial_b R = 1 - \frac{2GM_{\text{MS}}}{c^2 R}.$$

Thus in the preferred metric-only construction, spherical q is a composite geometric scalar and its vacuum profile is a first integral of the metric equations. It is not an auxiliary field and it is not an additional propagating mode.

For a static asymptotically flat vacuum, $M_{\text{MS}} = M$ and Birkhoff’s theorem gives

$$ds^2 = - \left(1 - \frac{2GM}{c^2 r} \right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r} \right)^{-1} dr^2 + r^2 d\Omega^2,$$

so

$$q_{\text{geo}} = N^2 = 1 - \frac{2GM}{c^2 r}.$$

This is the nonperturbative action-level realization of the bounded capacity relation in the strongest sector where an invariant local definition is currently available.

The $q = 0$ surface. The equation $q_{\text{geo}} = 0$ locates a marginal sphere. A well-posed exterior variational problem is obtained first on a stretched timelike boundary $q = \epsilon > 0$ with the usual GHY term and fixed induced data, and then by taking the null limit with the corresponding null and joint terms. The Einstein action determines that universal gravitational boundary book-keeping. It does not determine a new substrate boundary Hamiltonian, microscopic reflectivity, or the rule that excises $q < 0$.

Accordingly, the restriction

$$\mathcal{M}_q = \{q_{\text{geo}} > 0\}$$

is a physical postulate about the domain of the capacity EFT, not a consequence of varying the Einstein action. The exterior solution and horizon location are closed within spherical metric reduction; the claim that the classical interior is absent, and the dynamics experienced at the saturation surface, remain conditional on a microscopic boundary theory. Appendix F and Appendix N state this boundary between result and interpretation explicitly.

Status of the construction. The Schwarzschild exterior, its standard Hawking temperature, and ordinary exterior perturbation equations follow from the metric parent. In the factorized $j = 3$ completion, the one-bit cut and coherent EPRL area now fix the channel area and Immirzi normalization (Appendix F.5); the older Green-response 1/4 identity remains a separate normalization cross-check. Rotating and charged GR exteriors remain valid solutions of the baseline parent action, but a covariant capacity scalar that identifies their saturation surface has not yet been constructed. The universal strong-field result is therefore spherical and exterior; boundary microphysics, dynamical saturation, nonspherical capacity geometry, and any controlled departure from GR remain open.

22. Many-Pasts: The History-Space Ontology

Many-Pasts makes a conservative operational claim and a distinct ontological claim. Laboratory records obey standard quantum mechanics. The ontology assigns a conditional measure to the decoherent past histories compatible with the one realized present.

The operational weight. For a decoherent family, the joint history weight and its record-conditioned form are

$$p(h, P) = \mathcal{D}(h, h), \quad p(h | P) = \frac{\mathcal{D}(h, h)}{\sum_{h' \in \mathcal{H}_P} \mathcal{D}(h', h')}.$$

Equivalently, one may define $D(h, P) = -\ln p(h, P)$ on the support of the decoherent joint measure, so that $p(h | P) \propto e^{-D(h, P)}$. The exponential is a reparameterization of a normalized quantum probability, not an additional classical measure placed on interfering paths. Projective measurements and general quantum instruments therefore retain their Born probabilities. Appendix G gives the construction and the no-signaling proof.

Branch realization without many worlds. The weight also answers what it means for one outcome to be realized. There is no forward branching into co-real worlds and no collapse event. A definite present is a present with definite macroscopic records, and the histories the weight supports are exactly those compatible with those records; alternative outcomes correspond to alternative present records, not to coexisting branches. Probability is the measure this weighting assigns over the admissible pasts of the one realized present.

Record retention and the resolution of the past. The physical degrees of freedom that distinguish histories are present records. A durable record can be a detector state, an environmental correlation, or the exported history register of the renewal dilation. Such a record keeps distinct those past alternatives that remain distinguishable in the present record algebra. If orthogonal fine records R_i are later merged into a coarser record $\bar{R} = \sum_{i \in I} R_i$, decoherence and additivity give

$$p(h, \bar{R}) = \sum_{i \in I} p(h, R_i), \quad p(h | \bar{R}) = \frac{\sum_{i \in I} p(h, R_i)}{\sum_{h'} \sum_{i \in I} p(h', R_i)}.$$

The history measure therefore loses resolution by ordinary quantum coarse-graining when the present loses a record. The ontology contains no separate archival copy of a distinction that has disappeared from every physical register. This does not erase consequences already carried into the current state. It identifies earlier alternatives with identical surviving records as the same present physical state.

No future pull. The class operators and unitary maps determine the joint measure before conditioning. The factor $p(h | P)$ updates the description of earlier alternatives after P is recorded; it does not enter the Hamiltonian, the channel, or the class operator that produced P . A record that has not yet formed supplies no conditioning event in the realized present. Many-Pasts therefore adds no retrocausal force, final-boundary dynamics, or Born-rule bias. It gives a physical reading to retrodictive conditioning already present in standard quantum mechanics.

Familiar quantum examples. In a double-slit experiment, the alternatives through the two slits remain inside one coarse class operator until a durable which-path record decoheres them. Their cross terms are therefore retained before the record and suppressed after it. In an EPR or Bell experiment, the present is a joint record of the pair and the detectors. The joint measure

gives the usual nonclassical correlations, while the local marginals remain independent of the remote setting. Measurement creates a stable record and thereby identifies the decoherent family on which conditional probabilities can be used. These are standard quantum calculations with a record-conditioned history-space reading.

The arrow of time. The same framework proposes to orient time through conditional typicality. The maximum-caliber replacement process does not solve this problem. At stationarity it obeys detailed balance identically, $p(b)K_*(b, b') = p(b')K_*(b', b) = p(b)p(b')$, and is time-reversal symmetric as a stochastic process. The reversible dilation also has an inverse. Histories with a low-entropy past and increasing future entropy dominate only after a past-boundary condition and a substrate mixing or large-deviation theorem suppress Boltzmann-fluctuation histories. This is not an added law of laboratory probability; it is the open typicality result stated precisely in Appendix G.7.

Bianconi supplies a useful compatibility result, but not that missing theorem. In her low-curvature matter- and radiation-dominated Friedmann approximations, the local geometric-relative-entropy density decreases as the universe dilutes while the integrated entropy grows with the expanding volume and the integrated energy approaches a constant [25]. This demonstrates in a concrete information-geometric action that local ordering and a global entropy increase need not conflict. It does not select a low-entropy boundary, establish substrate mixing, or show that the Many-Pasts conditional measure favors ordinary histories over Boltzmann fluctuations. The process-level arrow therefore remains open exactly where Appendix G.7 places it.

No external cycle ledger. If a proposed cosmological evolution reaches a state whose present record algebra contains no witness of an earlier macroscopic era, the descriptions “first beginning” and “return to the same beginning” are not distinct states within this ontology. Distinguishing them would require a register that survives the record-free interval or an external time parameter that counts passages. Many-Pasts supplies neither. This conditional observation does not derive a cyclic universe, a unique beginning, or a nucleation rate. It removes an otherwise hidden meta-history from the ontology.

Where the weight is used. Many-Pasts supplies the history-space realization in which the electron dressing operates, while faithful full-support resolution proves the independence of successive passes. Appendix H shows that this condition is equivalent to maximum path entropy and uniquely selects the replacement process; the lightest-defect criterion selects the same temporal independence together with independent channel layers. The record-conditioned viewpoint also enters the proposed macroscopic arrow of time and the conditional cosmological extensions. It changes no laboratory law.

Local renewal and reversible history export. Complete local renewal has a precise quantum form. If the new cell must contain no information about the old cell even when the old cell is entangled with a reference, the one-tick channel is uniquely

$$\mathcal{E}_*(\rho) = \rho_* \text{Tr } \rho.$$

This local channel need not destroy information globally. A reversible dilation transfers the old cell into an environment register while a fresh admissible register becomes the new present. Many-Pasts interprets the exported correlations as history degrees of freedom. The identification is structural: Stinespring dilation guarantees an environment, while Postulate III supplies its history-space reading. It does not derive the decoherence functional or show that an indefinitely long history can be stored in finite microscopic resources.

Coherence under renewal. The replacement channel acts only on the renewed closure register. Its Heisenberg dual is

$$\mathcal{E}_*^\dagger(O) = \text{Tr}(\rho_* O)I,$$

so no later observable of that register can recover an input off-diagonal. The global dilation can nevertheless retain coherent information in its complement. If two spatial branches export discarded states $|e_x\rangle$ and $|e_y\rangle$, their reduced off-diagonal is multiplied by $\langle e_y|e_x\rangle$. A cell-addressed mark record makes those states orthogonal and would destroy position coherence in one update. The allowed free vertex instead transports the marked fiber as part of the coherent defect system and leaves the discarded renewal record branch-independent. Ordinary environmental interactions may then reduce the overlap in the usual way. Appendix H.11 gives the proof and distinguishes this condition from the weaker statement that a record merely “moves with” a worldline.

The retained quantum sector. Appendix H.11 now settles the local question. The nine-state fusion complement is the physical marked fiber, so restricting the displayed Lorentzian action to that complement fixes both its moving-frame connection and its nondegenerate 6:5:3 mismatch spectrum. On the real one-mark phase space these operations generate $\mathfrak{su}(9)$, select a unique positive complex structure, and give $\mathcal{H}_{\text{mark}} \simeq \mathbb{C}^9$. The complete quadratic response algebra identifies normalized states exactly up to common phase, yielding $\mathbb{CP}^8$ and its Fubini–Study geometry. The marked tangent metric independently equals one quarter of the Fisher metric obtained from the fresh closure statistics. Caticha’s Hamilton–Killing theorem is therefore a consistency and continuum guide rather than the premise carrying the local reconstruction [26, 27].

Status. The operational measure remains mathematically complete because it imports the standard global decoherence functional and quantum instruments; a substrate derivation of that full functional is still missing. Locally, however, the marked Hilbert and projective geometry and its full finite-dimensional unitary algebra are derived within the selected quadratic action. The projected two-mark bridge is a genuine entangling interaction and generates $\mathfrak{su}(81)$ with the two local algebras; extension to a connected marked network is conditional only on simultaneous retained marks and survival of that bridge under the final host projection. Long-time history storage, continuum/Fock-space quantum fields, electromagnetic dynamics and charge, and the thermodynamic arrow retain their open or conditional grades.

23. Microstructure Hamiltonian and Underlying Dynamics

The UV closure chain now has an explicit finite action on the scale-setting side and a controlled geometric witness at the semiclassical level. Appendix H derives the replacement process, its quantum channel, the state-weighted determinant transfer, and the decorated native-cell vertex used below.

The selected factorized geometric completion is not obtained by identifying the finite capacity alphabet with the magnetic indices of the geometric spin network. The microscopic one-cell space is instead taken to factor as

$$\boxed{\mathcal{H}_{\text{cell}} = \mathcal{H}_{\text{geom}} \otimes \mathcal{H}_{\text{cap}} \otimes \mathbb{C}_{\text{orient}}^2},$$

Here $\mathcal{H}_{\text{geom}}$ carries the gauge-reduced simplicial GFT/spin-foam data, while $\mathcal{H}_{\text{cap}}$ carries the finite routed capacity alphabet and its marks. Appendix Q shows that this factorization removes the Gauss–Casimir contradiction without changing $\Omega = 1680$, the K^2 spectrum, η_* , $g_{\text{share,eff}}$, ζ_* , Z_e , L_* , or G_* . An equivariant relational-frame lock relates the two selected copies of V_3 . Once

the lock exists, Schur's lemma fixes it up to phase. The two factors are bookkeeping components of one substrate, and Postulate I restricts physical configurations to their locked sector.

On the capacity factor, Appendix B proves that positive oriented matching has V_3 as its unique zero-cost transmitted sector and that the displayed homogeneous action has a unique $J = 3$ minimum. Maximal fusion then gives the exact tree-independent canonical blocking ray on the selected maximal-channel branch

$$F_N : V_3^{\otimes N} \rightarrow V_{3N}, \quad F_N |3, \mathbf{n}\rangle^{\otimes N} = |3N, \mathbf{n}\rangle.$$

The microscopic representation stays fixed while blocks enter the large-spin regime along this canonical ray. Several external constructions support the geometric limit. Proper-EPRL amplitudes remove unwanted asymptotic sectors at the vertex level and retain the Regge exponential [99, 100]. Han-type spin-foam models provide a coupled large-spin/refinement witness to the Einstein sector; in the controlled linearized construction, the low-energy excitations exhaust the smooth linearized Einstein solutions and the two graviton helicities [97, 98]. Area-Regge continuum analysis gives the same leading graviton dynamics with a first length-metric correction of order $a^2 C^2$ [101]. Applying the Han-type refinement result to the decorated proper-EPRL factor remains a coupled-embedding audit.

The scalar capacity deficit is a coordinate on the Hamiltonian-constraint response, not an additional radiative gravitational field. Gauge-reduced vacuum propagation remains transverse-traceless. A GFT condensate may still realize the microscopic state and clock [60, 61], but Einstein propagation does not require a separate light scalar condensate mode. Appendix H.10 lifts any geometric rigging map through the finite capacity factor and proves clock-test positivity and the causal boundary value from strong-resolvent convergence. On the selected regular metric-Regge branch, Appendix H.10 proves (H_{TT}) : the Einstein TT infrared limit, together with an exact fixed- $j = 3$ primitive realization whose nonmaximal complement remains uniformly gapped. It also proves that the present capacity premises leave the geometric gluing tensor and measure underdetermined. A new geometric principle must fix those data before (H_{phase}) , dynamical selection of the Einstein branch, becomes a well-posed derived theorem. A spin-3-regularized canonical host obeys the distinct condition (H'_{can}) on its zero fiber; identifying the two physical sectors would require a separate theorem and is not assumed here.

On the defect side, the closure ensemble fixes the stationary marginal p_{η_*} and maximum path entropy selects

$$K_*(b, b') = p_{\eta_*}(b').$$

This fixes the dimensionless history process but cannot produce seconds. The electron, already the theory's single dimensionful anchor and its lightest charged one-bit defect, supplies the clock through the positive transfer spectrum below.

Requiring the output cell to decouple from every reference system forces the replacement channel $\mathcal{E}_*(\rho) = \rho_* \text{Tr } \rho$. The decorated vertex chooses the diagonal maximum-entropy completion

$$\rho_* = \sum_b p_*(b) |b\rangle \langle b|, \quad p_*(b) = Z^{-1} e^{-\eta_* K^2(b)},$$

and prepares it from the amplitude $A_*(b) = \sqrt{p_*(b)}$. A reversible register permutation moves the old cell into history and the fresh amplitude into the present. For the complete 1680-state tetrahedral register this is one native circuit layer. A direct circuit made only from disjoint face comparisons still requires the three perfect matchings of the four-face graph; the decorated action uses the complete tetrahedron as its local gate.

On the factorized seven-channel history space let

$$|v^{(7)}\rangle = \bigotimes_{m=-3}^3 |\sqrt{p_{\eta_*}}\rangle_m, \quad P^{(7)} = |v^{(7)}\rangle\langle v^{(7)}|.$$

On the commutative cell algebra, likelihood multiplication by $p_*(b)$ has the state-weighted determinant

$$\Delta_{\tau_p}(\mathbf{R}) = \exp\left(\sum_b p_b \ln p_b\right) = e^{-g_{\text{share,eff}}}.$$

The seven-channel determinant is $r = e^{-7g_{\text{share,eff}}}$. It is the almost-sure geometric transfer rate of the record-conditioned renewal history; the annealed equality probability would instead involve the collision entropy. Compressing the one-bit charged loop to its determinant line gives the positive scalar survival transfer

$$T_{\text{surv}} = 1 - r.$$

The Euclidean transfer Hamiltonian

$$H_{\text{surv}} = -\frac{\hbar}{\tau_*} \ln T_{\text{surv}}$$

therefore has the exact raw gap

$$E_{\text{raw}} = -\frac{\hbar}{\tau_*} \ln(1 - r).$$

The closure amplitude has an exact Gaussian linearization. Projecting the same amplitude through the two directed scalar returns gives $u = 8\eta_*/49$, and its canonical unitary dilation supplies the factor $(1 - u)^{21/2}$. The nine present/history closure polarizations give the marked weight ζ_* of Section 13.4. The electron graph contributes $Z_e = (1 + \zeta_*)(1 + 7\zeta_*^2)$, so the dressed transverse identification is

$$E_e = \frac{3}{2} Z_e E_{\text{raw}}.$$

Identifying this lowest charged transfer gap with the electron rest energy fixes the update time,

$$\tau_* = -\frac{3}{2} Z_e \frac{\hbar}{m_e c^2} \ln(1 - r).$$

Analytic continuation supplies the phase frequency $E_e/\hbar$. The identification of the same spectral tick with the causal cell scale gives $L_* = c\tau_*$.

The causal length of that spectral update is

$$L_* = c\tau_* = -\frac{3}{2} Z_e \lambda_e \ln(1 - e^{-7g_{\text{share,eff}}}).$$

The baseline rare-event correction $-\ln(1 - r) = r[1 + O(r)]$ differs from r only at order 10^{-23} . The finite marked response changes the scale by $Z_e - 1 = 0.00530828$, which is the physically relevant correction resolved by the microscopic vertex.

Retained temporal information raises the recurrence mass by e^{I_t} , so electron lightness independently selects $I_t = 0$. Fermionic exclusion caps the occupied channel count at seven and subadditivity gives $\Delta_k \geq 0$, hence the lightest resolved one-bit defect has $k = 7$ and $\Delta_7 = 0$ at fixed per-channel marginal. The unit mixing gap, charged survival gap, and geometric spin-2 Hessian remain distinct objects. The decorated transfer action fixes the first two and the marked edge Hessian. In the ordinary geometric branch the source-coupled scalar response must be the nondynamical Hamiltonian-constraint combination; any additional scalar fluctuation of

a particular condensate realization must be gapped, relaxational, gauge/constraint, or source-orthogonal. The physical radiative support is the two TT graviton modes.

The finite scale-setting dynamics is therefore specified end to end: renewal fixes the fresh marginal, the determinant fixes the baseline recurrence, the decorated vertex fixes the marked event and routing, positivity fixes the transfer gap, and the electron fixes the cadence. The complete edge-Hessian and graph-enumeration audits are finite and reproducible. Long-time history capacity and a first-principles derivation of the carrier-resolved metric contact remain open; the ordinary semiclassical Einstein branch is supplied independently by the factorized spin-foam/GFT geometric sector.

The cell-order calculations show that the closure weight, refresh, history tilts, and conserved capacity field do not supply vacuum curvature stiffness. Under the factorized completion this is expected rather than a missing scalar mechanism: vacuum spin-2 stiffness belongs to the geometric factor, while the finite capacity sector supplies the source, records, and coefficient chain. The capacity variables do couple geometry to persistent closure failure, and Section 24 measures that local response. The CDT simulations remain an independent numerical host/interface rather than a derivation of the EPRL refinement limit. Postulate III conditions histories within a specified state and does not select the nonperturbative gravitational vacuum.

Part VII. The Substrate on a Dynamical Lattice

This part tests the cell ensemble numerically on a dynamical simplicial geometry, and then applies the resulting joint measure to the equilibrium vacuum and the cosmological term.

24. Lattice Tests: Compatibility, Defect Response, and Transport

The single-cell coefficient chain of Part II can be evaluated in closed form. Its many-cell consequences require numerical tests: survival of the vacuum ensemble on fluctuating geometry, local strain around a persistent closure failure, and transport toward the $1/r$ deficit of Section 12. The calculation places the unchanged cell ensemble on a dynamical simplicial geometry. Appendix J gives the ensemble definitions, controls, and measured tables.

24.1 The host geometry and the cell identification

The host is a causal-dynamical-triangulations ensemble: four-dimensional triangulations of $S^1 \times S^3$ weighted by the Regge action [79]. CDT is used because it independently sustains an extended four-dimensional de Sitter-like phase [80, 81]. That phase is an external host datum for the substrate test, not a state selected by Many-Pasts.

The identification is immediate, and it respects a separation the theory requires. Each spatial tetrahedron of a slice hosts one cell's boundary data: its four triangles carry the labels $m = -3, \dots, 3$ of the $j_{\text{eff}} = 3$ sector of Section 5, each slice triangle is shared by exactly two cells, and the slice gluing supplies adjacency. Nothing else of the lattice enters. The theory's cell is its label configuration, a dimensionless unit of capacity; no size is ever assigned to it, and the granularity stays in the capacity, as Section 1.2 requires. The lattice simplices are regulator scaffolding, as they are throughout lattice gravity. The identification is nevertheless geometric in the sense that matters: the joint admissibility statistics of the labels depend on how the slice is glued, so the closure weighting can in principle tell one geometry from another, and that channel is the entire coupling between the theory and the host.

Two boundaries of the design are stated at the outset. First, the host supplies a dynamical, curvature-carrying simplicial geometry with the theory's cell structure, and nothing more is

asked of it. The calculation does not assume the CDT ensemble is the theory’s own vacuum, and it withholds any claim that the substrate itself derives four-dimensional emergence. At the tuned host couplings used below, increasing N_{41} from about 40,000 to 100,000 raises both the Hausdorff estimator and the concentration of volume in an extended region, as expected when moving farther into that finite-volume phase; the order of the visible phase boundary and the joint continuum limit remain open (Appendix J.1). Second, the weighting coupled to the host is the paper’s own: the admissibility energy of Sections 5–6 at the closed point, with $\eta = \eta_*$ and injectivity enforced. It is not a spin-foam vertex amplitude. The distinction drawn in Section 29.8 was kept operational: a spin-foam amplitude was coupled first as a neighboring-theory control, and the failures exposed by that exercise supplied the control methodology used below (Appendix J.3). The design’s two-level closure structure — geometric closure exact by construction in the host, label closure soft — was once its most criticized feature; under the factor separation of Appendix Q it is the predicted architecture, with the exact level carried by the geometric/gauge host and the soft level carried by the separate finite capacity factor.

24.2 The coupled ensemble and its controls

The coupled system is the joint Gibbs measure

$$\pi(g, m) \propto \exp\left(-S_{\text{Regge}}(g) - \varepsilon(N_{41} - \bar{N})^2 - \beta \sum_{\text{cells } c} [E_c(m) - \mu]\right),$$

$$E_c(m) = \eta_* K^2(m_c) + \lambda n_{\text{coll}}(m_c),$$

where the sum runs over the slice cells, K^2 is the closure invariant of Appendix B, n_{coll} counts label collisions (injectivity is imposed as a penalty whose hard limit is the constraint, with the residual collision fraction reported so the softness stays visible), ε pins the slice volume, and μ is the per-cell label free energy, computed by thermodynamic integration so that the closure term cannot masquerade as a shift of the bare cosmological coupling. The labels carry a uniform base measure, so at $\beta = 0$ the label entropy cancels exactly and the bare host is recovered identically. The physical weighting fixed by Part II is $\beta = 1$ at η_* : once that chain is accepted there is no coupling dial left free, and the intermediate β values serve only as a diagnostic interpolation.

Every run opens by recomputing the single-cell chain on its own tables and refuses to proceed unless $g_{\text{share,eff}} = 7.4198$ and $\langle K^2 \rangle_{\eta_*} = 3/(2\eta_*) = 50.223$ are reproduced, so the object coupled to the lattice is verifiably the object counted in Part II. Every quoted triangulation is connected, simplicial, and correctly foliated, and comparisons use matched total volume at the same regulator. The principal control shuffles the 210 label-orbit energies while preserving their values and permutation symmetry. This destroys the closure structure without changing the energy histogram or sampling machinery, so only a difference from that shuffled control is attributed to closure. These validity conditions and the interpretation of each possible outcome were recorded before the corresponding data were examined (Appendix J.3).

24.3 What the closure sector cannot supply: vacuum stiffness

Three cell-order calculations bound the closure sector’s contribution to vacuum stiffness. The static closure weight induces only about one percent of the bare geometric coupling. The selected memoryless kernel, modeled as cell-by-cell redraws, has a closed stationary state with vacuum admissibility 0.536 and no detectable curvature action. A separate closure-class history tilt, solved by a Doob transform on a ring, also remains short-ranged and well below the required coupling. These are calculations of specified effective models; they do not derive the refresh kernel from Postulate III.

Two further calculations extend the exclusion beyond the label sector, to the conserved capacity field that Section 24.7 introduces as the carrier of the long-range sector. Even a field the labels

cannot see might rank geometries on its own, because integrating out a conserved Gaussian field induces a purely geometric action: $\frac{1}{2} \ln \det' L(g)$ per channel, the spanning-tree entropy of the slice graph by the matrix-tree theorem. Computed on degree-matched proxies for smooth extended and for crumpled geometry, and on real engine slices, that entropy is nearly universal at fixed coordination: the per-cell differential is a few times 10^{-4} , and with all seven channels the phase-tipping force is of order 10^{-3} against the bare $k_0 \simeq 2.2$. The theory's own non-Gaussianity closes the loophole tighter still. The finite budget of Postulate I gives the field a saturation mass $m^2 = 2/g_{\text{share,eff}} \simeq 0.27$, and the massive determinant suppresses precisely the soft modes that carried the residual sensitivity, shrinking the differential by a further factor of three at the budget mass and toward zero beyond. What the field retains is a local renormalization of the host's couplings, of order 0.08 per cell in the seven-channel count: enough to relocate the host's phase boundaries, and unable to create the phase.

Within the tested models, static weight, refresh, history tilts, and conserved fields do not supply vacuum stiffness. The refresh does couple geometry to the density of closure failure, which the defect experiment measures. The same pass verifies the locality fracture: injectivity-preserving unit shifts split the 1680 states into 48 components of 35 states. This excludes that local realization of the full-entropy kernel; it does not derive the nonlocal physical operator.

24.4 The externally hosted vacuum and the role of conditioning

The exclusions show that the specified substrate does not dynamically select its vacuum geometry. Two questions called emergence of spacetime must therefore be separated. The first is kinematic encoding: the closure invariant K^2 measures the failure of the four oriented faces to close, adjacency carries proximity, and the marked-transfer dictionary of Section 13 converts entanglement increments into meters. The cells are dimensionless capacity units rather than sites of a preferred spatial grid. The second question is dynamical selection among crumpled, branched, and smooth extended geometries. Section 24.3 finds that none of the tested substrate mechanisms performs that ranking.

Many-Pasts cannot fill that dynamical gap. Appendix G normalizes the probabilities of alternative present records before conditioning, so $p(h \mid P)$ describes histories compatible with an already specified present and does not select which P occurs. In the simulations the smooth extended vacuum is supplied by the chosen Regge/CDT host phase as an external background datum. Many-Pasts may condition the compatible decoherent histories within that datum, but it neither ranks candidate geometries nor explains why the extended present is realized. The coefficients of Part II are computed on that host state, and the medium dresses it rather than generating it. A substrate derivation of the prior state or of a probability measure over alternative vacuum geometries remains open.

Regge/CDT is a useful external host because it supplies tetrahedral cells, dynamical curvature, no rigid preferred spatial grid, and a demonstrated extended four-dimensional phase [80, 81]. This choice is operational, but the completed fixed-regulator runs are not a vacuum derivation by the substrate. The host supplies the geometry on which the label theory is tested; the substrate then produces quantitative dressing of its couplings. Integrating out the labels gives $S_{\text{eff}} = S_{\text{host}} - \log Z_{\text{label}}$. Appendix J.6 reports the predicted extensive volume shift and its measured scaling family, including the independently predicted shuffled-control line. The corresponding curvature-sector shifts remain to be tested statistically. The hierarchy is explicit: cell-level non-closure reweights the glued tetrahedra, and integrating out the labels converts that reweighting into corrections to the host's volume- and curvature-sector couplings. This is presently a quantitative interface with an externally supplied vacuum geometry. Section 24.9 states the additional critical-surface test that would promote the interface to a continuum embedding.

24.5 Compatibility: the weighting on dynamical geometry

At $N_{41} \simeq 100,000$, three eighty-slice ensembles were evolved from the same tuned host with respectively zero closure coupling, the physical weighting $\beta = 1$, and $\beta = 1$ after shuffling the closure energies among label orbits (Appendix J.5). Their total-simplex counts agree within 0.6%, and every final triangulation remains connected, simplicial, and correctly foliated. The label sector orders exactly as the closure weight demands: the collision fraction falls from 0.654 at $\beta = 0$ (against the uniform-measure prediction $1 - 840/2401 = 0.650$) to 0.077 under the physical weighting, while the shuffled control stays high at 0.736. The Hausdorff values remain 3.55–3.59 and the volume profiles remain extended across the matched ensembles. Repeating the physical weighting with forty rather than eighty slices gives collision fraction 0.078 and $d_H = 3.74$. Thus the microscopic closure structure is strongly active without destabilizing the host geometry, and the shuffled comparison shows that the ordering follows the closure structure rather than the energy histogram or sampler.

24.6 The defect experiment: geometry responds to the theory’s mass

The theory’s matter is persistent closure failure: committed capacity, maintained against the refresh (Sections 3.2 and 23). The experiment inserts it by hand and repeats the comparison with two independent random seeds. In each repetition, one ensemble holds one hundred well-separated cells in maximal closure failure (all four faces at $m = 0$: six collisions, $K^2 = 48$), while a control ensemble holds the same number in the best-closed injective configuration ($m = \{0, 1, 2, 3\}$, the minimum $K^2 = 40.67$). Anchoring, protection from geometry moves, and every other update rule are identical. Every final triangulation remains connected and correctly foliated, and all one hundred marked cells survive in each ensemble. Subtracting the closed-cell control therefore isolates the physical effect of failure content. The final analysis pools the two seeds and uses across-seed scatter where replicated and integrated-autocorrelation-time-corrected standard errors otherwise.

The capacity response is measured with high statistical significance. At the first shell the mean closure energy is 1.6802 ± 0.0026 around failure pins and 1.7260 ± 0.0008 around closed pins, a difference -0.0458 at 16.8σ . The collision fraction changes from 0.0783 ± 0.0002 to 0.0698 ± 0.0009 , a 9.2σ separation. These label observables largely return by the second shell, as the short closure correlation length requires. The geometric channel appears in a different observable and farther out: at the third shell the mean coordination is lower around failure pins by 0.0398, a 4.0σ separation from the closed-cell control after autocorrelation and seed scatter. Shell-cell counts do not separate significantly at this precision, and the first- and second-shell coordination differences are consistent with zero. The data establish a local geometric response, although several preview signals disappear under the completed error treatment.

Persistent closure failure measurably strains the surrounding capacity state and changes the local coordination field. The closed-cell control carries the same anchoring without the failure, isolating the failure content as the source. This finite-lattice result supplies the microscopic source-to-strain seed used by the weak-field sector. The defect-only experiment reaches three lattice steps and establishes neither continuum curvature nor a value of G . The separately conserved instrument of Section 24.7 tests transport beyond that short closure range.

24.7 Reaching Newtonian range: the conservation requirement

The strain of Section 24.6 dies within a lattice step, as the sub-cell correlation length requires. Macroscopic transport needs a different operator. If free capacity is only a per-cell budget, local maximum-entropy re-equilibration gives $(2z - L)\delta\mu = -m/\chi$ on the slice adjacency ($z = 4$ faces per cell, L the slice Laplacian). That operator has no small-momentum pole; on a real slice

the response falls eight orders of magnitude within thirteen steps. A locally rebalanced budget therefore screens at the substrate scale.

Committed stock and maintenance throughput. Particle formation and persistence are different operations. Let $J_{c,A}^\mu$ carry the committed stock belonging to source A . Its balance law is

$$\nabla_\mu J_{c,A}^\mu = \Gamma_{\text{form},A} - \Gamma_{\text{rel},A}.$$

For an already-existing stable defect both rates vanish, so its mass-equivalent committed stock remains constant. The defect nevertheless draws a maintenance throughput

$$\mathcal{P}_{\text{maint}} = \alpha_{\text{maint}} \rho_{\text{def}}.$$

This is the rate at which the surrounding renewal bandwidth services persistence; it is not continuing conversion into more matter. Spatial free-bandwidth flux enters the defect event, while the reversible update exports the previous replaceable closure state along an internal history edge. The history register is an output of the renewal circuit, not an ordinary spacetime fluid, so no four-current J_{hist}^μ is assumed. In free propagation that output carries no cell address of the persistent mark. This stock–throughput distinction removes the apparent secular growth of a static particle.

Augmented-graph Ward identity. The following result holds for the refresh-bandwidth completion. Form the directed renewal event graph whose edges are spatial bandwidth transport, ordinary temporal continuation, and the history output of a maintenance event. Let B be its incidence matrix and j_e the oriented edge throughput. In the absence of formation, release, or terminal absorption, each local reversible gate has one incoming active slot for every outgoing active or history slot. Its exact Kirchhoff identity is

$$\Delta_t n_v + \sum_e B_{ve} j_e = 0.$$

At a stable defect the history-edge current is $j_v^H = \mathcal{P}_{\text{maint},v}$; it is an outflow from the present register, not an increase of committed stock. Summing over any region cancels every internal edge and leaves only boundary transport and the explicitly counted history outputs.

On a fixed spatial slice, a local linear constitutive law has the quadratic operator

$$L_f = B_s W B_s^T,$$

where B_s is the spatial incidence matrix and W contains positive face conductances. Since $B_s^T \mathbf{1} = 0$,

$$\boxed{L_f \mathbf{1} = 0.}$$

For a connected slice the constant mode is the only exact zero mode. Locality and spatial isotropy then give

$$\lambda_f(k) = D_q k^2 + O(k^4)$$

near $k = 0$; a nonzero constant term would violate the incidence identity and represents leakage. The steady equation on a compact slice is

$$L_f \mu_q = \alpha_{\text{maint}} (\rho_{\text{def}} - \bar{\rho}_{\text{def}}),$$

where the subtraction removes the constant mode. In an extended three-dimensional limit its pseudoinverse has

$$G_f(r) \sim \frac{1}{4\pi D_q r},$$

so a persistent maintenance demand produces the unscreened $1/r$ profile without tuning a scalar mass. Conservation proves the gapless transport pole within this completion. It does not prove that refresh bandwidth is the physical carrier, that defects couple only to it, or that the resulting scalar is the metric capacity coordinate.

The continuum junction condition. Let the renormalized static response be $\delta q = \chi_q \mu_q$, $J_q^i = -D_q \nabla^i \mu_q$, and $\mathcal{P}_{\text{maint}} = \alpha_{\text{maint}} \rho_{\text{def}}$. Away from the compact zero-mode subtraction,

$$\nabla^2 q = \frac{\chi_q \alpha_{\text{maint}}}{D_q} \rho_{\text{def}}.$$

The static bridge $q = 1 + 2\Phi/c^2$ and $\nabla^2 \Phi = 4\pi G_* \rho$ therefore require

$$\boxed{\frac{\chi_q \alpha_{\text{maint}}}{D_q} \longrightarrow \frac{8\pi G_*}{c^2}}$$

after renormalized-operator and lattice-spacing matching. The three quantities on the left are lattice outputs. The electron-derived G_* enters only after their dimensionless regulator dependence has been removed, so the continuum comparison is target-blind rather than a fit.

This transport experiment is performed on a separate forty-slice realization at $N_{41} \simeq 100,000$ (Appendix J.8). The field begins at the vacuum anchor 7.4198, moves only through antisymmetric face fluxes, and is recycled through the compact zero mode; its mean remains 7.4198 through all 4000 sweeps. Absorption reads the persistent excess failure above the measured vacuum dressing, so the source strength is produced by the label dynamics rather than assigned by the transport law. Five fixed source values, $m = 0, 1, 2, 3, 6$, are represented by thirty-two well-separated defects each. The $m = 0$ value measures commitment-independent boundary dressing; the positive values test the source and response laws independently. Within-pin series use Sokal integrated-autocorrelation corrections, and final errors include across-pin and across-level scatter.

Four distinct measurements give a consistent finite-lattice source-to-field chain. First, conservation is exact. Second, after subtracting the $m = 0$ dressing, the emergent maintenance charge is additive,

$$Q - Q(0) = (0.1440 \pm 0.0025)m, \quad Q(0) = 0.0075.$$

Third, the shell-one deficit divided by that independently measured charge is constant across the positive source values,

$$\left. \frac{\Delta f_1 - \Delta f_1(0)}{Q - Q(0)} \right|_{m=1,2,3,6} = 4.9965 \pm 0.5244,$$

where the uncertainty is the larger of propagated error and across-level scatter. Sources of different strength therefore couple through one finite-lattice junction ratio. The dimensionful continuum coefficient $8\pi G_*/c^2$ still requires a continuum scaling and operator-matching calculation. Fourth, the dressing-subtracted profile remains positive through eight shells and fits

$$\Delta f(d) \propto d^{-0.62 \pm 0.11}.$$

The screening diagnostic gives $d \ln(\Delta f d)/dd = +0.1062 \pm 0.0551$, statistically consistent with zero and implying the finite-range bound $\xi > 4.6$ lattice steps. The exact Ward identity and the absence of detected screening therefore agree: the tested carrier is conservation-protected and long-ranged over the usable graph radius.

Two qualifications are visible in the same data. The proportional source fit has a maximum relative residual of 0.242, driven mainly by the $m = 6$ source falling below the extrapolation from

weak sources; this is finite-capacity saturation, not precision linearity at arbitrary multiplicity. The exponent differs from the ideal three-dimensional massless Green-function value -1 ; the closed-slice zero mode and finite slice radius flatten the measured tail, but a larger-volume scaling study must determine whether the exponent tends to -1 . These measurements establish the tested finite-lattice behavior while leaving the continuum scaling and absolute junction normalization open.

24.8 What the finite-lattice results establish

The fixed-lattice results establish four points. The admissibility sector orders on a dynamical host without destabilizing its matched geometry. Persistent closure failure produces a replicated local capacity response and an autocorrelation-corrected local coordination response. Conditional on refresh bandwidth as the carrier, the exact augmented-graph identity protects the long-range pole. The separate transport calculation then verifies conservation, approximately additive source strength, one lattice junction ratio across the tested source values, a power-law tail, and no detected screening. None of these mechanisms selects the vacuum geometry; CDT supplies that host state externally, while the substrate measurably dresses its N_{41} coupling direction. The exact Regge volume projection of that direction is given in Section 25. The remaining CDT questions are qualitatively different: whether the joint system lies in the required extended phase with a massless TT sector, whether it also reaches an optional continuous cutoff-removal trajectory, how its lattice spacing and operators match to continuum observables, and whether the absolute junction coefficient tends to $8\pi G_*/c^2$. Section 26 records those distinct grades.

24.9 The capacity-decorated continuum target

The present runs compare coupled and control ensembles at fixed regulator. Because L_* is a physical microscopic length, existence of the effective theory does not require a second-order point at which the regulator spacing vanishes. The necessary geometric condition is instead membership in an extended semiclassical phase whose long-wavelength transfer matrix contains a massless transverse–traceless sector. A continuous critical surface is a separate, stronger universality and cutoff-removal test. For a causal triangulation T , let $Z_{\text{cap}}[T; \mathbf{g}_{\text{cap}}]$ be the trace over its closure labels, defect marks, renewal gates, source-labelled committed allocations, and history outputs. The joint partition function is

$$Z(\mathbf{g}) = \sum_{T \in \mathfrak{T}_{\text{CDT}}} \frac{1}{C_T} e^{-S_{\text{CDT}}[T; \kappa_0, \Delta, \kappa_4]} Z_{\text{cap}}[T; \mathbf{g}_{\text{cap}}],$$

and integrating out the capacity sector gives

$$S_{\text{eff}}[T] = S_{\text{CDT}}[T] - \ln Z_{\text{cap}}[T].$$

The theory fixes $\mathbf{g}_{\text{cap}}$ at its microscopic values. Phase membership must then be tested at the tuned host couplings with three observables:

1. a Hausdorff estimator tending to four, $d_H(N_{41}) \rightarrow 4$, as the volume grows;
2. a de Sitter-like volume profile, $N_3(t) \propto \cos^3(t/s_0)$ with $s_0 \propto N_4^{1/4}$;
3. a gapless TT transfer mode, $\lambda_{\text{TT}}(k) = 1 - c_{\text{TT}} k^2 a^2 + O(k^4)$ as $k \rightarrow 0$.

The completed volume pair bears only on the trend toward the first condition and on a coarse extended-volume proxy: increasing N_{41} from about 42,000 to 102,000 raises d_H from 3.31 to 3.56 and the blob score from 1.46 to 1.57. At about 100,000 simplices, the three matched ensembles remain connected and correctly foliated and give $d_H = 3.55$ – 3.59 . These results are consistent with movement farther into the finite-volume extended region, but they do not establish $d_H \rightarrow 4$;

no de Sitter-profile scaling fit or TT transfer-spectrum measurement has yet been performed for the decorated ensemble. The third test is decisive: extended-looking volume data with a gapped TT sector would not supply the physical Einstein branch.

Two capacity-register numbers reported elsewhere are deliberately excluded from this phase test. The refresh model's stationary admissibility 0.536 and the history-channel correlation length $\xi_{\text{hist}} \simeq 0.57$ cells characterize the finite register dynamics, not the host's geometric correlation length or graviton spectrum.

The additive vacuum term and the finite continuum volume coupling are distinct coordinates of this approach, but the volume normalization is not an adjustable unknown. Section 25 derives the exact Regge operator

$$\mathcal{V}_4 = a^4(v_{41}N_{41} + v_{32}N_{32})$$

and its corresponding source direction in the (κ_4, Δ) plane. On a fixed-ratio trajectory the scalar matching is $t_4 = \bar{v}_4 a^4 \rho_{\text{vol,R}} + o(a^4)$, with $\bar{v}_4 = (v_{41} + \xi v_{32})/(1 + \xi)$. The continuum calculation must therefore measure the limiting α , ξ , lattice scale, and mixed critical eigenoperator, rather than fit a free Z_V . Vacuum normalization removes the complete capacity-sector bulk coefficient at every fixed regulator. Consequently a finite source-independent capacity density cannot hide in t_4/a^4 : it would already be an extensive term before the limit and would have been included in μ_{cap} . If the common-parent critical limit exists, Postulate I then excludes an additional independent host volume action and gives $\Lambda_{\text{R}}^{\text{empty equilibrium}} = 0$. The currently external CDT host does not satisfy that antecedent by assumption, so its own cosmological coupling remains a regulator input rather than a result of these runs.

If cutoff removal is additionally required, κ_4 can be tuned toward infinite volume and the remaining regulator couplings can be tested for a continuous transition. CDT supplies evidence that candidate continuous lines can occur and can support this stronger construction [87, 88]. Let $\mathfrak{C}_*$ denote the manifold on which the geometric correlation length diverges in lattice units. The distinct symbol keeps this geometric locus separate from the coherence observable $\mathcal{C}$. The cutoff-removal embedding succeeds only if the fixed capacity slice intersects that manifold,

$$\boxed{\mathcal{P}_{\text{cap}} \cap \mathfrak{C}_* \neq \emptyset, \quad \frac{\xi_{\text{geom}}}{a} \longrightarrow \infty.}$$

This requirement introduces no capacity fit. Failure of the fixed slice to intersect $\mathfrak{C}_*$ would reject the cutoff-removal embedding, but would not by itself reject a finite- L_* theory whose extended phase and massless TT sector had already been demonstrated. Geometric criticality removes the ultraviolet regulator. The augmented-current Ward identity has a different job: it protects the infrared transport pole. Neither result supplies the other.

Appendix J.9 states the phase-membership, optional critical-surface, finite-size-scaling, operator-mixing, current, and junction tests in full. A successful capacity-decorated CDT transfer matrix would provide an independent nonperturbative geometric embedding of the finite marked vertex. Until the phase and TT tests are passed, CDT remains the external host used by the simulations; the factorized EPRL/GFT construction of Section 23 and Appendices B, H, and Q supplies the separate controlled semiclassical Einstein route.

25. Equilibrium Vacuum and Cosmological Term

The normalization $q = 1$ defines the unstrained vacuum reference. The regulated joint ensemble makes the corresponding subtraction exact, rather than leaving it as an unspecified cancellation.

At fixed triangulation let

$$Z_{\text{cap}}[T] = \mathbb{E}_{m \sim \text{unif}} \exp \left[-\beta \sum_c E_c(m) \right]$$

and define its homogeneous bulk free energy per cell by

$$\mu_{\text{cap}}(a, \beta) = - \lim_{N_{\text{cell}} \rightarrow \infty} \frac{1}{\beta N_{\text{cell}}} \ln Z_{\text{cap}}^{\text{vac}}(a, N_{\text{cell}}).$$

Its dependence on the homogeneous regulator couplings is suppressed in the notation; the definition is applied along the trajectory whose continuum limit is being tested. The vacuum-normalized capacity factor is

$$\widehat{Z}_{\text{cap}}[T] := e^{\beta \mu_{\text{cap}}(a, \beta) N_{\text{cell}}(T)} Z_{\text{cap}}[T].$$

By construction,

$$\boxed{- \lim_{N_{\text{cell}} \rightarrow \infty} \frac{1}{\beta N_{\text{cell}}} \ln \widehat{Z}_{\text{cap}}^{\text{vac}} = 0.}$$

This is the lattice grand-potential statement. In the thermodynamic limit $F_{\text{cap}} = \mu_{\text{cap}}(a, \beta) N_{\text{cell}} + o(N_{\text{cell}})$, so the centered bulk potential

$$\Omega_{\text{cap}} = F_{\text{cap}} - \mu_{\text{cap}}(a, \beta) N_{\text{cell}}$$

has zero density in the homogeneous reference state. The cancellation is invariant under an arbitrary change of microscopic energy origin. If

$$E_c \mapsto E_c + C, \quad \mu_{\text{cap}} \mapsto \mu_{\text{cap}} + C,$$

then

$$\boxed{\widehat{Z}_{\text{cap}}[T] \mapsto \widehat{Z}_{\text{cap}}[T]}$$

exactly. When the bulk cell count admits the usual thermodynamic reading, Gibbs–Duhem writes the same relation as $\Omega_{\text{cap}}/V = -P$; an isolated equilibrium reference has $P = 0$ [78]. The partition identity above is stronger for the regulated model because it does not require the conditional refresh current to serve as that thermodynamic charge. A large homogeneous source-independent zero-point term therefore cannot reappear as a capacity-sector observable; it cancels against the uniquely shifted bulk free energy, while defect and geometry-dependent differences remain.

Integrating out the normalized capacity sector gives

$$S_{\text{eff}}[T] = S_{\text{CDT}}[T] - \ln \widehat{Z}_{\text{cap}}[T].$$

The homogeneous extensive capacity term is absent from this action. Only the geometry- and defect-dependent free-energy difference remains to source strain, as required by the information–geometry and mass–entropy postulates. This result does not use the conditional refresh-current Ward identity. Transport conservation protects the infrared pole; vacuum normalization follows from the equilibrium partition measure. They are different conserved structures.

Appendix J has already tested the finite-regulator algebra. Omitting the centering restores the term $+\beta \mu_{\text{cap}} N_{\text{cell}} = +(\beta \mu_{\text{cap}}/2) N_{41}$ in the geometric action. Against the quadratic volume pin it predicts

$$\Delta N_{41} = -\frac{\beta \mu_{\text{cap}}}{4\varepsilon},$$

and the measured uncentered-minus-centered displacement is -62.8 against -62.2 . The run does not prove a continuum cosmology, but it verifies the coefficient and sign of the subtracted microscopic bulk term. One bookkeeping refinement matters here. In the standard CDT action basis,

$$S_{\text{CDT}} = -(\kappa_0 + 6\Delta)N_0 + \kappa_4(N_{41} + N_{32}) + \Delta(2N_{41} + N_{32}),$$

the restored N_{41} term is the coupling displacement $\delta\Delta = +\beta\mu_{\text{cap}}/2$, $\delta\kappa_4 = -\beta\mu_{\text{cap}}/2$, not a pure shift of κ_4 . At fixed simplex ratio it contains the expected volume chemical potential; its orthogonal component renormalizes the regulator asymmetry. The measured displacement therefore verifies the predicted N_{41} direction, which is the exact statement licensed by the control.

The same conclusion is independent of the centering convention. The leading exponential growth of the decorated canonical sum defines a critical surface in the full (κ_4, Δ) plane. An extensive change of microscopic energy origin translates the bare coupling vector and that critical surface by the same displacement displayed above. Their normal difference is invariant, and it vanishes at the infinite-volume surface. On a trajectory that holds Δ fixed after the other relevant directions have been projected out, this normal coordinate is the familiar

$$t_4 := \kappa_4 - \kappa_4^c \longrightarrow 0.$$

Thus critical tuning absorbs the additive bulk term without turning an energy-origin choice into an observable. The remaining question is the normalization of the physical volume direction, to which we now turn.

The continuum volume matching can now be made explicit instead of being hidden in an unspecified Z_V . After Wick rotation, the exact Regge four-volumes of the two simplex types are [81]

$$\begin{aligned} V_{41} &= a^4 v_{41}(\alpha), & v_{41}(\alpha) &= \frac{\sqrt{8\alpha - 3}}{96}, \\ V_{32} &= a^4 v_{32}(\alpha), & v_{32}(\alpha) &= \frac{\sqrt{12\alpha - 7}}{96}, \end{aligned}$$

with $\alpha > 7/12$ so both Euclidean simplex types are nondegenerate. Here a is the spatial regulator edge length. In the hosted reading it is scaffolding with no fixed relation to L_* ; under the face-wise lock of Appendix H.9a, paragraph 4a, every spatial face carries the channel area $a_{\text{ch}} = 4 \ln 2 L_*^2$ and the edge is physical, $a^2 = (16 \ln 2 / \sqrt{3}) L_*^2$, $a = 2.530 L_*$ (Appendix J.10). The integrated volume operator is therefore the kinematic identity

$$\boxed{\mathcal{V}_4[T] = a^4 [v_{41}(\alpha)N_{41} + v_{32}(\alpha)N_{32}].}$$

Let $\rho_{\text{vol,R}}$ denote the coefficient of $\mathcal{V}_4$ in the dimensionless Euclidean effective action. If G_{R} is the continuum Newton coefficient and $L_G^2 := \hbar G_{\text{R}} / c^3$, then

$$\rho_{\text{vol,R}} = \frac{c^3 \Lambda_{\text{R}}}{8\pi \hbar G_{\text{R}}} = \frac{\Lambda_{\text{R}}}{8\pi L_G^2}.$$

On the matched weak-field branch $G_{\text{R}} = G_*$ and therefore $L_G = L_*$. Keeping L_G visible until that junction passes prevents the volume matching from assuming the gravitational result it is meant to join. The two action-conjugate volume couplings are therefore

$$\boxed{t_{41} = a^4 v_{41}(\alpha) \rho_{\text{vol,R}}, \quad t_{32} = a^4 v_{32}(\alpha) \rho_{\text{vol,R}}.}$$

Equivalently, in the (κ_4, Δ) basis a pure volume-source displacement obeys

$$\boxed{\begin{pmatrix} \delta\kappa_4 \\ \delta\Delta \end{pmatrix} = a^4 \rho_{\text{vol,R}} \begin{pmatrix} 2v_{32} - v_{41} \\ v_{41} - v_{32} \end{pmatrix}.}$$

Substitution returns $\delta(\kappa_4 + 2\Delta) = a^4 v_{41} \rho_{\text{vol,R}}$ and $\delta(\kappa_4 + \Delta) = a^4 v_{32} \rho_{\text{vol,R}}$, so this is an operator identity, not a fitted matching. At the isotropic point $\alpha = 1$, $v_{41} = v_{32} = \sqrt{5}/96$ and the Δ component vanishes, providing a direct check.

The earlier one-parameter formula is the fixed-ratio compression of this two-operator statement. If $\xi = N_{32}/N_{41}$ and $N_4 = N_{41} + N_{32}$, then

$$\bar{v}_4(\alpha, \xi) = \frac{v_{41}(\alpha) + \xi v_{32}(\alpha)}{1 + \xi}, \quad t_4 = \bar{v}_4(\alpha, \xi) a^4 \rho_{\text{vol,R}} + o(a^4).$$

Thus the geometric part of the previously unnamed factor is fixed:

$$\boxed{Z_V^{\text{geom}}(a) = \bar{v}_4(\alpha(a), \xi(a)).}$$

The critical calculation still has to show that $\alpha(a)$ and $\xi(a)$ approach finite limits, set a by a target-blind observable, and project out any mixed relevant eigenoperator. Those are continuum-existence and scale-setting tests; they are no longer freedom to choose the volume normalization.

This matching does not by itself identify a CDT regulator simplex with the physical capacity cell. In the $x^0 = ct$ convention one native cell has four-volume L_*^4 . Appendix J.10 records that this convention and the fixed face area are in tension by a factor 1.9: a regular spatial cell of edge $2.530L_*$ has three-volume $1.91L_*^3$, so one cell swept through one tick is $1.91L_*^4$, and the event measure must be fixed by one of the two readings. The average number of regulator simplices in such a block is

$$n_*(a) = \frac{L_*^4}{a^4 \bar{v}_4},$$

and its volume coupling is

$$\boxed{t_* := n_*(a) t_4 = \rho_{\text{vol,R}} L_*^4 = \frac{\Lambda_R L_*^4}{8\pi L_G^2} \xrightarrow{G_R=G_*} \frac{\Lambda_R L_*^2}{8\pi}.}$$

The cutoff and the simplex factor cancel. CDT can therefore supply arbitrarily fine scaffolding while L_* remains the finite physical capacity scale, as required by Section 24.1.

There is then no surviving capacity-sector 0/0 ambiguity. The definition of $\mu_{\text{cap}}(a, \beta)$ removes the complete thermodynamic bulk coefficient at every fixed regulator, not merely the part that diverges as $a \rightarrow 0$. If a nonzero finite $\rho_{\text{cap,R}}$ remained, then at any fixed a the normalized vacuum action would contain

$$\rho_{\text{cap,R}} a^4 [v_{41} N_{41} + v_{32} N_{32}],$$

which is still linear in the number of cells and therefore contradicts the defining zero bulk density of $\hat{Z}_{\text{cap}}^{\text{vac}}$. Hence, on every regulator and on every continuum subsequence for which the volume operator has a limit,

$$\boxed{\rho_{\text{cap,R}}^{(\text{eq})} = 0, \quad \Lambda_{\text{cap,R}}^{(\text{eq})} = 0.}$$

The complete equilibrium bulk term must be normalized before the regulator is removed. This order avoids inferring a continuum density from the ratio of two quantities that separately vanish.

What about an independent host cosmological term? In the present simulations it remains possible because CDT is deliberately an external scaffold: $\hat{Z}_{\text{cap}}[T]$ normalizes the capacity sector at fixed T , not the externally supplied host measure. In a completed theory, however, Postulate I says that geometry and capacity are the same substrate, so the vacuum normalization must apply to the bulk coefficient of the *joint* geometry–capacity measure. That joint coefficient is the leading exponential growth removed when the cosmological coupling is placed on its infinite-volume critical surface. Retaining another source-independent host volume action after

this common-parent normalization would split geometry back off from capacity and undo the already-used foundational identification. Writing $\mathfrak{C}_*$ for the common-parent critical manifold defined in Section 24.9, the continuum statement is consequently the conditional theorem

$$\boxed{(\mathcal{P}_{\text{cap}} \cap \mathfrak{C}_* \neq \emptyset) \text{ and Postulate I} \implies \Lambda_{\text{R}}^{\text{empty equilibrium}} = 0.}$$

This adds no new premise. It applies Postulate I to the exact vacuum normalization once a common-parent continuum limit exists. The existence of that decorated critical limit remains to be demonstrated empirically and mathematically, so the currently externally hosted lattice calculation does not yet license an unconditional claim about the observed Universe. Nor does equilibrium cancellation imply a matter-tracking residual, a relaxation law, evolving dark energy, or $w \neq -1$. Any observed homogeneous acceleration must come from a state-dependent, boundary, or nonequilibrium sector with covariantly conserved stress energy, or else falsify the common-substrate equilibrium completion; its equation of state must be derived separately.

A controlled nonequilibrium target. The external results also identify the correct next object more tightly than the phrase “state-dependent energy” alone. Let $C_0 > 0$ be the covariance of a regulated Gaussian capacity block in the centered equilibrium state and $C > 0$ the covariance of another state on the same support. The dimensionless relative covariance is

$$\mathcal{G} = C_0^{-1/2} C C_0^{-1/2}.$$

The Kullback–Leibler divergence between the corresponding centered Gaussian measures is the basis-independent identity

$$\boxed{\Gamma_{\text{rel}}(C \| C_0) = D_{\text{KL}}(\mathcal{N}(0, C) \| \mathcal{N}(0, C_0)) = \frac{1}{2} \text{Tr} [\mathcal{G} - I - \ln \mathcal{G}].}$$

If $\lambda_i > 0$ are the eigenvalues of $\mathcal{G}$, each contribution $\lambda_i - 1 - \ln \lambda_i$ is nonnegative and vanishes only at $\lambda_i = 1$. Thus

$$\Gamma_{\text{rel}} \geq 0, \quad \Gamma_{\text{rel}} = 0 \iff C = C_0,$$

and for $\mathcal{G} = I + X$,

$$\Gamma_{\text{rel}} = \frac{1}{4} \text{Tr} X^2 + O(X^3).$$

Appendix H.10 derives this functional rather than merely recognizing it: at finite regulator it is the quadratic-source Legendre transform of the equilibrium-normalized determinant of any positive dressed/reference Gaussian Hessian pair on a common physical support. The trace-log and positive mismatch forms are therefore closed as functional identities. The substrate must still derive the physical operator, its support and trace multiplicity, its dynamics and dimensions, and the conserved covariant response used in cosmology. This introduces no new premise or fitted potential: it is the exact dimensionless relative-information functional of any positive Gaussian fluctuation sector. Multiplication by the appropriate thermodynamic scale turns it into a relative free energy. Bianconi independently promotes the same operator-convex form to a covariant geometric-relative-entropy theory,

$$\Lambda_G = \frac{1}{2\beta} \text{Tr} [G - I - \ln G],$$

and obtains a low-energy Einstein limit and a local thermodynamic first law [25]. The result therefore supplies a well-motivated target class for the homogeneous influence functional: equilibrium has zero relative energy, while a distinguishable state can carry a positive mismatch energy without restoring a source-independent vacuum constant.

The missing cosmological action must still derive the relevant covariance or response operator, its normalization and dimensions, its time evolution, and a conserved metric stress tensor. Those results must determine $w(z)$. Bianconi’s G -field has its own derivative terms in the modified field equations, so it cannot be identified with q , varied as a new scalar, or imported wholesale without contradicting the metric-only decision of Section 11. Relative information nevertheless supplies a consistent local action, energy, and thermodynamic language for departures from the maximum-information vacuum. The microscopic theory here must still determine which departure the Universe occupies.

Appendix O develops the conditional dark-energy completion this leaves open. The primitive event count of the marked vertex supplies a fixed spacetime measure, so the continuum variation is unimodular and the cosmological term enters as an integration constant with $w = -1$ exactly, not as a local vacuum-energy coupling; constant shifts of the matter Lagrangian shift the constant without independently curving spacetime. The equilibrium normalization above supplies the initial condition: the centering removes every extensive bulk term at each regulator, its volume conjugate is the initial constant, and $\Lambda_i = 0$ follows conditionally on the volume-conjugacy lemma and the common continuum gate (Appendix O.5).

The source of later growth is collective loss of a baryonic source-clock carrier. Identity-free committed capacity cannot record stream assignment. When gas from several progenitors becomes irreversibly mixed in one retained bound object, matter no longer resolves progenitor membership and the still-labelled committed allocation is orphaned. Faithful renewal fixes complete rerouting at that event, and Appendix O.13 supplies the coefficient-free nine-state dilation. The astrophysical variable is $\epsilon_{\text{mix}} = 1 - I(A; Y)/H(A)$, measured from Lagrangian progenitor tags. A cosmological zoom must fix the progenitor geometry and K_{mix} independently before the final likelihood is evaluated. Refresh-first K^2 independence still assumes the post-refresh ledger. Once the release history is fixed, $w = -1$ between events and approaches it from below during active release.

Part VIII. Closure Status, Falsifiability, and Comparisons

26. Closure-Status Table

The closure bookkeeping is concentrated here in one place; the rest of the text states results and refers here for their status.

The leading word in each status uses a fixed vocabulary. *Closed* means derived within the stated postulates and ensemble. *Fixed* means no phenomenological freedom remains once the named branch is adopted. *Conditional* means the result follows if a named reading or completion holds. *Frontier* or *open* means the piece is structured but incomplete. *Empirical support* denotes comparison with data rather than derivation, and *audit task* marks an independent check still required. A conditional premise does not demote every theorem downstream of it. Rows therefore separate exact identities and within-model lemmas from the microscopic or empirical status of their premises.

Quantity / Claim	Sector	Status	Type of Support	Where Established
$\Omega_{\text{tet}}, g_{\text{share,max}}$	UV counting	Closed	exact combinatorics	Part II, App. B
η_*	admissibility closure	Closed	stationary normalized closure evidence $\ln Z + \frac{3}{2} \ln \eta$ maximized on the exact K^2 spectrum; the 3/2 is the determinant weight of the three closure-defect components	Part II, App. B
$g_{\text{share,eff}}$	UV entropy	Closed	exact weighted evaluation	Part II, App. B

Quantity / Claim	Sector	Status	Type of Support	Where Established
Memoryless refresh kernel	information / scale theorem	Closed under the foundational faithful full-support condition	applying that same condition to histories gives maximum path entropy, not a new premise: $H(B_{t+1} B_t) = g_{\text{share,eff}} - I(B_t; B_{t+1})$ is maximal iff $I = 0$, uniquely giving $K(b, b') = p_{\eta_*}(b')$	Part I, App. D, G–H
Renewal / retained marked quantum sector	quantum-channel interface	Exact channel, overlap, and local Hilbert reconstruction; network extension conditional	$\mathcal{E}_*^\dagger(O) = \text{Tr}(\rho_* O)I$ makes the renewed output coherence-blind; the retained Q fiber inherits the primitive quadratic action, generating $\mathfrak{su}(9)$, a unique positive complex structure, and $\mathbb{C}^9/\mathbb{C}\mathbb{P}^8$; the projected two-mark bridge gives $\mathfrak{su}(81)$ before the final host projection	§3.3, §22, App. H.11, P.6, Q.9
Decorated charged marked-transfer vertex	UV dynamics	Closed finite transfer and selected-branch Einstein TT theorem; geometric host underdetermined	Gaussian closure amplitude fixes $\sqrt{\eta_*}$; the marked transfer, routing, edge Hessian, and finite graph audits are closed inside the displayed action; Appendix B adds exact $J = 3$ phase selection and a canonical maximal-channel blocking ray; the selected metric-Regge branch has the massless TT infrared limit, while the underdetermination theorem proves that the capacity premises do not fix $\mathcal{G}_v$ or its measure; the empty-capacity rigging lift is exact; under the face-wise lock of H.9a the host is the equilateral CDT class with predicted $\kappa_0 = 2 \ln 2$ (App. J.10)	§13, §23, App. B, H, Q
Gaussian GFT-geometric-relative-entropy bridge	UV / continuum interface	Exact functional and factorized capacity-rigging theorems; selected-branch TT theorem closed	the normalized determinant and $\frac{1}{2} \text{Tr}'(\mathcal{G} - I - \ln \mathcal{G})$ are exact; the equilibrium capacity transfer has one stationary direction; any positive geometric rigging map lifts through Π_* , and the clock-test densities and causal boundary value follow from strong-resolvent convergence; (H_{TT}) is closed, unique determination of $\mathcal{G}_v$ is excluded, and a derived (H_{phase}) awaits a fixed geometric measure	§23, §25, App. B, H.10, Q
Maximal-fusion / spin-foam refinement	geometric continuum	Exact canonical maximal-channel map and selected-branch TT theorem; geometric measure not derived	$V_3^{\otimes N} \rightarrow V_{3N}$ is tree-independent on the maximal channel and maps coherent products exactly; the locked link has an exact $T^*SU(2)/\mathbb{Z}_3$ coherent phase space; the metric-Regge symbol and perfect-action pullback prove (H_{TT}) , while the host-underdetermination theorem excludes deriving a unique $\mathcal{G}_v$ from the capacity premises; the face-wise lock is the candidate principle and yields a predicted native point whose phase is a registered test (J.10)	§23, App. B, H.9a–H.10
Substrate length L_* (marked-transfer map)	scale setting	Fixed within the decorated transfer action and electron anchor	state-weighted determinant gives $r = e^{-7g_{\text{share,eff}}}$; the marked vertex gives $Z_e = (1 + \zeta_*)(1 + 7\zeta_*^2)$; positivity and the electron anchor give $L_* = -(3/2)Z_e \lambda_e \ln(1 - r)$	Part I, App. D, H
$J_{\text{bare}}, J_{\text{eff}}^{\text{tree}}$	UV edge kernel	Closed	tetrahedral isotropy identity	Part II, App. C
$\Sigma_{\text{ret}} = 65/9$	finite-loop UV	Conditional minimal return-sector completion	seven diagonal return channels plus one projected singlet are specified; an explicit microscopic return operator must derive their relative weights and exclude further motifs	Part II, App. C
$J_{\text{eff}}^{(\text{ren})}$	finite-loop UV	Conditional on the minimal return operator	algebraic Dyson resummation once Σ_{ret} is specified	Part II, App. C

Quantity / Claim	Sector	Status	Type of Support	Where Established
γ	continuum stiffness	Conditional loop-dressed value	Euclidean normalization is exact for the one-sublattice edge convention; the numerical value inherits the minimal return operator	Part II, App. C
Green-matched source projection	UV source map	Closed in the canonical weak-field branch	exact defect counting + tetrahedral on-site Green function	App. C
κ/γ	source-to-stiffness ratio	Closed in the canonical weak-field branch	$\sigma_{\text{def}} = \rho/\kappa_m(L_*)$ plus $G_{\text{tet}}(0)$ and tetrahedral 4/3 projection	Part III, App. C–D
Weak-field action / bridge law	EFT / gravity	Closed for the ordinary longitudinal branch	Einstein–Hilbert + GHY reduced in Newtonian gauge gives I_{Newton} ; $\delta S = -2S_\infty \Phi/c^2$ gives $I_{\text{cap}}^{\text{static}} = Z_S I_{\text{Newton}}$ with no extra scalar	Part III, App. D, N
G_*	gravitational scale	Parameter-free output of the decorated scale branch; historical postdiction	$G_* = (9/4)(\hbar c/m_e^2)Z_e^2 \ln^2(1 - e^{-7g_{\text{share,eff}}}) = 6.6742890772 \times 10^{-11}$, or -0.073σ relative to CODATA; the residual was known before the vertex was constructed	Part I, App. D, H, L
Matched G	weak-field gravity	Algebraic identity in the chosen normalization; not an independent determination	substituting the closed source map and S_∞^{cell} into $G = c^2\kappa/(8\pi\gamma S_\infty)$ gives identically $G = c^3 L_*^2/\hbar = G_*$	Part III, App. C–D
Electron anchor	mass / length sector	Fixed empirical elementary anchor	one-bit fermionic defect supplies λ_e for length setting and $m_e/\ln 2$ for mass–entropy map	Part III, App. D
Seven-sector additivity and lightest branch	UV scale theorem	Closed entropy theorem; exact conditional mass minimum	subadditivity gives $H_k = kg_{\text{share,eff}} - \Delta_k$ with $\Delta_k \geq 0$; at fixed admissibility marginals, the adopted recurrence mass $m_k \propto e^{\Delta_k - kg_{\text{share,eff}}}$ is minimized uniquely at the fermionic ceiling $k = 7$ and $\Delta_7 = 0$	App. D, H
a_0	galactic EFT	Exact conditional consequence; loading and horizon coupling open	the compact transverse doublet supplies phase volume $(2\pi)^2$; $a_0 = [g_{\text{share,eff}}/(2\pi)^2]cH_0$ follows after loading one sharing entropy into a phase cell and coupling it reversibly to the horizon bath	Part IV, App. N
Carrier nesting	galactic source map	Closed within the specified leading EFT; microscopic projector open	active carriers form an antichain and $\Pi_P X = \chi_P \mathbb{E}[X \mathcal{R}_P]$ obeys the tower rule; descendants remain in the full Einstein source but do not receive separate transverse halos	Part IV, App. N
RAR law and static action	galactic EFT	Closed within a conditional leading completion	$H_{\text{cell}} = k_B T_H x : N$ gives $\partial F_{\text{th}}/\partial E = n_B(x)$; the QUMOND-type auxiliary action gives the conservative nonspherical field equation and the spherical exponential RAR; $E/(k_B T_H) = x$ and the cell Hamiltonian await a microscopic derivation	Part IV, App. N
Baseline no slip / lensing	weak-field metric	Closed for the ordinary longitudinal branch	varying the unreduced Einstein scalar constraint gives $\nabla^2(\Phi - \Psi) = 0$ and hence $\Phi = \Psi$ under asymptotic flatness	Part III, App. D, N
Transverse metric and lensing	galactic metric response	Closed within the specified quasistatic contact; nonlinear covariance open	$h_{\mu\nu}^\perp = -2\phi_\perp(\bar{g}_{\mu\nu} + 2u_\mu u_\nu)/c^2$ gives no slip, one dynamical/lensing density, and no extra vacuum graviton pole; covariant variation of the record projector remains to be derived	§16, App. N
Solar-System and wide-binary limit	local gravity	Closed at leading quasistatic order in the carrier-resolved EFT; nonlinear preferred-frame audit open	the Milky Way is the active carrier, so localized Solar and binary sources remain purely Einstein; $Q_2^{\odot,\perp} = 0$, wide binaries are Newtonian apart from the Galactic tide, and the vacuum tensor kinetic term is unchanged	Part III–IV, App. F, N

Quantity / Claim	Sector	Status	Type of Support	Where Established
Telegrapher relation $D/\tau_0 = c^2$	transport	Closed in canonical transport branch	causal closure	Part V, App. E
Canonical $\tau_0^{-1} = H_0$ branch	transport	Fixed in the minimal transport closure	no-new-IR-scale choice	Part V, App. E
Hubble-tension mechanism	cosmology	Structurally supported extension	homogeneous trace-coupled mode	Part V, App. E
Equilibrium capacity-vacuum subtraction	cosmological constant / lattice dynamics	Exact regulator identity; conditional continuum theorem	the vacuum-normalized factor $\hat{Z}_{\text{cap}} = e^{\beta\mu_{\text{cap}}N_{\text{cell}}} Z_{\text{cap}}$ removes the complete capacity-sector bulk coefficient at every regulator and is energy-origin invariant; the uncentered control verifies the predicted N_{41} coupling displacement; the exact two-simplex Regge volume fixes $Z_V^{\text{geom}} = \bar{v}_4$ and, on the matched $G_R = G_*$ branch, gives $t_* = \Lambda_R L_*^2/(8\pi)$ without setting $a = L_*$; if the decorated common-parent critical limit exists, Postulate I excludes an independent host volume term and implies $\Lambda_R^{\text{empty equilibrium}} = 0$; continuum existence and dark-energy dynamics remain open	§25, §24.9, App. J.2, J.6, J.9
Cosmological term from fixed measure and collective source-clock release	dark energy	Conditional continuum and astrophysical branch; zero-mode and renewal lemmas exact within their stated premises	fixed primitive event measure gives a unique volume singlet; only the homogeneous marked drain changes Λ , while $\delta\rho_\Lambda(k \neq 0) = 0$; carrier loss creates no new material component and the metric-visible remainder stays in source-associated capacity; the absorbing first-entry F_Z and three-dimensional passive-tracer surrogate are executable, while the cosmological zoom, post-refresh ledger, and full likelihood remain open	§20, §25, App. O, P.7, Q.9a
Gauge compatibility of the capacity spine	UV consistency	Same-factor Gauss reading excluded; separate-factor covariance closed kinematically	the certified floor $K^2 \geq 122/3 > 0$ excludes a Gauss-Casimir reading; the microscopic completion uses $\mathcal{H}_{\text{geom}} \otimes \mathcal{H}_{\text{cap}} \otimes \mathbb{C}_{\text{orient}}^2$, so every capacity operator commutes strongly with the geometric gauge action; an equivariant V_3 frame lock preserves global rotational covariance without changing 1680, the K^2 spectrum, or any downstream coefficient	§6, §23, App. Q
Orientation doublet	UV geometry / matter interface	Persistent orientation excluded by renewal; pure-gravity decoupling closed; fermion audit open	a persistent lift would retain at least $\ln 2$ of mutual information and is therefore incompatible with full-support renewal; the two $\mu = +1$ lifts share the metric and Regge phase, so the proper-gravity vertex acts as the identity on the refreshed doublet; the matter completion must still verify tetrad-sign invariance of the Dirac and parity-odd fermion couplings	§23, App. H.9, Q

Quantity / Claim	Sector	Status	Type of Support	Where Established
Record dynamics, organization, and computation	history sector	Exact algebraic core plus named conditional interpretations	renewal archives exactly; the canonical defect's source cost and record-discrimination rate are one likelihood ratio; weak fixed-marginal correlations give $\mathcal{I}_H \propto D_+^2$ while the growing mode rises; the primitive face interaction generates $\mathfrak{su}(16)$ and composes to $\mathfrak{su}(4^N)$ on connected networks; identity-free capacity cannot record stream assignment; collective baryonic progenitor mixing supplies the conditional carrier-loss history, and the one-signed drain also inherits O.13's ledger and orientation premises	App. P, O.10–O.13
Saturated-phase committed capacity	cosmology	Conditional transition; zero-pressure constraint theorem closed within the pinned coherent branch	source-labelled saturation is $\sigma_A = 1$ while absolute availability remains nonzero; one coherent phase advancing per electron-calibrated tick gives $X = 1/2$ rather than adding it separately; the constraint action then gives $p = 0$, $c_s^2 = 0$, and $\rho \propto a^{-3}$; this is a capacity stress form, not a particle ontology; recruitment and coherence onset remain open	Part V, §20, App. M
Committed-capacity abundance $\Omega_c/\Omega_b = 1/\epsilon$	cosmology	Conditional on the transverse normalization, pinned reading, and per-defect bookkeeping	$\sigma_A = \epsilon M_{c,A}/M_A$ is defined on each Lagrangian source label; over-demanded sources are assumed to reach $\sigma_A = 1$ and allocations add, giving $\rho_c = \rho_b/\epsilon$; the recruitment dynamics, epoch, and joint Boltzmann evolution remain open	Part V, §20
$a_0(\Omega_c/\Omega_b) = cH_0$	cross-sector structure	Exact conditional identity independent of the cell value	multiplication cancels ϵ ; current central values give $a_0(\Omega_c/\Omega_b)/(cH_0) = 0.983 \pm 0.022$; the acceleration loading and committed-abundance premises remain separate	Part IV, Part V, §20
Post-fixation tests of ϵ	empirical support	Supported across heterogeneous tests; no combined significance assigned	a_0 is +2.6% from the local RAR scale; Ω_c/Ω_b agrees at 0.7σ ; all twenty-four X-COP inversions obey the capacity ceiling; two redshift samples report evolution of a_0 in the predicted direction; these comparisons share theory premises and are not statistically independent	§15, §18, §20, §27.3
Collective source-clock carrier loss / conditional conservation	cosmology	Exact no-stream, renewal, and volume-projection lemmas within stated premises; 3-D tracer surrogate completed; cosmological kernel open	shell crossing and phase drift do not release identity-free committed capacity; irreversible loss of baryonic progenitor information triggers complete renewal; the unique volume singlet receives the homogeneous η_* drain, while all $k \neq 0$ release transfer is internal to source-associated capacity and $\delta\Gamma$ cancels from its leading metric perturbation; the cosmological zoom must still fix $K_{\text{mix}}(\tau; M, z)$ before the final likelihood	Part V, §20, App. O.7–O.14
Diffuse source projection $\epsilon = g_{\text{share,eff}}/4\pi^2$	cluster sector	Conditional, with no additional coefficient	inherits the compact two-phase transverse normalization used for a_0/a_H ; the cluster extension adds no further dial	Part V, §18
Hot-atmosphere factor B_{bath}	cluster sector	Operationally closed; microscopic origin open	$M_{\text{hot,vir}}/(f_{b,\text{cos}} M_{500})$ from X-ray/SZ; no per-system fitted knob	Part V, §18

Quantity / Claim	Sector	Status	Type of Support	Where Established
Cluster residual $\mathcal{R}_{\text{rel}} = 1 + (W_{\text{bath}} - 1)f_{\text{cont}}$	cluster sector	Hook morphology and trend direction supported; linear lift candidate excluded; lift function open	linear candidate's ceiling 1.81 lies below measured peak residual 3–5 [4, 16, 17]; the total-source cap is $W_{\text{bath}}^{\text{max}} = 1 + 1/\epsilon = 6.32$, giving $\mathcal{R}_{\text{cap}} = 1 + f_{\text{cont}}/\epsilon \simeq 5.7\text{--}5.9$ for developed baths; all twenty-four X-COP source-weight inversions respect the cap; coherence-growth profile open	Part V, §18
Relaxed vs. merger source expressions	cluster sector	Conditional — two regimes, one coefficient	residual rides on virialized bath; Bullet on shocked gas + decoupled clumps; regime boundary not yet derived	Part V, §18
Channel-selection rule (suppression + collective lift)	cluster sector	Conditional — suppression open; linear lift excluded	transverse-suppression premise (participation $\Rightarrow$ weight ϵ) not yet derived from the source map; linear form of the lift excluded by the measured peak residual; the excess cap $1/\epsilon$ plus the ordinary unit response fixes the total endpoint at $1 + 1/\epsilon$; coherence-growth profile between the endpoints not yet derived	Part V, §18
Bullet gas/lensing inversion	cluster sector	Structurally supported; consistent but untested (ϵ not yet measured)	ϵ brings gas/galaxy to near-parity, compactness completes the inversion; existing flexible reconstructions non-discriminating because gas weight is halo-degenerate	Part V, §18
Resolved cluster lensing-map test	cluster sector	Open — principal empirical task	three-component $\kappa \propto (1 + (1 - \epsilon)B_{\text{bath}})\Sigma_{\text{bath}} + \epsilon\Sigma_{\text{shock}} + \Sigma_{\text{dec}}$; predicted best-fit $\epsilon \simeq 0.19$, floated on baryonic maps with free dark haloes disallowed; not yet performed	Part V, §18
Bounded capacity q , $N^2 = q$	strong field	Fixed static constitutive rule; covariant generalization open	multiplicative composition and weak-field matching fix the static lapse map; a generic lapse is foliation dependent	Part VI, App. F, N
Former ADM multiplier action	strong field	Excluded as a parent completion	with no independent q dynamics, variation gives $\lambda = 0$ and only renames the foliation-dependent lapse; adding dynamics introduces an extra mode	Part VI, App. F, N
Spherical parent reduction	strong field	Closed in the metric-only branch	exact two-dimensional reduction gives $q_{\text{geo}} = (\nabla R)^2 = 1 - 2GM_{\text{MS}}/(c^2 R)$ as a composite first integral and returns Schwarzschild in static vacuum	Part VI, App. F, N
Capacity-exhaustion horizon $q = 0$	strong field	Geometric zero closed in spherical symmetry; domain termination conditional	$q_{\text{geo}} = 0$ is the marginal sphere; identifying it with substrate saturation and excluding $q_{\text{geo}} < 0$ requires the bounded-domain postulate and boundary theory	Part VI, App. F, N
Hawking temperature and exterior ringdown	strong field	GR-matching branch closed; capacity boundary condition open	Schwarzschild exterior fixes the wave operator and Euclidean temperature; $\mathcal{R} = 0$ follows if future-horizon regularity is retained, while $\Gamma_{\partial q}$ must determine the physical substrate reflectivity	App. F
Bekenstein–Hawking / channel-area bridge	strong field	Channel-area normalization closed within factorized $j = 3$ embedding	a cut carries $\ln 2$ of capacity entropy while the geometric boundary puncture remains $j = 3$; coherent/maximal-fusion area additivity gives $Z_A(N) = \sqrt{3N/(3N+1)}$, and matching $N \ln 2 = A_N/(4L_*^2)$ fixes $\gamma_{\text{BI}} = \ln 2/(6\pi)$ and $a_{\text{ch}} = 4 \ln 2 L_*^2$; $n_{\text{hor}} S_{\infty}^{\text{cell}} = 1/4$ remains an independent response identity	App. B, C, F, Q
Horizon formation and boundary microphysics	strong field	Geometric marginal-surface formation closed; saturation dynamics open	GR collapse can form $q_{\text{geo}} = 0$; a bounded causal q_{cap} evolution, its equality to q_{geo} , the boundary action, and relaxation spectrum remain to be derived	Part VI, App. F

Quantity / Claim	Sector	Status	Type of Support	Where Established
Rotating / charged stationary exteriors	strong field	Baseline metric solutions closed; capacity map open	Einstein / Einstein–Maxwell gives Kerr / Reissner–Nordström / Kerr–Newman, but no nonspherical covariant capacity scalar or domain rule has yet been derived	App. F
Many-Pasts Born compatibility	quantum foundations	Local marked kinematics derived; global measure conditional on stated record hypotheses	H.11 derives $\mathbb{C}^9/\mathbb{CP}^8$ and the full local projector algebra; Gleason then fixes every normalized noncontextual additive measure on that record-projector lattice; the global state, record completeness, continuum/Fock composition, and substrate origin of the decoherence functional remain premises	§3.3, §22, App. G, H.11
No-signaling in operational branch	quantum foundations	Exact within the operational branch	summing a remote quantum instrument gives a trace-preserving map, leaving the local marginal independent of the remote setting	Part VI, App. G
Record-retention reading of Many-Pasts	quantum foundations	Closed as an ontological consequence of Postulate III and the imported record measure	the realized present includes its physical records; erasing fine record distinctions pushes the joint measure forward by ordinary coarse-graining; $p(h P)$ is formed after the joint dynamics and supplies no future-to-past force	§3.3, §22, App. G.6
Arrow-of-time account	quantum foundations	Open conditional extension	requires a Substrate Past Hypothesis, elapsed time below relaxation and recurrence, and a mixing/large-deviation theorem showing ordinary entropy-increasing histories dominate the conditioned ensemble	Part VI, App. G
Microstructure Hamiltonian	UV realization	Finite marked-transfer action and capacity rigging lift closed; selected-branch Einstein theorem closed; host selection underdetermined	the native capacity vertex and marked routing are finite and audited; the selected V_3 phase has an exact maximal-fusion map, locked coherent phase space, and, conditional on the factorized realization, a unique equivariant frame lock; (H_{TT}) and an exact primitive realization are closed, while a unique geometric gluing tensor is excluded as a consequence of the present capacity premises; a face-wise reading of the lock fixes the host class and bare couplings (H.9a, J.10)	Part VI, §23, App. B, H, Q
Charged-lepton spectrum	particle-sector extension	Fixed within the decorated marked-transfer and shell action; no fitted lepton coefficient	closure-spectrum collapse gives three shells; the baseline shell algebra is dressed by $Z_\mu = 1 + \zeta_*$ and $Z_{\tau,2} = 1 + (2/7)\zeta_*$, giving $m_\mu/m_e = 206.768280237$ and $m_\tau/m_e = 3477.343310$, both within 1σ	§13, App. H–I
Abelian / weak gauge hosting	gauge sector	Coherent extension; Standard Model identification external	baseline redundancy supplies the Abelian gauge template; primitive spin data carry an $SU(2)$ action, while weak chirality, doublets, and hypercharge assignments are not derived	App. I.2
Persistent open-route color algebra	gauge sector	Conditional derivation from one open-route premise	a reference-preserving open three-route record is a reversible qutrit channel; quotienting the channel phase gives $PU(3)$ with $\mathfrak{su}(3)$ Lie algebra and eight traceless endpoint operators; phase-invariant channel-return fidelity gives the adjoint Wilson form	App. I.2
Primitive color transfer $t_8 = 13/14$	gauge sector	Fixed within the minimal identity-kernel completion	primitive $j_0 = 3/2$ recoupling gives $G = (9/10)P_1 + (21/20)P_2$; maximum faithful throughput and route-frame isotropy give $\mathcal{E} = \mathbf{1}_1 \oplus (13/14)\mathbf{I}_8$	App. H.9, I.2

Closure-status table continued

Quantity / Claim	Sector	Status	Type of Support	Where Established
Diamond causal color regulator	gauge sector	Conditional on the minimal product causal stack and equal primitive heat times	four-link mixed loops, six-link spatial loops, the embedded diamond volume, and electric-magnetic isotropy give $a_t/d = 4\sqrt{2}/3$ and $g_{\text{graph,HK}}^2 = 2\sqrt{6} g_{\text{HK,step}}^2 = 0.24203562353$; graph-to-continuum matching remains open	App. I.2
Physical QCD matter sector	gauge / particle sector	Open and outside the present premises	the global $SU(3)$ endpoint lift and central $U(1)$ identification, triality-carrying quark defects, two weak species per generation, hypercharge, a lattice fermion action, finite scheme matching, confinement, and hadron observables are not derived by the projective pure-gauge route construction	App. I.2
Vacuum stiffness from the substrate	lattice dynamics	Excluded, five branches	induced curvature coupling $c_0 = +0.019$ against bare $k_0 \simeq 2.2$; refresh stationary state geometry-blind to exponential accuracy; history-tilt ceiling twenty times below requirement; free conserved field induces near-universal spanning-tree entropy; budget saturation mass suppresses the residual soft modes	§24.3, App. J.4
Vacuum geometry in the lattice realization	lattice dynamics	Open; externally hosted	tested substrate mechanisms do not select an extended phase; the Regge/CDT host supplies the vacuum geometry, while Many-Pasts conditions histories only after a present record is specified	§24.4, App. G, J.4
Capacity-decorated CDT phase / continuum tests	geometric embedding	Open, with explicit tests; finite-volume host behavior measured	at the same tuned couplings, increasing N_{41} from about 42,000 to 102,000 raises d_H from 3.31 to 3.56 and the blob score from 1.46 to 1.57; this is consistent with movement farther into the extended finite-volume region but does not establish phase membership; $d_H \rightarrow 4$, de Sitter-profile scaling, and especially a massless TT transfer sector remain to be measured, while intersection with a continuous critical manifold is the separate stronger cutoff-removal test; the predicted native point ($2 \ln 2, \simeq 0$) at $\bar{\alpha} \simeq 1.05$ is a registered phase test with criteria fixed before data (J.10)	§24.9, App. J.1, J.9, J.10
Ensemble fracture under local dynamics	lattice dynamics	Verified	explicit construction: 48 components ($4! \times 2$) of 35 states each under injectivity-preserving unit shifts, conserved (ordering, parity) charge	§24.3, App. D.4, H.6, J.4
Substrate contribution to the host volume term	lattice dynamics	Measured as a scaling family, demonstration volume	closure free energy per cell, computed from η_* alone, displaces the equilibrium volume by the predicted amount: single point -62.2 predicted, -62.8 measured; eight coupling-volume-penalty combinations span -10 to -158 at ratios 0.96 – 1.07 , with the concave β -curve and $1/\varepsilon$ scaling confirmed; the shuffled control lies on its separately predicted line at ratio 1.00 ; curvature-sector shifts remain to be measured with comparable statistics	§24.4, App. J.6

Quantity / Claim	Sector	Status	Type of Support	Where Established
Admissibility weighting on dynamical geometry	lattice dynamics	Finite-lattice compatibility measured	at $N_{41} \simeq 100,000$, three matched eighty-slice ensembles remain connected and correctly foliated with 0.6% total-volume spread; the collision fraction changes from 0.654 without the weighting to 0.077 with it, while the shuffled-energy control remains 0.736; the corresponding d_H values remain 3.55–3.59, and a forty-slice repetition gives collision fraction 0.078	§24.5, App. J.5
Local response of geometry to persistent closure failure	lattice dynamics	Demonstrated in two independent matched comparisons	for each of two random seeds at $N_{41} \simeq 100,000$, one hundred failure defects are compared with one hundred identically anchored closed cells, with every marked cell surviving; first-shell closure-energy and collision separations are 16.8σ and 9.2σ ; third-shell coordination separates at 4.0σ ; shell-cell count and first-/second-shell coordination do not separate significantly after autocorrelation and seed corrections	§24.6, App. J.7
Long-range propagation of the strain field	lattice dynamics	Conditional Ward theorem; finite-lattice source-to-field behavior measured; continuum junction open	on a separate forty-slice realization at $N_{41} \simeq 100,000$, conservation is exact; $Q - Q(0) = (0.1440 \pm 0.0025)m$; the dressing-subtracted shell-one deficit per charge is 4.9965 ± 0.5244 across $m = 1, 2, 3, 6$; the profile is $d^{-0.62 \pm 0.11}$ through eight shells and the screening fit is consistent with zero, giving $\xi > 4.6$; the 24.2% maximum source-linearity residual and nonideal exponent are explicit finite-size and saturation diagnostics, while carrier identification, critical scaling, and $\chi q \alpha_{\text{maint}}/Dq \rightarrow 8\pi G_*/c^2$ remain open	§24.7–24.9, App. J.8–J.9
Numerical consistency checks	validation layer	Supportive audit layer	cross-sector consistency tests and executable reproduction block	App. K, R
EFT consistency checklist	field-theory audit	Supportive audit layer	ordinary longitudinal branch has no extra scalar degree of freedom; static source sign and quadratic form checked; telegrapher stability belongs to the conditional transport completion	App. D

This table is the epistemic map used for the rest of the discussion.

Cosmological row. The saturated phase enters the ledger as follows. The per-source variable σ_A separates saturation of recruitment from exhaustion of absolute capacity. Conditional on $\sigma_A = 1$ and one coherent tick-normalized phase, $X = 1/2$ follows and the zero-pressure constrained-capacity stress form is a theorem. The abundance $1/\epsilon = 5.321$ stands against the measured 5.364 ± 0.065 and inherits the transverse normalization together with the recruitment and per-defect assumptions. The dynamics that drive $\sigma_A \rightarrow 1$, establish coherence, and account for conversion energy remain open.

27. Falsifiability and Observational Tests

27.1 Static weak-field falsifiers

The static galactic sector has five direct checks. Rotation curves test the exponential transition and baryonic Tully–Fisher limit [1, 45]; resolved disks test the geometry corrections in the nonspherical Poisson equation; galaxy–galaxy lensing tests the no-slip metric; Cassini tests the

carrier nesting rule; and dispersion-supported dwarfs test the boundary of the equilibrium carrier phase.

Stacked weak lensing first established a low-acceleration continuation and reported an early/late-type split [46]. A later joint analysis extends the kinematic RAR by about 2.5 decades in acceleration and finds a common early/late relation after imposing strict isolation and using consistent stellar and gas masses [47]. Those controls remove the earlier split. A persistent mismatch between dynamical and lensing accelerations in an isolated sample would falsify the leading no-slip contact.

The local prediction is sharper. The Milky Way is the active transverse carrier, while the Sun and wide binaries are resolved Einstein descendants. The branch predicts $Q_2^{\odot,\perp} = 0$ and Newtonian wide-binary dynamics apart from the smooth Galactic tide. Cassini gives $Q_2 = (1.6 \pm 1.8) \times 10^{-27} \text{ s}^{-2}$; current universal total-field implementations are in 3–15 σ tension with that result [38]. Wide-binary analyses remain statistically divided [35, 36, 37]. A confirmed order-unity binary boost would reject carrier nesting as stated, while a significant Solar quadrupole correlated with the Galactic field would reject its local projection rule.

At lower baryonic mass, the GravSphere dwarf sample of Júlio et al. [39] lies systematically above the extrapolated rotating-galaxy RAR and shows multivalued loci and large scatter. The current EFT restricts the exact exponential law to equilibrium active carriers; it must derive a phase-selection boundary before that restriction can explain the dwarf data. A universal application of the same law to dispersion-supported ultrafaint dwarfs is already disfavored by these observations.

27.2 Dynamical falsifiers

The dynamical extension has two independent observational tests. *Source-projection* tests ask whether relaxed clusters follow the predicted hook profile—near unity in BCG-dominated centers, maximal where the virialized bath dominates, and lower in the deep outskirts [4, 16, 17]—and whether resolved merger maps prefer $\epsilon \simeq 0.19$. The linear lift candidate already fails on amplitude (Section 18). *Transport* tests ask whether $D/\tau_0 = c^2$ evolves those source weights correctly through a merger. Systems such as the Bullet Cluster [2] probe both. A failure of the propagation law would reject the causal completion even if the static branch survived.

27.3 Cosmological falsifiers

Cosmology presents a different kind of test. The question there is whether a full Boltzmann treatment allows the trace-coupled homogeneous mode to reduce the sound horizon without spoiling the CMB or structure-growth observables. If it cannot, the cosmological extension fails on its own terms. The empirical target is set by the current measurement spread: early-universe inferences near 67.4 [51], distance-ladder determinations ranging from $\simeq 70$ [53] to 73 [52], and a tension whose proposed resolutions are reviewed in Di Valentino et al. [54].

The relation $a_0(z) = cH(z)g_{\text{share,eff}}/(4\pi^2)$ predicts measurable redshift evolution. For a Planck-like background, $H(2.3)/H_0 \simeq 3.47$. A disk with $g_{\text{bar}} \simeq 2 \times 10^{-10} \text{ m s}^{-2}$ then has $g_{\text{obs}}/g_{\text{bar}} \simeq 2.02$, compared with 1.39 for an epoch-independent scale. Observed outer rotation curves at these redshifts indicate strong baryon dominance [40, 44], although pressure-support and stacking systematics leave the comparison unsettled. Controlled observations showing no stronger boost than matched $z = 0$ systems would falsify $a_0 \propto H(z)$.

The equilibrium-vacuum route has separate continuum requirements, now with no free volume normalization. The capacity-decorated trajectory must approach a common-parent critical limit, show finite limiting α and simplex ratio ξ , set a by a target-blind observable, and re-

cover the exact two-simplex volume source after mixed-eigenoperator projection. A nonzero source-independent *capacity* bulk coefficient after vacuum normalization would directly falsify the regulator identity. A continuum completion that still requires an independent host cosmological action would instead falsify the Postulate I common-substrate identification. Only if both conditions hold does the conditional theorem $\Lambda_R^{\text{empty equilibrium}} = 0$ apply. The value or constancy of w tests whatever state-dependent or nonequilibrium cosmological dynamics is eventually supplied; it does not test the exact equilibrium partition identity by itself.

A MUSE sample of 79 star-forming galaxies at $0.33 < z < 1.44$ finds a radial-acceleration scale that rises with redshift [42]; a resolved low-redshift HI sample tentatively reports the same direction [43]. An epoch-independent a_0 predicts no such evolution. The comparison is already quantitative. The reported scale at $z \sim 1$, $a_0 = 2.38_{-0.10}^{+0.12} \times 10^{-10} \text{ m s}^{-2}$, is an enhancement of 1.98 over the local value, which $a_0 \propto E(z)$ produces at $z = 1.18$; the prediction sits at -2.3σ if the effective redshift of the high- z data is 1.0, -1.0σ at 1.1, and $+0.3\sigma$ at 1.2, so the verdict turns on the sample’s effective redshift. The reported linear slope $a_1 = (1.59 \pm 0.10) \times 10^{-10}$ likewise exceeds the $E(z)$ -anchored effective slope of about 1.19 over the sampled range, but $E(z)$ is convex and a linear fit absorbs curvature, so the slope comparison settles nothing by itself. The clean form of the test is the per-bin $a_0(z)$ values compared directly with $E(z)$: parameter-free, computable from published data, and able to fail. Current mass-to-light and pressure-support uncertainties remain comparable to the effect and enter that comparison as stated by the authors.

Saturated-phase falsifiers. Three tests bind the saturated phase: a full Einstein–Boltzmann implementation must jointly evolve the homogeneous mode, commitment transition, and constrained fluid with abundance fixed at $1/\epsilon$; the recruitment derivation must determine its epoch and survive scrutiny of the pinned reading and per-defect bookkeeping; and linear observables must remain compatible with the inherited constrained-fluid dynamics. The subsequent release channel is more selective than collapse. The completed 3-D hydro surrogate already shows that full-halo progenitor placement loses information too slowly and fails after delayed convolution. The decisive cosmological zoom must therefore show that recoverable Lagrangian progenitor information is lost mainly during retained first-galaxy assembly, remains negligible in minihaloes, and produces a frozen I_{rel} in the narrow background-compatible region without using cosmology to choose the progenitor geometry. Failure of that timing or normalization falsifies the release channel without changing the saturated-phase abundance calculation.

Dark-energy falsifiers. The sector of Appendix O adds four direct failure modes. A confirmed low-redshift crossing to $w > -1$ would close the mechanism, which approaches -1 from below during one-signed accumulation; a reconstructed $\rho_\Lambda(z)$ exceeding today’s value at an earlier epoch would do the same. The completed 3-D surrogate has already falsified the identification of Poisson coverage with progenitor-information loss and excludes its own full-halo delayed history. The remaining astrophysical test is two-sided: an independent cosmological zoom fails the collective trigger if it finds substantial minihalo-era loss, a slow full-halo-like kernel, or excessive early mixing that overshoots the background-compatible release integral. A background fit of the frozen hydro history must remain competitive without introducing a tunable mixing amplitude, delay, or progenitor concentration. Finally, an independently propagating determinant mode in the coarse-grained condensate Hessian would falsify the unimodular reduction itself.

27.4 Correlated-constant falsifiers

The inferred L_* , induced G_* , and matched weak-field G are not independent legs: the matched route carries the same electron length calibration. Their agreement is a normalization audit

rather than a separate falsifier. The marked action adds two independent tests because the same ζ_* enters the muon and tau through different graph polynomials. A future independent determination of L_* would provide a fourth test of the shared vertex.

27.5 Many-Pasts status

Many-Pasts imports ordinary quantum instruments, so it predicts no laboratory departure from the Born rule or no-signaling. Its discriminating burdens are theoretical and now separable. The substrate must reproduce the decoherence functional rather than assume it; the retained (P, Φ) sector must derive the symplectic and Fisher-compatible metric structures needed for the Caticha Hamilton–Killing bridge; and a conditional-typicality calculation must suppress high-entropy-past and Boltzmann-fluctuation histories. Failure of the first two would remove the proposed microscopic quantum completion, while failure of the third would remove the proposed arrow; none would constitute a new experimental violation of standard quantum mechanics.

28. What the Theory Would Have to Get Wrong to Fail

The failure modes are not all equally severe, and they are ordered here by how much of the theory each would remove.

Kills the core ontology. A demonstrated failure of mass–entropy equivalence, an internal incoherence in the Many-Pasts weighting, or evidence that geometry cannot be read as the long-wavelength form of an entanglement-capacity substrate would remove the foundations on which everything else rests.

Kills the static capacity-response closure or its transverse completion. A weak-field UV coefficient chain that cannot be reconciled with an independently validated microscopic derivation would break the ordinary static closure. An RAR transition shape that departs from the proposed bosonic law, persistent low-acceleration slip, unacceptable residual PPN effects, or violation of the metric Ward identity would falsify the transverse completion while leaving the ordinary Einstein/capacity equivalence intact.

Kills an extension only. If the cosmological trace-coupled homogeneous mode cannot survive a full Boltzmann likelihood confrontation, the cosmology sector fails while the static weak-field branch stands. If the saturated phase fails any of its commitments — the dark-to-baryonic abundance departing from the capacity ceiling, linear-regime observables departing from the inherited form, or no committed component surviving on cluster infall streams — the saturated committed-capacity reading fails in the same contained way. If the source clock is not carried by baryonic Lagrangian matter, if a tracer simulation finds insufficient or prematurely early progenitor-identity loss, or if the resulting mixing-weighted history fails a background-data fit, the dark-energy release extension fails without touching the weak-field branch. If q_{cap} does not track q_{geo} during collapse, or if the derived boundary response conflicts with black-hole observations, the bounded-domain interpretation fails while the spherical Einstein reduction remains. None of these touches the weak-field core.

Requires modification, not death. A fuller graph calculation may revise the separate conditional stiffness self-energy, and strong-field boundary spectroscopy may change without altering the Einstein exterior. A failure of the decorated charged vertex would be more serious because the same ζ_* enters the electron scale and both heavier-lepton ratios. The positive-spectrum clock conversion survives only if another microscopic charged action replaces it.

One scale-setting commitment cuts across these tiers. The high-precision G_* rests on seven-fold additivity, the record-conditioned determinant transfer, and the decorated marked vertex. A microscopic Hessian with a collective edge mode, persistent endpoint polarization, or off-diagonal charged propagation would fail the finite action and displace the correlated G and lepton results. If the electron identification or mass–entropy ontology failed, the scale-setting derivation of Appendix H would not go through.

29. Comparison with Other Approaches

The galactic excess and cosmological abundance are assigned to two phases of one medium. The comparisons below distinguish that proposal from nearby alternatives while retaining the conditional grades of Sections 15 and 20.

29.1 Relative to Λ CDM

The contrast with Λ CDM begins at the level of ontology. Here visible matter is interpreted as localized defects in a vacuum-capacity medium. The ordinary longitudinal response is a reduced representation of Einstein gravity; the proposed extra galactic response is carried by a transverse substrate sector. The UV entropy enters both branches, while the galactic branch additionally requires its phase-cell, carrier, and metric-vertex matching conditions. Its leading lensing response is specified; cosmology remains a separate extension.

29.2 Relative to MOND-like interpolation programs

MOND-like programs [18, 59, 21] usually begin from an acceleration law or interpolation function. Here the same law comes from a thermal occupation contact and a carrier-resolved auxiliary action. The source-space nesting rule removes the Solar cross-term found in universal functions of the total field, while the no-slip metric contact fixes the leading lensing response. The comparison now turns on the microscopic derivation of those structures and on resolved nonspherical, dwarf, and lensing tests.

29.3 Relative to Verlinde-style emergent gravity

Verlinde-style emergent-gravity programs share the broad intuition that gravity may be entropic [22, 58, 29, 30], but they are usually formulated through thermodynamic reasoning or horizon-inspired force laws. The present framework specifies tetrahedral counting, admissibility closure, edge coupling, return dressing, and Euclidean normalization before reaching the continuum EFT. Its correctness is an empirical question.

29.4 Relative to TeVeS and other multi-field modified gravities

Multi-field relativistic MOND completions such as TeVeS [20] introduce additional scalar and vector fields alongside the metric to obtain relativistic lensing and cosmology. Here the ordinary longitudinal response adds no field beyond the Einstein metric. The transverse sector is a carrier-conditioned metric contact with no additional vacuum graviton polarization; its leading stationary tensor structure gives no slip, while nonlinear covariance and the microscopic vertex remain open.

29.5 Relative to AeST

The closest modern comparator is the AeST theory of Skordis and Złośnik [31], which combines the metric with a dynamical timelike vector and a scalar field and can reproduce MOND-scale galaxy phenomenology while fitting the CMB power spectrum. The present construction differs

in its finite counting input and is less developed cosmologically: the joint Boltzmann treatment remains open here.

29.6 Relative to scalar-tensor gravity

The reduced capacity functional of Section 10 can be mistaken for the scalar part of a Brans–Dicke-type theory [56]. The action reconstruction shows otherwise: in the ordinary static branch it is the Einstein constraint action after a field redefinition and carries no independent scalar. A scalar–tensor or disformal theory is retained only as a control case for sectors that might genuinely require an additional mode; if adopted, ordinary fifth-force, slip, and PPN constraints would apply.

29.7 Relative to quantum-mechanical interpretations

Because Many-Pasts occupies the role of an interpretation of quantum mechanics (Sections 3.3, 22), it should be placed against the standard options. It posits one realized present, adds no collapse term, and introduces no hidden sharp values. Its operational probability theory is the decoherent-histories formalism: unresolved alternatives retain amplitudes, decoherent record histories receive diagonal probabilities, and conditioning on the present occurs only after the alternative records are normalized. The new content is the ontology assigned to that conditional measure and its proposed substrate realization. Born statistics and no-signaling are inherited from the quantum instruments; a substrate derivation of the decoherence functional remains open.

29.8 Relative to CDT, spin foams, and group field theory

The construction uses discrete tetrahedral boundary data, with primitive $j_0 = 3/2$ pairs whose positive oriented matching transmits the seven-state V_3 sector; this places it near the quantum-tetrahedron vocabulary of simplicial spin networks [62, 63, 64]. Its selection principle remains distinct from ordinary spin-foam/GFT programs: the finite capacity sector is fixed by a boundary-counting and faithful-resolution problem rather than by using a gravitational vertex amplitude as its statistical weight.

The earlier identification of the four capacity labels with the open Peter–Weyl magnetic indices is not required and is not adopted in the completed embedding. Appendix Q instead uses

$$\mathcal{H}_{\text{cell}} = \mathcal{H}_{\text{geom}} \otimes \mathcal{H}_{\text{cap}} \otimes \mathbb{C}_{\text{orient}}^2.$$

The geometric factor carries the standard gauge and intertwiner data of the simplicial amplitude; the finite capacity factor carries the injective four-port alphabet, K^2 , the Gibbs weight, marks, and records. Their selected V_3 frames are related equivariantly, so geometry–capacity equivalence does not identify two distinct operators or force the capacity count to discard geometric intertwiners. Appendix Q excludes the Gauss-index identification and proves the strong gauge invariance of the numerical spine.

This selected factorized completion supplies a direct spin-foam/GFT witness for the vacuum geometry. The supporting external results have different jobs and are not combined into a single published theorem. Proper-EPRL-type amplitudes isolate the gravitational Regge exponential in the semiclassical block sector [99, 100]. The capacity matching supplies an exact canonical maximal-fusion ray $V_3^{\otimes N} \rightarrow V_{3N}$ on the selected channel, and the block-lock lemma propagates the chosen one-cell geometry–capacity lock along that ray. Han-type semiclassical–continuum constructions independently reach Einstein gravity; in the controlled linearized construction the low-energy excitations give all smooth linearized Einstein solutions and the two graviton helicities [97, 98]. The area-Regge continuum calculation gives the same leading graviton dynamics and

an $O(a^2 C^2)$ correction [101]. Appendix H.10 turns that selected-branch symbol into the strong-resolvent theorem (H_{TT}) and pulls it back to an exact fixed- $j = 3$ primitive realization. It also proves that the finite capacity premises cannot determine the independent geometric gluing tensor or measure. A derived (H_{phase}) therefore requires an additional geometric principle before the phase test can even be posed. Appendix H.9a, paragraph 4a, names the one candidate already in the postulates, the face-wise reading of the relational lock, under which the host is the equilateral CDT class at predicted couplings (Appendix J.10). The factorized capacity decoration inherits any positive host spectrum by identity; an alternative nonfactorized vertex must preserve the same interface properties.

Section 24 supplies a separate numerical interface rather than a duplicate of this spin-foam argument: the unchanged finite capacity ensemble is coupled to a CDT host [80, 81]. In the completed runs the host supplies the dynamical vacuum geometry and the paper tests how its capacity weighting dresses it. Section 24.9 and Appendix J.9 first ask whether the fixed capacity slice lies in a stable extended phase with a massless TT transfer sector. Intersection with a continuous geometric critical surface is the stronger optional cutoff-removal test, not the existence condition at physical L_* . Success or failure of that CDT route is therefore a separate embedding test and does not alter the block-scale spin-foam witnesses.

29.9 Relative to algebraic and information-geometric gravity

Three continuum results address separate claims made here. The observer-dressed de Sitter algebra establishes a maximum-entropy empty gravitational state. The horizon modular calculation equates the relative information of a coherent excitation with its Killing energy flux and, after the area input, with Einstein curvature. The geometric-relative-entropy action gives a local bulk information functional with an Einstein limit, a positive Legendre energy, cosmological thermodynamics, and de Sitter area scaling [24, 23, 25]. They support the continuum ontology but do not solve the ultraviolet problem. This paper must still derive the local bound, matter commitment, dimensional scale, and response coefficients from one finite microstructure. Bianconi's independent dynamical G -field belongs to a modified-gravity theory and is not imported into the metric-only longitudinal branch.

The coefficient comparison is explicit. On the matched branch $L_*^2 = \hbar G_*/c^3$, the horizon normalization gives

$$S_\infty = \frac{A_{\text{dS}}}{4L_*^2} = \frac{c^3 A_{\text{dS}}}{4\hbar G_*}.$$

The observer-dressed de Sitter construction uses this area coefficient and gives $S_{\text{max}} - S_{\text{gen}}(E) = \beta_{\text{dS}} E + O(E^2)$ for a small central excitation. The horizon modular calculation uses $S_{\text{rel}} = c^3 \delta A / (4\hbar G_*)$ and obtains

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G_*}{c^4} \langle T_{\mu\nu} \rangle.$$

The bulk route derived here gives

$$\nabla^2 \delta S = -\frac{\kappa}{\gamma} \rho, \quad \frac{\Phi}{c^2} = -\frac{\delta S}{2S_\infty}, \quad G_* = \frac{c^2 \kappa}{8\pi \gamma S_\infty},$$

and hence $\nabla^2 \Phi = 4\pi G_* \rho$. The two continuum routes therefore use the same coupling. Appendix F.5 now fixes the microscopic channel-to-area normalization within the factorized coherent $j = 3$ embedding, so the former area-normalization rider is removed; the result is a consistency matching, not independent evidence for the area law.

At Gaussian order, the relationship to Bianconi is sharper. Appendix H.10 proves that the normalized determinant of any positive dressed GFT Hessian relative to its equilibrium Hessian

on the same physical support has exactly the trace-log form used by geometric relative entropy, while its quadratic-source Legendre transform is exactly the positive covariance mismatch $\frac{1}{2} \text{Tr}'(\mathcal{G} - I - \ln \mathcal{G})$. In the factorized completion the normalized equilibrium capacity trace contributes one to the empty vacuum amplitude, while the selected coarse geometric factor supplies the TT Einstein Hessian. This closes the Gaussian functional bridge and shows that the capacity decoration preserves the ordinary low-energy support. The selected metric-Regge branch satisfies (H_{TT}) . The remaining covariant geometric task is logically prior to a phase calculation: fix $\mathcal{G}_v$ and its measure by a new geometric principle, then test (H_{phase}) . The distinct canonical completion retains (H'_{can}) .

The cosmological scaling also agrees at the level of form. With $R_A = c/H$,

$$S_\infty = \frac{\pi c^2}{H^2 L_*^2}, \quad S_{\text{GfE}} \simeq \frac{\bar{\omega}_{[1]} c^2}{\ell_P^4 H^2}, \quad \frac{S_{\text{GfE}}}{S_\infty} \simeq \frac{\bar{\omega}_{[1]} L_*^2}{\pi \ell_P^4}.$$

Bianconi therefore obtains the same H^{-2} dependence, but her couplings do not determine the finite-cell coefficient used here.

30. Conclusion

One physical picture runs through the paper. The vacuum is a finite medium of entanglement capacity; matter is that capacity tied up in stable, localized defects; a particle's mass measures the entanglement its defect commits; and gravity is the capacity strain the surrounding medium carries once that commitment is made. The theory derives general relativity as the medium's low-energy geometry. On galactic scales, the same medium produces the additional long-range response usually attributed to dark matter.

The central result is a tightly specified static weak-field construction with its remaining assumptions exposed. The tetrahedral ensemble fixes the sharing entropy; faithful renewal and the decorated marked-transfer vertex fix the electron-anchored substrate length; the edge kernel and Green-matched source map fix the ordinary response; and the separate minimal loop operator supplies the stated conditional stiffness correction. The weak-field bridge then rewrites the Einstein constraint sector in the capacity variable.

Newtonian gravity is the point-source limit of the Einstein/capacity action equivalence, and baseline lensing and PPN values follow from that parent. The carrier-resolved galactic EFT now supplies the thermal Bose contact, conservative nonspherical action, source nesting rule, no-slip metric response, point-source lensing law, causal transport poles, and leading Solar-System and wide-binary limits. The one-entropy horizon loading, $E/(k_B T_H) = x$, carrier projector, and no-slip metric vertex remain matching premises awaiting a microscopic GFT derivation. The marked-transfer action gives G_* at -0.073σ and the muon and tau ratios at -0.535σ and $+0.481\sigma$. These are historical postdictions from one zero-continuous-fit action, not three blind predictions.

The same medium extends into sectors the paper holds more tentatively. Transport gives the field a finite propagation speed and the lag that clusters and mergers require. The cluster sector reads lensing anomalies as a phase-dependent projection of an already-fixed coefficient. The cosmological mode reduces the sound horizon, allowing a higher inferred Hubble constant. In the saturated early universe the medium becomes a conserved committed density that gravitates as zero-pressure committed capacity. In strong fields the exact spherical reduction identifies the composite q_{geo} and recovers Schwarzschild, while domain termination, boundary microphysics, and the nonspherical capacity map remain open. These are extensions and frontier completions, not closed results, and the closure table marks the distinction.

The microscopic construction has also been tested on a dynamical simplicial lattice. At matched $N_{41} \simeq 100,000$, the admissibility weight strongly orders the microscopic labels while the host geometry remains extended and nearly unchanged. Two independent comparisons between identically anchored failure defects and closed cells give local capacity separations of 16.8σ and 9.2σ and a third-shell coordination separation of 4.0σ after autocorrelation and seed corrections. A separate conserved-field calculation finds exact conservation, approximately additive emergent source charge, one junction ratio across the tested source strengths, a power-law tail through eight shells, and no detected screening. Increasing the host volume from about 42,000 to 102,000 raises both the Hausdorff estimator and the concentration of volume in the extended region. Extended-phase membership, a massless TT transfer sector, the optional critical-surface test, and the absolute G junction remain to be measured. Section 26 records the present grades.

The joint measure also closes a sharper equilibrium-vacuum statement. Normalizing by the complete homogeneous capacity free energy at each regulator makes the theory invariant under arbitrary extensive shifts of microscopic energy origin, and the uncentered control verifies the predicted N_{41} coupling displacement. Conditional on the decorated common-parent continuum limit, the fixed primitive event measure removes an independent host volume action and gives zero cosmological term for the empty equilibrium branch. Appendix O now supplies a more restrictive subsequent dynamics. The fixed-event history space has a unique volume-conjugate singlet, so only the homogeneous projection of the universal marked drain changes Λ ; a local $\Lambda(x)$ branch would require additional cell-by-cell volume couplings absent from the construction. Collective baryonic loss of progenitor identity triggers complete renewal, but it does not create a new material or scalar component. The metric-visible remainder stays in source-associated capacity of the retained coarse carrier, while the orthogonal marked-history output has no independent leading local stress. The absorbing first-entry excursion-set construction fixes the assembly history and the three-dimensional passive-tracer surrogate supplies an explicit delayed mutual-information kernel. For nonzero spatial modes, release-hazard fluctuations only redistribute the internal source labels and cancel from the leading total metric source, leaving $\delta\rho_\Lambda = 0$ and no new short-scale gradient force. The remaining astrophysical input is therefore sharply isolated: a self-gravitating cosmological progenitor-tagged zoom must determine $K_{\text{mix}}(\tau; M, z)$ independently, after which the background and perturbation likelihood contains no fitted release amplitude or bias parameter.

Three independent continuum constructions support different parts of the proposed ontology. The type II₁ de Sitter algebra realizes an entropy-maximizing empty gravitational state. The horizon modular calculation relates an excitation's relative information to energy flux and, after the area input, to Einstein curvature. The geometric-relative-entropy action gives a local bulk information dynamics with an Einstein limit, a positive mismatch energy, a first law, and H^{-2} de Sitter entropy [24, 23, 25]. Section 25 derives the same positive mismatch functional from a Gaussian covariance identity. These results support the continuum ontology but do not derive the finite cell, coefficients, cosmological state, or ultraviolet completion used here.

The ordinary geometric sector now has an exact primitive kinematic spine and a selected-branch Einstein theorem. Positive oriented matching gives a unique $J = 3$ transmitted phase, maximal fusion gives an exact canonical $V_3^{\otimes N} \rightarrow V_{3N}$ ray, and multiplicity one propagates the selected one-cell lock along that ray. Proper-EPRL isolates the desired Regge asymptotic sector, Han-type refinement reaches the Einstein sector in a controlled model and yields the two graviton helicities in its linearized theorem, and area-Regge gives the leading Einstein dynamics with $O(a^2 C^2)$ corrections. Appendix H.10 completes the strong-resolvent step on that metric branch and the gapped perfect-action pullback supplies a positive cylindrically consistent fixed- $j = 3$ realization. These results prove (H_{TT}) . The host-underdetermination theorem then proves that the present capacity premises cannot select the independent geometric gluing tensor or measure: a hosted Einstein completion exists, but a derived completion requires a new geometric principle before

(H_{phase}) can be tested. On the displayed factorized branch, renewal, the positive marked trace, and the exact rigging-map lift close the capacity pole, residue, positivity, causality, preferred-frame, and refresh-order audits. The canonical and covariant hosts remain distinct selected completions, not proved equivalent representations.

Many-Pasts assigns a record-conditioned ontology to the decoherent histories compatible with the present. Its operational branch changes no laboratory prediction: no-signaling follows from standard quantum instruments, and the Born form is the unique normalized noncontextual additive measure on a sufficiently rich record-projector lattice. The local marked kinematics are no longer wholly imported. The retained Q complement inherits the displayed Lorentzian action; its transport and 6:5:3 phase splitting generate $\mathfrak{su}(9)$ on a real canonical phase space, select a unique positive complex structure, and yield $\mathbb{C}^9/\mathbb{CP}^8$. The fresh-state response metric agrees exactly with one quarter of the Fisher metric, and a projected neighboring bridge supplies the finite entangling algebra before the final host projection. The full continuum/Fock space, global decoherence functional, initial state, and record completeness remain premises. The past is represented at the resolution retained by present physical records; erasing a record coarsens the same joint measure, and conditioning on a record after it forms supplies no future-to-past dynamics. Faithful full-support resolution selects the memoryless dressing kernel. The decorated vertex prepares its diagonal fresh closure state, reversible dilation exports the old replaceable register into history, and the marked spectrum plus electron anchor fixes the clock and phase frequency. The complementary-channel calculation shows how renewal can coexist with interference: the marked position fiber remains in the coherent system, and the discarded free-renewal record carries no path label. Durable history capacity, electromagnetic charge and Maxwell dynamics, relativistic quantum fields, and the Past-Hypothesis/mixing package remain open. None is a new founding premise.

The particle appendix also contains a conditional projective color sector: a persistent open tetrahedral route, transported reversibly, gives the $PU(3)$ qutrit-channel group, its $\mathfrak{su}(3)$ algebra, the adjoint Wilson form, and the fixed primitive transfer $t_8 = 13/14$, and the same primitive fusion reproduces the nine-state representation required by the marked present/history fiber (Appendix I.2). The global $SU(3)$ endpoint lift, chiral quark matter, continuum-scheme matching, confinement, and hadron observables remain outside the completed derivation.

The remaining work is specific. In ordinary gravity, the selected metric-Regge branch has the Einstein TT infrared theorem and an exact fixed- $j = 3$ perfect-action realization, while the factorized capacity decoration, empty-capacity rigging lift, channel-to-area normalization, worldline source, and WEP are closed at their stated grades. The host-underdetermination theorem shows that the current capacity premises cannot fix the independent geometric gluing tensor and measure. A derived completion needs a geometric principle to determine those data, followed by a nonperturbative phase proof; the face-wise reading of the relational lock is the candidate, and it predicts the CDT bare couplings, $\kappa_0 = 2 \ln 2$ at $\tilde{\alpha} \simeq 1.05$, whose phase is a registered finite-volume test (Appendix J.10). The provisional $L_{\text{block}} \sim 3.3\text{--}5.3L_*$ bracket now has a computed representative, $L_{\text{block}} \simeq 3.4L_*$ from the area-Regge mismatch gap (Appendix J.10), which a decorated-vertex measurement must still confirm, and block-scale Lorentz invariance remains a named premise. The refreshed orientation doublet is inert on the pure-gravity branch, while its fermion tetrad-sign audit remains open. A canonical-covariant identification would be an additional theorem. The CDT calculations are compatibility checks; an Einstein-phase claim would require a massless TT transfer sector, and a continuous critical surface would be the stronger cutoff-removal test. In the galactic sector, the remaining theory work is a microscopic derivation of the cell Hamiltonian, carrier projector, and metric vertex; full nonlinear variation of the carrier records; and a bath calculation of the equilibration rate and noise correlation volume. Resolved nonspherical SPARC and lensing maps and the ultrafaint-dwarf phase boundary are the immediate empirical tests. The finite-loop stiffness operator still needs its microscopic

return-operator audit. Cosmology needs a self-gravitating progenitor-tagged zoom to freeze the mixing kernel, followed by the full DESI+CMB+Dovekie background and one-metric perturbation likelihood. The cluster lift needs its coherence profile and resolved lensing-map test, and the strong-field boundary needs its saturation dynamics and spectroscopy. Durable history capacity and the Past-Hypothesis/mixing theorem remain open in the foundations sector. The color branch still needs a derived chiral matter action and graph-to-continuum matching before comparison with physical QCD.

One ultraviolet count fixes the coefficients of the closed sector in advance. A measurement inconsistent with any of those linked coefficients would falsify that sector. The calculations above specify those comparisons.

Appendix A: Symbol Dictionary and Canonical Conventions

Appendix A gathers the conventions used throughout the technical material that follows, fixing the units, field definitions, and couplings in one place before the denser calculations begin.

Plain-language terms. Several physical words recur throughout the paper and are collected here in plain form before the symbols:

- *Capacity* — the entanglement support locally available in the vacuum medium.
- *Defect* — a stable, localized commitment of that capacity; coarse-grained, a particle.
- *Deficit* — capacity no longer freely available near a defect, $\delta S = S_\infty - S_{\text{ent}}$.
- *Capacity strain* — the extended deficit profile whose fractional value gives the weak-field potential and whose gradient gives the gravitational field.
- *Committed capacity* — capacity locked into transferring on behalf of a defect; the conserved carrier of the saturated phase (Section 20).
- *Dressing* — the cloud an elementary defect builds by resolving the boundary sectors; its determinant transfer and marked response set the length in the decorated scale branch.
- *Saturation* — the regime in which the capacity bath has no slack left, with the available transfer channel at its ceiling.
- *Pinned reading* — the statement that the saturated phase holds exactly at that ceiling and stays there.
- *Admissibility* — the weighting that favors boundary configurations close to a regular, isotropic local cell.
- *Closure* — the condition that the four oriented face data sum to zero, so the cell closes into a regular volume; K^2 measures the failure of closure.
- *Many-Pasts* — the postulate that one recorded present is supported by its compatible decoherent pasts. Their operational probabilities are the diagonal decoherence-functional weights conditioned on that present; $e^{-D(h,P)}$ is only shorthand for those normalized quantum probabilities.

A.1 Units, signature, and entropy normalization

All dimensional quantities are expressed in SI units unless noted otherwise. The metric signature is $(-, +, +, +)$. Covariant spacetime integrals use $x^0 = ct$, so the Einstein–Hilbert coefficient is $c^3/(16\pi G)$; after writing $dx^0 = c dt$, the ADM coefficient is $c^4/(16\pi G)$. Entropies are measured in nats, so Boltzmann’s constant is absorbed into the entropy normalization. The canonical UV cell has spatial scale L_* and volume $V_* = L_*^3$; it is the bipartite primitive cell containing the two paired tetrahedral sites, so the site density is $2/L_*^3$ and the four-cell $\Delta V_4 = L_*^4/c$ is assigned one per primitive cell — the convention under which $\gamma_Q = 4\hbar c J/(3L_*^2)$ and $\kappa/\gamma = 3L_*/(4G_{\text{tet}}(0)\kappa_m(L_*))$ are simultaneously exact (Appendix C.4–C.5). In the decorated electron-anchored marked-transfer branch,

$$L_* = -\frac{3}{2} Z_e \lambda_e \ln(1 - e^{-7g_{\text{share,eff}}}), \quad \lambda_e = \frac{\hbar}{m_e c}, \quad Z_e = (1 + \zeta_*)(1 + 7\zeta_*^2).$$

The conventional Planck length $L_P = \sqrt{\hbar G/c^3}$ is used only as a comparison scale or in standard gravitational thermodynamic expressions after the gravitational scale has been identified.

These conventions matter because the argument repeatedly moves between a dimensionless ultraviolet counting problem and a dimensionful continuum EFT. The units and signature make those descriptions directly comparable.

A.2 Core scalar variables

The canonical continuum variable is the vacuum-relative coarse-grained entanglement field

$$S_{\text{ent}}(x),$$

with vacuum baseline S_∞ and deficit

$$\delta S(x) = S_\infty - S_{\text{ent}}(x).$$

For nonlinear work the bounded occupancy fraction is

$$q(x) = \frac{S_{\text{ent}}(x)}{S_\infty} = 1 - \frac{\delta S}{S_\infty} \in [0, 1].$$

The absolute entropy unit is fixed only after choosing a cell or horizon normalization. Under a constant rescaling of S_{ent} , the quantities S_∞ and κ/γ rescale together, leaving $\delta S/S_\infty$ and $\kappa/(\gamma S_\infty)$ invariant. The source channel is

$$\chi(x) = -\frac{T^\mu{}_\mu}{c^2},$$

which is the continuum trace channel of the localized defect sector and reduces to the ordinary mass density ρ in the nonrelativistic static limit.

A.3 Couplings and derived observables

The main-text conventions are

$$\gamma : \text{entanglement-field stiffness}, \tag{10}$$

$$\kappa : \text{continuum defect-entropy coupling}, \tag{11}$$

$$\kappa_m(\ell) : \text{mass-per-entropy map at scale } \ell, \tag{12}$$

$$\zeta_* \equiv 9e^{-g_{\text{share,eff}}} \left(1 - \frac{8\eta_*}{49}\right)^{21/2}, \tag{13}$$

$$Z_e \equiv (1 + \zeta_*)(1 + 7\zeta_*^2), \tag{14}$$

$$L_* \equiv -\frac{3}{2} Z_e \lambda_e \ln(1 - e^{-7g_{\text{share,eff}}}) \quad \text{in the decorated marked-transfer branch}, \tag{15}$$

$$G_* = \frac{c^3 L_*^2}{\hbar} = \frac{9}{4} \frac{\hbar c}{m_e^2} Z_e^2 \ln^2(1 - e^{-7g_{\text{share,eff}}}), \tag{16}$$

$$G_{\text{tet}}(0) : \text{tetrahedral on-site Green constant}, \tag{17}$$

$$g_{\text{share,max}} = \ln(1680), \tag{18}$$

$$g_{\text{share,eff}} : \text{admissibility-weighted sharing entropy}, \tag{19}$$

$$J_{\text{bare}}, J_{\text{eff}}^{\text{tree}}, J_{\text{eff}}^{(\text{ren})} : \text{UV edge couplings}, \tag{20}$$

$$a_0 = \frac{cH_0 g_{\text{share,eff}}}{4\pi^2}. \tag{21}$$

The canonical weak-field bridge and Newton closure are

$$\frac{\Phi}{c^2} = -\frac{\delta S}{2S_\infty}, \quad \frac{\kappa}{\gamma} = \frac{3L_*}{4G_{\text{tet}}(0)\kappa_m(L_*)},$$

$$G = \frac{c^2\kappa}{8\pi\gamma S_\infty}.$$

Collected in one place, these formulas also make clear which quantities are downstream of the closure chain. The UV data determine the stiffness and source-to-stiffness ratio first; the observable weak-field constants appear after the bridge and fixed S_∞ normalization are applied.

A.4 Notation map

One notation set is used throughout. The effective sharing entropy is denoted $g_{\text{share,eff}}$, the scalar variable is always the vacuum-relative field S_{ent} or its deficit δS , and the weak-field bridge is used in the single form stated above. The principal extension-sector symbols are:

$$\epsilon \equiv \frac{g_{\text{share,eff}}}{4\pi^2}, \quad \nu(x) \equiv \frac{1}{1 - e^{-x}},$$

$$x \equiv \frac{E_\perp}{k_B T_H},$$

W_{bath} : cluster bath source weight,

B_{bath} : measured bath-development fraction,

D : capacity diffusivity, τ_0 : transport relaxation time,

$D/\tau_0 = c^2$,

$\sigma_* \equiv \pi/g_{\text{share,eff}}$,

$a_{\text{UV}} \equiv 1/\text{Var}_{\eta^*}(K^2).$

In the transverse EFT, $\mathcal{R}_P$ is the retained record algebra of carrier P , Π_P its conditional-expectation source projector, $\Phi_{b,P}$ its Newtonian auxiliary potential, and $\phi_{\perp,P}$ its excess potential. The variables $z_P = |\nabla\Phi_{b,P}|^2/a_0^2$ and $x_P = z_P^{1/4}$ enter the thermal occupation $n_B(x_P)$ and the auxiliary function $\mathcal{Q}_\perp(z_P)$. The one-capacity Hessian symbols ΔC , u_C , A_L , A_T , and $C_\times$ are retained only as static thermodynamic consistency data in Appendix N; they are not mode frequencies.

Appendix B: UV Boundary Ensemble and Admissibility Closure

The ultraviolet counting problem is finite. A seven-state tetrahedral ensemble, weighted by its closure defect, determines a unique admissibility-closed entropy. Sections 5–6 use this result.

B.1 Tetrahedral package and logical status

The canonical ultraviolet cell is a tetrahedron, the minimal volumetric simplex in three spatial dimensions. Its four faces are distinguishable relational ports. Each side of a shared face carries a primitive half-integer representation V_{j_0} , and the two sides are compared only after one has been transported and orientation-reversed into the frame of the other. Appendix B.4 shows that the positive oriented-matching operator on

$$V_{j_0} \otimes V_{j_0}$$

transmits the maximal coupled multiplet V_{2j_0} .

The value of j_0 has two possible grades. Conditional on identifying the independently constructed marked present/history fiber with the complete nonmaximal fusion record, the representation content fixes

$$j_0 = \frac{3}{2}, \quad V_{2j_0} = V_3, \quad |M| = 7.$$

Without that dynamical incidence identification, each half-integer j_0 defines a discrete zero-parameter branch, and Appendix B.5 compares those branches through the same downstream map.

The counting convention is then fixed by the cell ontology. A letter is the occupation of one channel in a single-copy cell resource, so one letter cannot be routed through two ports. The ports remain distinguishable, and the two global orientations are distinct. The state count and its raw entropy are therefore

$$\Omega_{\text{tet}} = 2P(7, 4) = 1680, \quad g_{\text{share}, \text{max}} = \ln(1680) = 7.42654907240.$$

The closure weighting below acts on this finite ensemble. Appendix B.4 derives the primitive pair selection and the conditional $7+9$ complement structure; Appendix B.5 records the neighboring discrete counts and preserves their postdictive status.

B.2 Closure invariant, kernel, and unique fixed point

The canonical scalar closure invariant is

$$K^2(b) = 48 - \frac{1}{3}(S^2 - \Sigma^2), \quad S = \sum_{i=1}^4 m_i, \quad \Sigma^2 = \sum_{i=1}^4 m_i^2.$$

The admissibility family is

$$p_\eta(b) = \frac{1}{Z(\eta)} e^{-\eta K^2(b)}, \quad Z(\eta) = \sum_{b \in B} e^{-\eta K^2(b)}.$$

Stationary normalized closure evidence fixes the admissibility precision η . Classical closure would set the three-component oriented-face sum $\mathbf{c}(b) = \sum_i \hat{n}_i m_i \in \mathbb{R}^3$ to zero. Its squared magnitude is $|\mathbf{c}(b)|^2 = (4\Sigma^2 - S^2)/3$. The invariant K^2 used here is instead the quantum expectation of the squared closure operator. Each face carries the seven-state $j_{\text{eff}} = 3$ representation ($2j + 1 = 7$), and the product boundary state $|b\rangle = \bigotimes_{i=1}^4 |j, m_i\rangle_{\hat{n}_i}$ uses the tetrahedral normal frame $\hat{n}_i \cdot \hat{n}_j = -\frac{1}{3}$. The closure operator $\hat{\mathbf{C}} = \sum_i \hat{\mathbf{A}}_i$ then obeys

$$K^2(b) = \langle b | \hat{\mathbf{C}}^2 | b \rangle = \sum_i j(j+1) + 2 \sum_{i < k} m_i m_k \hat{n}_i \cdot \hat{n}_k = 4j(j+1) - \frac{1}{3}(S^2 - \Sigma^2),$$

with $4j(j+1) = 48$ for $j = 3$. The decomposition

$$K^2 = |\mathbf{c}(b)|^2 + \sum_i [j(j+1) - m_i^2]$$

exhibits K^2 as the mean nonclosure plus the irreducible quantum variance of the four face operators. The expectation is taken in the unprojected product state. This implements the soft-closure convention: the evidence weight $e^{-\eta K^2}$ imposes closure statistically, whereas projection onto the gauge-invariant intertwiner subspace would give different expectation values. The seven-label alphabet and the constant 48 are fixed together by the $j_{\text{eff}} = 3$ product-state boundary realization and the tetrahedral normal frame.

The evidence factor follows from the same operator. The closure operator $\hat{\mathbf{C}}$ has three components, and the normalized isotropic quadratic evidence kernel on that three-dimensional defect

space carries the determinant weight $\eta^{3/2}$, so the normalized closure evidence is $\eta^{3/2}Z(\eta)$ and its logarithm is

$$\mathcal{F}(\eta) = \ln Z(\eta) + \frac{3}{2} \ln \eta.$$

With $\partial_\eta \ln Z = -\langle K^2 \rangle_\eta$, the stationary condition $\mathcal{F}'(\eta) = 0$ is the closure relation

$$\langle K^2 \rangle_\eta = \frac{3}{2\eta},$$

so the factor $3/2$ is the determinant weight of the three independent closure components, not an equipartition rule imported onto the bounded discrete spectrum; the discreteness, multiplicities, and positive floor of the spectrum remain inside the exact finite sum $Z(\eta)$. The second derivative is $\mathcal{F}''(\eta) = \text{Var}_\eta(K^2) - 3/(2\eta^2)$, and on the exact spectrum $\mathcal{F}'$ has a single interior zero,

$$\eta_* = 0.0298668443935,$$

at which $\text{Var}_{\eta_*}(K^2) = 15.69$ is dwarfed by $3/(2\eta_*^2) = 1681.6$, giving $\mathcal{F}''(\eta_*) = -1665.9 < 0$: η_* is the unique local maximum of the normalized closure evidence. Because the parity-symmetric ensemble is finite, both $Z(\eta)$ and $\langle K^2 \rangle_\eta$ are exact finite sums over the spectrum. The distinct closure-defect values and their degeneracies are

K^2	$\frac{122}{3}$	$\frac{134}{3}$	$\frac{142}{3}$	$\frac{146}{3}$	$\frac{152}{3}$	$\frac{154}{3}$
mult	96	96	96	288	192	144

K^2	$\frac{158}{3}$	54	$\frac{164}{3}$	$\frac{166}{3}$	$\frac{170}{3}$
mult	384	192	48	96	48

with total multiplicity 1680 as required. In particular,

$$Z(\eta) = \sum_a n_a e^{-\eta K_a^2}, \quad \langle K^2 \rangle_\eta = \frac{\sum_a n_a K_a^2 e^{-\eta K_a^2}}{\sum_a n_a e^{-\eta K_a^2}},$$

where (K_a^2, n_a) run over the table above. The closed-branch value η_* is therefore the unique interior maximum of an exact finite-spectrum functional, not an unseen numerical fit. The corresponding effective sharing entropy is

$$g_{\text{share,eff}} = - \sum_{b \in B} p_{\eta_*}(b) \ln p_{\eta_*}(b) = 7.41980002357.$$

The closed-branch moments used in the UV stiffness discussion are

$$\begin{aligned} \langle K^2 \rangle_{\eta_*} &= 50.2229154254, \\ \text{Var}_{\eta_*}(K^2) &= 15.6889750078, \\ a_{\text{UV}} &\equiv \frac{1}{\text{Var}_{\eta_*}(K^2)} = 0.0637390269. \end{aligned}$$

These values quantify the local stiffness of the canonical closure point rather than a tunable phenomenological uncertainty. The closure-saturation product is

$$C_{\text{cl}} := \eta_* \langle K^2 \rangle_{\eta_*} = \frac{3}{2}, \quad C_{\text{cl}}^{-1} = \frac{2}{3}.$$

The value $C_{\text{cl}} = 3/2$ is an identity imposed by the stationarity condition, not an independent numerical cross-check. Its reciprocal is reused as the transverse export factor in the decorated scale-setting branch discussed in Appendix D.4.

The admissibility parameter stops being free here. The kernel introduces η , and the closure condition removes its arbitrariness again by demanding that the fluctuation scale produced by the weighting agree with the weighting itself.

B.3 Rooted reduction and local benchmarks

Rooting on the shared face reduces the exact parity-symmetric ensemble to 140 rooted microstates and 69 rooted closure classes. The rooted classes can be labeled by $\alpha = (m_\bullet, K^2)$, so the same reduced state space supports the local evaluation, the cavity benchmark, and the later shell propagation. Let X denote the root-face label and Y_r the boundary record after r rooted shells. The local information observable

$$\sigma_{\text{ind}}^{(r)} = \frac{H(X | Y_r)}{H(X)}$$

has the principal pre-nonlocal benchmarks

$$\sigma_{\text{ind}}^{\text{toy}} = 0.44997, \quad (22)$$

$$\sigma_{\text{ind}}^{\text{loc}} = 0.44708, \quad (23)$$

$$\sigma_{\text{ind}}^{\text{Bethe}}(J=0) = 0.44749. \quad (24)$$

Here the Bethe value is the homogeneous cavity evaluation on the 69×69 rooted-class interaction graph at zero transport coupling,

$$\mu_\alpha \propto w_\alpha \left(\sum_\beta U_{\alpha\beta}(0) \mu_\beta \right)^{z-1}, \quad \sum_\alpha \mu_\alpha = 1,$$

where $w_\alpha = n_\alpha e^{-\eta_* K_\alpha^2}$ is the rooted-class Gibbs weight, $z = 4$, and $U_{\alpha\beta}(0)$ is the rooted shared-face compatibility matrix before shell transport is turned on. Thus $\sigma_{\text{ind}}^{\text{Bethe}}(J=0)$ is the cavity-theory benchmark of the same explicit rooted ensemble. The three benchmark values are legacy single-instance calculations and remain provisional until they are independently rederived. One consequence is already stable enough to constrain the Appendix H.7 fusion layer: a pair coupling confined to the shared fused label carries at most $\ln 7$ of pair information. Induced fractions below 0.5 therefore require pre-fusion face structure with capacity up to $\ln 16$. The horizon target implied by the effective sharing entropy is

$$\sigma_* = \frac{\pi}{g_{\text{share,eff}}} = 0.42340665.$$

The gap between the local benchmarks and σ_* arises from shell and loop structure. The local admissibility closure already satisfies its own condition.

The remaining ultraviolet calculation propagates the fixed local closure ensemble through transport and return structure.

B.4 Channel ontology and derived selection

This section separates four statements that earlier versions ran together. Positive oriented matching selects one transmitted multiplet V_{2j_0} . The choice of that matching rule is part of the primitive ultraviolet architecture and has a spectral falsifier. The marked-fusion identification conditionally fixes $j_0 = \frac{3}{2}$ and the sharp kernel. If that incidence identification is not imposed, $j_0 = \frac{3}{2}$ remains the surviving member of the discrete audit in B.5.

B.4.1 Setup

Two cells share a face. Each contributes a primitive spin- j_0 slot, so the two-sided space is

$$\mathcal{H}_f = V_{j_0} \otimes V_{j_0} = \bigoplus_{J=0}^{2j_0} V_J.$$

The decomposition is multiplicity-free. The face exports a classical weight alphabet to the admissibility ensemble. The microscopic questions are which coupled sector is transmitted, how the untransmitted information is retained, and why the four port assignments are injective.

B.4.2 Positive oriented matching

Scope. Oriented gluing alone does not select j_0 : every half-integer j_0 defines a discrete member with $4j_0 + 1$ transmitted weights. Sections B.4.2e–f give a stronger result conditional on one physical identification: the marked present/history fiber is the faithful record of all nonmaximal primitive-fusion information. Under that identification the independently fixed spin-one closure response uniquely gives $j_0 = \frac{3}{2}$. No smallest-spin rule is used.

B.4.2a Face-sector Hessian

Proposition (exact block decomposition). If the gauge-fixed face Hessian commutes with the diagonal $SU(2)$ action, Schur’s lemma gives

$$\mathcal{H}^{(2)}(p) = \bigoplus_{J=0}^{2j_0} (Z_J p^2 + r_J) I_{V_J}.$$

Thus an $SU(2)$ -covariant quadratic kernel cannot mix different total-spin sectors. Its dynamical content is the finite spectrum $\{Z_J, r_J\}$.

For invariant block densities

$$\rho_J = \sum_{M=-J}^J |\sigma_{JM}|^2,$$

write the radial part of the homogeneous potential as

$$V_{\text{eff}} = \sum_J r_J \rho_J + \frac{1}{2} \sum_J u_J \rho_J^2 + \sum_{J < K} w_{JK} \rho_J \rho_K.$$

The first instability lies in the block with the smallest r_J . A pure J_* saddle, with $\rho_{J_*}^{(0)} = -r_{J_*}/u_{J_*}$, is locally stable against an unoccupied block J when

$$m_{J|J_*}^2 = r_J - \frac{w_{JJ_*}}{u_{J_*}} r_{J_*} > 0.$$

These are the general tests. The minimal matching action below evaluates them explicitly.

B.4.2b Exact coherent-matching kernel

The same geometric face has opposite outward orientations in its two incident cells. Let g_{LR} transport the right frame into the left frame and define the right generator in the common orientation by

$$\tilde{\mathbf{J}}_R = -\text{Ad}_{g_{LR}} \mathbf{J}_R.$$

Perfect matching compares equal oriented data. If

$$|j, \mathbf{n}\rangle = D^{(j)}(g_{\mathbf{n}})|j, j\rangle$$

is a spin coherent state, the matched pair is

$$|j, \mathbf{n}\rangle_L \otimes |j, \mathbf{n}\rangle_{\tilde{R}}.$$

The highest-weight identity

$$|j, j\rangle \otimes |j, j\rangle = |2j, 2j\rangle$$

and diagonal covariance imply

$$|j, \mathbf{n}\rangle \otimes |j, \mathbf{n}\rangle = F_{2j}^\dagger |2j, \mathbf{n}\rangle,$$

where $F_{2j} : V_j \otimes V_j \rightarrow V_{2j}$ is the maximal-channel Clebsch–Gordan coisometry.

The direction-independent positive Gram operator of exact matching is

$$\mathcal{G}_{\text{sharp}} = (4j + 1) \int_{S^2} \frac{d\Omega_{\mathbf{n}}}{4\pi} |j, \mathbf{n}; j, \mathbf{n}\rangle \langle j, \mathbf{n}; j, \mathbf{n}|.$$

The coherent-state resolution of the identity on V_{2j} gives

$$\boxed{\mathcal{G}_{\text{sharp}} = F_{2j}^\dagger F_{2j} = P_{2j}.}$$

Thus exact oriented matching transmits the maximal coupled multiplet. At $j_0 = \frac{3}{2}$,

$$\mathcal{G}_{\text{sharp}} = P_3, \quad \dim V_3 = 7.$$

The seven-state link is the image of the pair operator on the full sixteen-state space; it is not inserted as a separate alphabet after the tensor product has been formed.

The contraction alternative. Ordinary ε -contraction is a different microscopic operation. It pairs dual legs and leaves an unfused alphabet of $2j_0 + 1$ states rather than transmitting V_{2j_0} . At $j_0 = \frac{3}{2}$ this gives four letters and the 48-state branch in B.5. The two operations therefore define distinct gluing ontologies with distinct closure spectra. The present theory adopts oriented fusion because the face must continue to carry closure data after gluing.

B.4.2c Finite-width completion and its falsifier

The quantum mismatch of two perfectly aligned spin- j_0 slots has the irreducible floor $2j_0$. Subtracting that floor defines

$$Q_f = \frac{1}{2} \left[\left(\mathbf{J}_L - \tilde{\mathbf{J}}_R \right)^2 - 2j_0 I \right].$$

On V_J ,

$$Q_f|_{V_J} = \frac{1}{2} [(2j_0)(2j_0 + 1) - J(J + 1)] I_{V_J}.$$

Hence $Q_f \geq 0$ and its null space is exactly V_{2j_0} . The positive finite-width transfer

$$T_s = e^{-sQ_f}, \quad s > 0,$$

forms a semigroup and converges to P_{2j_0} as $s \rightarrow \infty$. For $j_0 = \frac{3}{2}$,

$$\boxed{Q_f = 6P_0 + 5P_1 + 3P_2, \quad T_s = e^{-6s}P_0 + e^{-5s}P_1 + e^{-3s}P_2 + P_3.}$$

A minimal homogeneous action on the complete primitive space is

$$\Gamma_f = \int d^d x \left[Z |\partial \Phi|^2 + r \Phi^\dagger \Phi + \kappa \Phi^\dagger Q_f \Phi + \frac{u}{2} (\Phi^\dagger \Phi)^2 \right], \quad Z, \kappa, u > 0.$$

Its potential is

$$V = r\rho + \frac{u}{2}\rho^2 + 6\kappa\rho_0 + 5\kappa\rho_1 + 3\kappa\rho_2, \quad \rho = \sum_{J=0}^3 \rho_J.$$

For $r < 0$, the fusion-sector support of the homogeneous minimum is uniquely $J = 3$:

$$\rho_3 = -\frac{r}{u}, \quad \rho_0 = \rho_1 = \rho_2 = 0.$$

The orthogonal blocks have positive masses

$$m_{0|3}^2 = 6\kappa, \quad m_{1|3}^2 = 5\kappa, \quad m_{2|3}^2 = 3\kappa.$$

At fixed total density, every nonmaximal occupation raises the action, so mixed- J support is excluded globally in this minimal model.

Theorem (maximal-fusion blocking ray). Let N transmitted cells each carry the selected irrep V_3 . The highest-spin irrep V_{3N} occurs with multiplicity one in $V_3^{\otimes N}$. Therefore the iterated maximal-channel Clebsch–Gordan coisometries define, after one consistent phase convention, a unique map

$$F_N : V_3^{\otimes N} \longrightarrow V_{3N}$$

independent of the binary fusion tree. Associativity here is not an additional dynamical assumption: every binary tree that always chooses the maximal coupled irrep ends in the same multiplicity-one copy of V_{3N} , so two such intertwiners differ only by an overall phase.

For spin coherent states the map is exact,

$$F_N |3, \mathbf{n}\rangle^{\otimes N} = |3N, \mathbf{n}\rangle.$$

Thus a fixed microscopic $j = 3$ alphabet is compatible with an arbitrarily large semiclassical block spin on the selected maximal-channel branch; the elementary representation does not run under this canonical blocking map.

Lemma (equivariant lock under maximal blocking). Let $L_3 : V_3^{\text{cap}} \rightarrow V_3^{\text{geom}}$ be the normalized one-cell equivariant lock of Appendix Q, and let F_N^{cap} and F_N^{geom} denote normalized maximal-channel fusion maps on the two factors. Because V_{3N} occurs with multiplicity one in $V_3^{\otimes N}$, the two normalized $SU(2)$ -equivariant maps from $(V_3^{\text{cap}})^{\otimes N}$ to V_{3N}^{geom} can differ only by a phase. Hence there is $e^{i\phi_N}$ such that

$$F_N^{\text{geom}} L_3^{\otimes N} = e^{i\phi_N} L_{3N} F_N^{\text{cap}}.$$

Thus, conditional on the selected one-cell lock, its propagation to every maximal block is unique and parameter free. This is a representation-theoretic theorem; it does not by itself prove that the full interacting coarse-graining map dynamically projects arbitrary blocks onto the maximal channel.

A dynamical attraction toward the same ray is exhibited within the displayed positive all-pairs diagnostic model

$$H_N = \kappa \sum_{a < b} (9 - \mathbf{J}_a \cdot \mathbf{J}_b) = \frac{\kappa}{2} [(3N)(3N + 1) - \mathbf{J}_{\text{tot}}^2].$$

It obeys

$$\ker H_N = V_{3N}, \quad \Delta_N \equiv E_{3N-1} - E_{3N} = 3\kappa N.$$

The increasing gap is a diagnostic of maximal-spin phase selection; the exact fusion map above does not require an all-to-all microscopic interaction.

The same conclusion is visible directly in the face mismatch. For equal spin j ,

$$Q_j = j^2 - \mathbf{J}_L \cdot \tilde{\mathbf{J}}_R,$$

and on coherent data separated by angle θ ,

$$\langle Q_j \rangle = j^2(1 - \cos \theta) = \frac{1}{2}j^2\theta^2 + O(\theta^4).$$

Hence e^{-sQ_j} has angular width $\Delta\theta \sim (j\sqrt{s})^{-1}$. On a maximal block $j = 3N$ the accepted geometric matching therefore sharpens rather than broadens as N grows.

Corollary (additive channel area and the coarse area operator). The selected factorized completion of Appendix Q locks each transmitted capacity multiplet V_3^{cap} to a geometric V_3^{geom} in the same relational orientation. On the coherent branch the geometric face variable is the flux vector itself: a spin- J coherent face carries the classical/Regge area proportional to J , while the kinematical LQG Casimir operator has eigenvalue proportional to $\sqrt{J(J+1)}$ [89, 90]. For N maximally fused primitive faces,

$$J_N = 3N.$$

Area additivity of the N fine facets therefore fixes the physical coarse area observable uniquely. If

$$\hat{A}_{\text{LQG}}(J) = 8\pi\gamma_{\text{BI}}L_*^2\sqrt{J(J+1)},$$

then the renormalized operator on the maximal-fusion ray is

$$\hat{A}_{\text{phys}}^{(N)} = Z_A(N)\hat{A}_{\text{LQG}}(3N), \quad Z_A(N) = \frac{3N}{\sqrt{3N(3N+1)}} = \sqrt{\frac{3N}{3N+1}}.$$

Hence

$$\hat{A}_{\text{phys}}^{(N)} = 24\pi\gamma_{\text{BI}}NL_*^2, \quad Z_A(1) = \frac{\sqrt{3}}{2}, \quad Z_A(N) \rightarrow 1.$$

The microscopic spin remains fixed. The factor $Z_A(N)$ renormalizes the coarse observable so that one irrep V_{3N} represents the additive area of N coherent V_3 facets. It reconciles the exact small- J Casimir spectrum with the linear area of the Regge phase, and approaches unity in the large-block limit. Appendix F.5 fixes the remaining dimensionless normalization γ_{BI} from the one-bit cut entropy.

Controlled semiclassical-refinement witness. The exact blocking ray can be embedded in the established spin-foam semiclassical–continuum limit of Refs. [97, 98]. To avoid collision with the couplings elsewhere in this paper, denote their typical large-spin parameter by Λ_{sf} . Their allowed family includes

$$\Lambda_{\text{sf}}(\mu) = \Lambda_0\mu^{-2+u}, \quad 0 < u < \frac{2}{5}, \quad \mu \rightarrow 0.$$

Choose $u = 1/4$ and identify the spin-foam large-spin scale only up to the fixed normalization carried by the blocked representation,

$$N(\mu) = \left\lfloor N_0\mu^{-7/4} \right\rfloor, \quad J_{\text{block}} = 3N, \quad \Lambda_{\text{sf}} \propto J_{\text{block}} = 3N.$$

The constant factor is asymptotically irrelevant and is absorbed into Λ_0 ; no equality between the symbol Λ_{sf} and the cell count is required. One admissible regulator choice is

$$\delta(\mu) = \delta_0\mu^{3/4},$$

which lies inside the semiclassical–continuum window for sufficiently small μ . The physically relevant refinement variable is the Regge edge length measured in the running infrared unit μ^{-1} ,

not an identification of one capacity cell with a fixed geometric edge. With $\ell_{\text{block}} \sim \sqrt{\gamma \Lambda_{\text{sf}}} L_P$ and $L_P(G_*) = L_*$ on the matched normalization,

$$\ell_{\text{rel}}(\mu) \equiv \mu \ell_{\text{block}} \propto \mu^{1/8} \longrightarrow 0$$

(up to the fixed dimensionless spin-foam normalization). This use of L_* fixes the gravitational unit; it does not assign a literal edge length L_* to the microscopic capacity cell. Along this explicit witness trajectory,

$$\boxed{\Lambda_{\text{sf}}^{-1} = O(\mu^{7/4}), \quad \Delta\theta = O(\mu^{7/4}), \quad \delta = O(\mu^{3/4}), \quad \ell_{\text{rel}}^2 = O(\mu^{1/4}).}$$

Along this trajectory, the finite-spin correction, capacity-matching width, transverse spin-sum regulator, and leading lattice-curvature correction all vanish while the microscopic constituent remains exactly $j = 3$. In the controlled linearized sector, Ref. [98] proves that the low-energy excitations exhaust the smooth solutions of the linearized Einstein equations: the two helicity-2 graviton modes. Ref. [97] gives the broader Lorentzian/Euclidean Einstein–Regge semiclassical-continuum construction. No general nonlinear convergence theorem is claimed here.

The microscopic test is finite. For the reduced physical Hessian define

$$h_J = \frac{1}{2J+1} \text{Tr}_{V_J} (P_J \mathcal{H}_f P_J).$$

The weak pass condition is

$$h_3 < h_2, \quad h_3 < h_1, \quad h_3 < h_0.$$

The minimal-kernel target is

$$\boxed{(h_0 - h_3) : (h_1 - h_3) : (h_2 - h_3) = 6 : 5 : 3.}$$

After condensation the remaining tests are $h_J + w_{J3}\rho_3 > 0$ for $J = 0, 1, 2$. The construction fails if another block is lighter, inequivalent J sectors mix after gauge reduction, a stable mixed- J minimum survives, or the primitive transfer is not positive.

B.4.2d Primitive tensor and effective pushforward

For a shared face (ab) , let Q_{ab} denote the transported mismatch operator above. A concrete finite-width primitive tensor is

$$\mathcal{G}_{v,s}^{(0)} = \mathcal{P}_v^{\text{inv}} \left[\bigotimes_{(ab) \in E(v)} e^{-sQ_{ab}} \right] \mathcal{P}_v^{\text{inv}},$$

where $\mathcal{P}_v^{\text{inv}}$ is the cellwise gauge/intertwiner projector. The zero-parameter primitive theory is defined by its sharp form,

$$\boxed{\mathcal{G}_{v,\sharp}^{(0)} = \mathcal{P}_v^{\text{inv}} \left[\bigotimes_{(ab) \in E(v)} P_3^{(ab)} \right] \mathcal{P}_v^{\text{inv}}.}$$

The finite- s family is a positive regulator and a spectral falsifier, not an additional coupling of the sharp theory.

Let

$$F_3 : V_{3/2} \otimes V_{3/2} \longrightarrow V_3$$

obey $F_3^\dagger F_3 = P_3$ and $F_3 F_3^\dagger = I_{V_3}$. On an isolated face,

$$\boxed{F_3 T_s F_3^\dagger = I_{V_3}}$$

for every $s > 0$. For the complete gauge-projected vertex, finite- s independence additionally requires $\mathcal{P}_v^{\text{inv}}$ to preserve the facewise coupled- J decomposition, or equivalently that the spectator/intertwiner dressing be channel-blind. This condition requires a reduced-kernel or commutator audit. The sharp P_3 support survives whenever its overlap is nonzero; finite-width spectral ordering remains an explicit test.

With $\mathcal{F}_v = \bigotimes F_3$, the post-fusion geometric tensor is

$$\mathcal{G}_v^{\text{eff}} = \mathcal{F}_v \mathcal{G}_{v,\sharp}^{(0)} \mathcal{F}_v^\dagger.$$

The marked-transfer vertex of Appendix H decorates this effective object. It is not itself the primitive pair kernel.

B.4.2e Exact marked complement and conditional sharp-kernel selection

Let a marked response be built from two spin- s strands. Then

$$\mathcal{H}_{\text{mark}}(s) = V_s^P \otimes V_s^H = \bigoplus_{J=0}^{2s} V_J.$$

Maximal fusion of two primitive spin- j slots transmits V_{2j} and leaves

$$Q(j) = (V_j \otimes V_j) \ominus V_{2j} = \bigoplus_{J=0}^{2j-1} V_J.$$

Theorem (unique complement match). The representations are isomorphic if and only if

$$\boxed{\mathcal{H}_{\text{mark}}(s) \cong Q(j) \iff j = s + \frac{1}{2}.$$

Proof. Both decompositions are multiplicity-free. The marked space contains one copy of each V_J from $J = 0$ through $2s$; the fusion complement contains one copy from $J = 0$ through $2j - 1$. They agree exactly when $2s = 2j - 1$. Equivalently,

$$\dim Q(j) = (2j + 1)^2 - (4j + 1) = 4j^2, \quad \dim \mathcal{H}_{\text{mark}}(s) = (2s + 1)^2,$$

and the positive solution is $2j = 2s + 1$. □

The marked closure response is independently a three-component vector and therefore has $s = 1$. Conditional on identifying that marked response with the complete fusion record,

$$\boxed{j_0 = \frac{3}{2}.$$

Explicit complement isometry. On $V_{3/2} \otimes V_{3/2}$ let

$$P_Q = P_0 + P_1 + P_2.$$

The blockwise equivariant isometry

$$\mathcal{I} : Q \longrightarrow V_1^P \otimes V_1^H$$

is defined by

$$\mathcal{I} |(j_0 j_0); J, M\rangle = |(1_P 1_H); J, M\rangle, \quad J = 0, 1, 2.$$

The independent present and history strands have ordered first-excitation basis

$$|a, b\rangle = |a\rangle_P \otimes |b\rangle_H, \quad \langle a, b | c, d \rangle = \delta_{ac} \delta_{bd}.$$

Let W map this basis to the nine hard-core marked states,

$$W|a, b\rangle = d_{ab}^\dagger |0\rangle.$$

The exact unit Gram matrix gives

$$W^\dagger W = I_9, \quad W W^\dagger = I_{\mathcal{H}_d}.$$

Define

$$L = W \mathcal{I} P_Q.$$

Then

$$\boxed{L^\dagger L = P_Q, \quad L L^\dagger = I_{\mathcal{H}_d}.$$

The complete sharp pair map is

$$U_{\text{sharp}} |\psi\rangle = F_3 P_3 |\psi\rangle \otimes |0\rangle_R + |\emptyset\rangle \otimes L |\psi\rangle.$$

The two flags are orthogonal, so

$$\boxed{U_{\text{sharp}}^\dagger U_{\text{sharp}} = P_3 + P_Q = I_{16}.$$

The seven-state link and nine-state marked record are therefore complementary outputs of an explicit reversible pair map.

Since V_0 , V_1 , and V_2 occur once, Schur's lemma implies that every other equivariant minimal complement isometry differs by one phase on each block,

$$L' = e^{i\phi_0} L_0 \oplus e^{i\phi_1} L_1 \oplus e^{i\phi_2} L_2.$$

If no microscopic operator compares the blocks coherently, these phases are marked-field conventions. A vertex that does compare them must derive their relative phases.

Why the unit marked weights select the sharp limit. For finite s , the system Kraus amplitude in the canonical dilation is $T_s^{1/2}$ and the complementary amplitude is $\sqrt{I - T_s}$. Its squared norms on the three nonmaximal blocks are

$$\ell_0^2(s) = 1 - e^{-6s}, \quad \ell_1^2(s) = 1 - e^{-5s}, \quad \ell_2^2(s) = 1 - e^{-3s}.$$

The H.9 marked vertex instead has one common quadratic normalization,

$$G_{\text{mark}} = I_9 = P_0 + P_1 + P_2.$$

Therefore, within the minimal incidence identification in which this marked vertex is the canonical fusion environment and no compensating J -dependent coupling is added, exact equality of the marked weights requires

$$\boxed{s \rightarrow \infty, \quad T_s \rightarrow P_3, \quad I - T_s \rightarrow P_Q.}$$

A finite physical width could survive only with additional block-dependent normalization. The sharp conclusion is conditional on the marked-fusion incidence identification; without it, the equal H.9 Gram matrix and the finite-width fusion transfer describe distinct objects and impose no constraint on one another.

B.4.2f Conditional dimensional endpoint

The closure defect is a spatial vector,

$$C_a, \quad a = 1, 2, 3.$$

In three spatial dimensions the vector representation of $SO(3)$ lifts to the spin-one irrep of $SU(2)$:

$$d = 3 \implies \mathbb{R}^3 \cong V_1 \implies s = 1.$$

With the marked-fusion incidence identification, the theorem above gives

$$s = 1 \implies j_0 = \frac{3}{2} \implies V_{2j_0} = V_3 \implies |M| = 7.$$

The minimal three-dimensional simplex has four faces, and the present/history vector record has $3^2 = 9$ states. Single-copy channel capacity makes the four port assignments injective, while the two global orientations give

$$\boxed{\Omega = 2P(7, 4) = 1680.}$$

The complete conditional integer chain is therefore

$$d = 3 \implies s = 1 \implies j_0 = \frac{3}{2} \implies \begin{cases} \text{ports} = 4, \\ \text{alphabet} = 7, \\ \text{record} = 9, \\ \Omega = 1680. \end{cases}$$

For the physical $d = 3$ rotation algebra one may summarize the evaluated identities as

$$\text{ports} = d + 1, \quad |M| = 2d + 1, \quad \dim \mathcal{H}_{\text{mark}} = d^2.$$

They are not a claimed continuation to arbitrary dimension. For $d \neq 3$, the vector representation belongs to $\text{Spin}(d)$ and need not be one $SU(2)$ spin. A dimension-general theorem would require repeating the complement analysis in that group.

B.4.3 Injectivity from single-copy channel capacity

The exclusion statement concerns a single cell resource, not Pauli antisymmetry among the four port labels.

Lemma. Assume:

1. the letters index a single copy of the cell-level fermionic channel family, with occupations $n_m \in \{0, 1\}$;
2. a face displaying m is, by definition, that channel occupation routed through the face's port;
3. the four ports are distinguishable relational registers.

Then the admissible boundary states are the injections $f : F_4 \rightarrow M$, and their number before orientation doubling is $P(|M|, 4)$.

Proof. If two ports displayed the same letter m , the one cell resource would require $n_m = 2$, contrary to its single-copy ceiling. The assignment is therefore injective. The ports are distinct interfaces to distinct neighbors, so permuting letters between ports changes the routing state; the assignments are ordered. Antisymmetry acts in the occupation algebra of each channel, not by quotienting the relational port index. $\square$

This lemma does not follow from generic fermionic statistics alone. Its physical content is that the four ports draw from one shared channel multiplet rather than four independent copies. That is the same single-copy channel ontology used by the one-bit defect and determinant sectors.

B.4.4 Uniform base measure

Lemma. If the selected face multiplet realizes its full Shannon capacity, its unconditioned base measure is uniform.

Proof. For a probability measure μ on M ,

$$H(\mu) \leq \ln |M|,$$

with equality only at $\mu(m) = 1/|M|$. Full use of the fixed alphabet therefore selects the uniform prior. $\square$

This statement concerns the reference measure, not the final closure-weighted ensemble:

$$p_\eta(b) = \frac{1}{Z(\eta)} \underbrace{\mu_0(b)}_{\text{uniform base measure}} e^{-\eta K^2(b)}.$$

Covariance alone does not force uniformity, because the face normal permits covariant deformations such as functions of $(\mathbf{J} \cdot \hat{n})^2$. Full-capacity saturation excludes those deformations at the prior level; the closure observable then supplies the physical nonuniformity.

B.4.5 Status ledger

Derived from the primitive matching rule. For any fixed half-integer j_0 , exact oriented coherent matching gives the transmitted block V_{2j_0} and an alphabet of $4j_0 + 1$ states. The single-copy channel ontology gives injectivity; relational ports preserve ordered assignments; and the global face orientation gives the binary factor.

Derived conditional on marked-fusion incidence. The exact relation

$$\mathcal{H}_{\text{mark}}(s) \cong Q(j) \iff j = s + \frac{1}{2}$$

combines with the three-dimensional vector response $s = 1$ to fix $j_0 = \frac{3}{2}$, seven transmitted states, nine complement states, and $\Omega = 1680$. The exact unit normalization of the marked vertex selects the sharp P_3 kernel within the minimal canonical dilation.

Microscopic premise. The primitive action implements positive oriented matching. The displayed sharp tensor $\mathcal{G}_{v,\sharp}^{(0)}$ is the minimal completion. A more fundamental simplicial calculation must still reproduce its $J = 3$ support or pass the weaker Hessian inequalities of B.4.2c.

Exact but not yet dynamically identified. The representation equivalence, $L^\dagger L = P_Q$, $LL^\dagger = I_9$, and $U_{\text{sharp}}^\dagger U_{\text{sharp}} = I_{16}$ are exact. What remains conjectural is that this complement map is precisely the complete H.9 marked dynamics, including its flags, hard-core mark, phases, and routing.

Data status. Without the incidence identification, $j_0 = \frac{3}{2}$ is selected by the postdictive discrete audit below. With it, the same audit checks an internally selected branch. In neither reading does a continuous parameter choose the alphabet.

B.5 Relaxation and branch audits

The following tables propagate nearby discrete constructions through one fixed downstream map. They are alternatives, not samples from a probability distribution, so “sigma” significances do not apply. The marked sector keeps nine response polarizations. A branch with alphabet size n uses $\binom{n}{2}$ pair records, the scalar factor $2/n$, and the second-return coefficient n . Because the Newton target informed the historical construction, Appendix L grades this as a conditional postdictive comparison.

B.5.1 Fusion and constituent-spin branches

branch	Ω	$g_{\text{share,eff}}$	$n g_{\text{share,eff}}$	G_*/G
$j_0 = \frac{1}{2}$ full reducible space	48	3.8712	15.5	7×10^{31}
$j_0 = \frac{3}{2}$ unfused contraction	48	3.8712	15.5	8×10^{31}
$j_0 = \frac{3}{2}, J = 2$	240	5.4785	27.4	2×10^{21}
$j_0 = \frac{3}{2}, J = 3$	1680	7.4198	51.9	1.0000
$j_0 = \frac{3}{2}, J = 5$	15840	9.6572	106.2	7×10^{-48}
$j_0 = \frac{3}{2}$ full reducible space	87360	11.2616	180.2	4×10^{-112}

The two 48-state rows have the same count but different representation content: the first mixes V_0 and V_1 , while the second carries the unfused $V_{3/2}$ alphabet. The family spans about 10^{143} in G_*/G . Holding the rest of the construction fixed, the seven-state branch is the only row near the measured Newton scale.

B.5.2 Counting conventions at seven letters

convention	Ω	$g_{\text{share,eff}}$	G_*/G	m_μ/m_e
$2P(7,4)$	1680	7.4198	1.0000	206.7683
drop orientation doubling	840	6.7267	1.7×10^4	207.82
allow repeated labels	4802	8.4483	5.5×10^{-7}	206.09
quotient port permutations	70	4.2417	3.2×10^{19}	231.01
quotient ports and orientation	35	3.5486	1.1×10^{24}	256.31

These rows change the counting convention while retaining the same seven-letter closure formula and downstream transport. The baseline is the only displayed convention that keeps the Newton and charged-lepton comparisons simultaneously near their measured values.

B.5.3 One continuous prior deformation

To test one nearby nonuniform prior, introduce the parity-even, face-axis-preserving tilt

$$w_\lambda(b) = \exp \left[-\lambda \sum_{i=1}^4 m_i^2 \right]$$

and re-solve the closure condition at each λ . At $\lambda = 0$, exact finite differences of the enumerated ensemble give

$$\frac{dg_{\text{share,eff}}}{d\lambda} = -0.278458, \quad \frac{d \ln G_*}{d\lambda} = 3.90135, \quad \frac{d \ln(m_\mu/m_e)}{d\lambda} = 1.37074 \times 10^{-3}.$$

With the present CODATA uncertainty and the baseline residual retained, the one-standard-deviation Newton interval along this one deformation is approximately

$$-5.3 \times 10^{-6} < \lambda < 6.2 \times 10^{-6}.$$

For the muon ratio, $|\Delta\lambda| \simeq 1.6 \times 10^{-5}$ produces one current experimental standard deviation locally. This is a sensitivity scale, not a symmetric bound about zero, because the baseline prediction is not exactly centered. These statements constrain one specified deformation conditional on the frozen downstream map; they do not directly measure the microscopic prior or exhaust all nonuniform measures. Parity-odd tilts have zero linear response by the $m \mapsto -m$ symmetry and require a separate quadratic audit.

Provenance. The entropy near 7.42 was recognized historically by inverting the measured Newton constant, and the decorated vertex was constructed after the remaining Newton and lepton residuals were known. Appendix L retains that chronology. The new result is structural: once the primitive oriented-matching rule and the marked-fusion incidence identification are adopted, the sequence

$$d = 3 \implies V_1 \implies j_0 = \frac{3}{2} \implies V_3 \implies 7 \implies 1680 \implies g_{\text{share,eff}} = 7.419800\dots$$

contains no adjustable continuous parameter. The branch tables show how sharply nearby stated constructions move the downstream observables; they do not turn the original comparison into a historically blind prediction.

Appendix C: Edge Kernel, Finite Renormalization, and Continuum Matching

Appendix C carries the local boundary ensemble of Appendix B into edge transport, loop dressing, and the continuum stiffness coefficient of the weak-field EFT.

C.1 Channel-averaged isotropy identity and tree coupling

Let $\hat{n}_i$ be the four face normals of a regular tetrahedron. The exact identity

$$\sum_{i=1}^4 \hat{n}_i \hat{n}_i^\top = \frac{4}{3} I_3$$

implies a channel-averaged transverse fraction of $2/3$. The bare edge stiffness is therefore

$$J_{\text{bare}} = \frac{2}{3} \eta_* = 0.0199112296.$$

For a rooted $z = 4$ coarse adjacency graph, the tree-to-lattice map gives

$$J_{\text{eff}}^{\text{tree}} = \frac{J_{\text{bare}}}{z-1} = \frac{2\eta_*}{9} = 0.0066370765.$$

This is the first place where local closure data become a transport law. The tetrahedral identity fixes the isotropic projection, and the rooted branching structure determines how much of the microscopic edge penalty survives as net outward propagation on the coarse graph.

C.2 Horizon target and shell convergence

The horizon-capacity target is

$$\sigma_* = \frac{\pi}{g_{\text{share,eff}}} = 0.42340665.$$

At the derived coupling the explicit shell values are

$$\sigma_{\text{ind}}^{(2)} = 0.42143, \quad \sigma_{\text{ind}}^{(3)} = 0.42166, \quad \Delta_{2 \rightarrow 3} = 0.00023.$$

The residual shift from the target is already small and stable by shell depth $r = 2$, isolating the remaining correction to the loopy local-return sector rather than a broad nonlocal ambiguity.

The shell calculation places the tree branch near the target and assigns the residual discrepancy to local returns. No broad long-range correction is required at this order.

C.3 Finite-loop self-energy closure

The minimal loopy correction is organized as a local Dyson dressing:

$$J_{\text{eff}}^{(\text{ren})} = \frac{J_{\text{eff}}^{\text{tree}}}{1 + J_{\text{eff}}^{\text{tree}} \Sigma_{\text{ret}}}.$$

The specified minimal return operator uses

$$\Sigma_{\text{ret}} = 7 + \frac{2}{9} = \frac{65}{9},$$

The two terms have distinct return-channel origins. A short return motif leaves a shared face, explores a local closed loop, and re-enters the same coarse edge before contributing to net long-range transport. In the canonical label basis $m = -3, -2, \dots, 3$, the minimal operator contains seven label-diagonal returns, one for each face-label channel, contributing

$$\text{Tr}(I_7) = 7.$$

In addition to these label-preserving loops, permutation symmetry allows one collective mode shared across all channels. Writing

$$P_{\text{sing}} = |u\rangle\langle u|, \quad u = \frac{1}{\sqrt{7}}(1, 1, \dots, 1),$$

this shared return is rank one. Permutation symmetry permits this singlet but does not fix its weight relative to the diagonal returns or exclude longer return motifs. The minimal completion assigns it the same $2/3$ transverse projection used in the tree coupling and the rooted return factor $1/(z-1) = 1/3$ on the $z = 4$ graph. The collective contribution is then

$$\text{Tr}\left(\frac{2}{3} \frac{1}{3} P_{\text{sing}}\right) = \frac{2}{9},$$

since $\text{Tr}(P_{\text{sing}}) = 1$. Equivalently,

$$R_{\text{ret}} = I_7 + \frac{2}{9} P_{\text{sing}}, \quad \Sigma_{\text{ret}} = \text{Tr}(R_{\text{ret}}) = 7 + \frac{2}{9}.$$

Thus $65/9$ is fixed inside the stated minimal return operator, not derived from the closure ensemble alone. A microscopic graph-return calculation must derive the relative diagonal and

singlet weights, test longer motifs, and determine whether the operator closes on these channels. Within this conditional completion,

$$c_{\text{loop}}^{(\text{ren})} \equiv \frac{J_{\text{eff}}^{(\text{ren})}}{J_{\text{eff}}^{\text{tree}}} = \frac{1}{1 + J_{\text{eff}}^{\text{tree}} \Sigma_{\text{ret}}} \approx 0.95426,$$

and

$$J_{\text{eff}}^{(\text{ren})} \approx 0.00633348.$$

This reproduces the shell-target crossing near $J_{\text{bare},\text{cross}} \sim 0.019$ at the stated level of agreement.

The local Dyson dressing corrects the tree branch for short motifs that recycle amplitude before it contributes to coarse transport. Summing those returns determines the renormalized coupling from the tree coupling.

C.4 Euclidean-action normalization and continuum stiffness

The lattice quadratic form is interpreted canonically as a Euclidean action weight,

$$\frac{I_E}{\hbar} = \frac{J_{\text{eff}}^{(\text{ren})}}{2} \sum_{a,i} (Q_a - Q_{a+L_* \hat{n}_i})^2,$$

where a runs over one sublattice representative of each bipartite primitive cell and i runs over its four outgoing bonds. Every undirected nearest-neighbor edge is counted once. This convention is essential: summing the four bonds from both sublattices would double the variation and the source normalization. The microscopic four-cell is

$$\Delta V_4 = \frac{L_*^4}{c},$$

a coarse-graining convention rather than derived cell geometry (Section 9; the abstract cell complex has no regular-tetrahedron Euclidean embedding). Because every edge occurs once, varying the discrete action gives $J_{\text{eff}}^{(\text{ren})} L_\diamond Q = s$ for a source term $-\sum_a s_a Q_a$, with $L_\diamond = 4I - A$. The diagonal inverse of this same unnormalized Laplacian is the $G_{\text{tet}}(0)$ used in C.5, so a point source obeys $Q(0) = s G_{\text{tet}}(0) / J_{\text{eff}}^{(\text{ren})}$. This removes the factor-of-two ambiguity between the stiffness and source conventions. The same tetrahedral identity then yields

$$\gamma_Q = \frac{4\hbar c}{3L_*^2} J_{\text{eff}}^{(\text{ren})}$$

for the occupancy field Q_{occ} . With the horizon-capacity normalization

$$S = \pi Q_{\text{occ}},$$

the canonical convention $\frac{\gamma}{2}(\partial S)^2$ gives

$$\gamma = \frac{4\hbar c}{3\pi^2 L_*^2} J_{\text{eff}}^{(\text{ren})} = \frac{4\hbar c}{3\pi^2 L_*^2} \frac{2\eta_*/9}{1 + (2\eta_*/9)(65/9)}.$$

Using the substrate-induced scale

$$G_* := \frac{c^3 L_*^2}{\hbar},$$

this is

$$\gamma = \frac{4J_{\text{eff}}^{(\text{ren})}}{3\pi^2} \frac{c^4}{G_*} \approx 8.556 \times 10^{-4} \frac{c^4}{G_*}.$$

This step converts the dimensionless lattice weighting into the dimensionful continuum stiffness used by the weak-field action, after Euclidean normalization and faithful sector-resolution scale setting.

C.5 Local defect insertion and the source-side lattice constant

The stiffness-side matching is not the only UV quantity that can be closed locally. For the canonical rigid defect insertion, excluding one of the seven admissible face labels from one face removes exactly one-seventh of the isotropically averaged local partition weight. Therefore the logarithm of the isotropically averaged partition ratio is exactly

$$\Delta S_{\text{def}} := -\ln \left\langle \frac{Z_{\text{def}}}{Z_{\text{vac}}} \right\rangle_{\text{iso}} = \ln \frac{7}{6}.$$

This is the exact isotropic source benchmark in the canonical seven-label ensemble. The isotropically averaged defect free-energy cost differs from it only at $O(10^{-5})$ because the admissibility weighting breaks label symmetry only weakly.

The local source benchmark $\ln(7/6)$ should not be confused with the elementary fermionic one-bit anchor $\ln 2$. The former is the isotropically averaged partition-ratio shift produced by removing one admissible label from the seven-label boundary ensemble. The latter is the intrinsic binary entropy of an elementary occupied/unoccupied fermionic face-exclusion defect. The source theorem uses $\ln(7/6)$ to normalize the local scalar insertion into the lattice response, while the electron anchor uses $\ln 2$ to fix the mass-entropy unit of the elementary fermionic defect.

Exclusion sufficiency lemma. The reduction of the 1680-state boundary ensemble to a single scalar source is, at the classical source level, exact rather than approximate. The canonical exclusion defect is the vacuum ensemble conditioned on the exclusion event A , $P_{\text{def}} = P_{\text{vac}}(\cdot | A)$, so its likelihood ratio is

$$\frac{dP_{\text{def}}}{dP_{\text{vac}}} = \frac{\mathbf{1}_A}{P_{\text{vac}}(A)},$$

a function of the exclusion indicator $X = \mathbf{1}_A$ alone. The indicator is therefore a sufficient statistic for distinguishing the defect ensemble from the vacuum, and coarse-graining onto it preserves the full classical relative information,

$$D(P_{\text{def}} \| P_{\text{vac}}) = -\ln P_{\text{vac}}(A) = D(P_{\text{def}}^X \| P_{\text{vac}}^X).$$

The scope of the lemma is exactly its statement: one scalar statistic carries all the local classical source information distinguishing the canonical exclusion-defect ensemble from the vacuum. It does not by itself establish that the condensate has a single infrared mode, that orientation or defect-species structure is dynamically irrelevant, or that spatial correlations carry no further information; those are dynamical questions, and the open one is stated in Appendix H.7. For independent exclusions the source is exactly additive, $-\ln P(\bigcap_i A_i) = \sum_i -\ln P(A_i)$, so the leading dilute source is linear in the defect number, with correlation corrections entering at pair order in the density.

To propagate that local defect into the lattice field equation one needs the on-site Green function of the tetrahedral/diamond nearest-neighbor Laplacian. The four-valent diamond graph is adopted as the coarse adjacency realization of the abstract face-sharing tetrahedral boundary complex; the complex enters combinatorially, since regular tetrahedra admit no face-to-face Euclidean tessellation (the dihedral angle $\arccos(1/3) \simeq 70.53^\circ$ does not divide 360°), so the adopted graph and its Green function are the working objects while cell volumes and face areas remain conventions of the coarse map. Eliminating the two-sublattice structure gives the standard Brillouin-zone representation for the diamond lattice Green function [5]

$$G_{\text{tet}}(0) = \frac{1}{(2\pi)^3} \int_{[-\pi, \pi]^3} \frac{4d^3k}{16 - |1 + e^{ik_1} + e^{ik_2} + e^{ik_3}|^2}.$$

This integral is the reproducible source-side lattice constant: it is the self-energy of a unit point insertion for the same scalar mode whose long-wavelength stiffness was matched in Appendix C.4. Joyce’s exact evaluation of the diamond-lattice Green function, in this normalization, gives [7, 5]

$$G_{\text{tet}}(0) = \frac{3\Gamma(1/3)^6}{2^{14/3}\pi^4} = 0.4482203943883814\dots$$

Thus the source-side graph constant is an exact lattice invariant rather than a fitted numerical coefficient. Direct quadrature with endpoint extrapolation reproduces the same value.

Green-tensor transverse response. The same graph response also fixes the transverse export weight $w_\perp$, the single graph quantity that propagates downstream into the horizon channel count of Appendix F.5 and into the global $\ln(3/2)$ closure-saturation factor of the scale-setting relation. We compute it directly from the four nearest-neighbor bond frame, with no fitting freedom.

Let d_i , $i = 1, \dots, 4$, be the four diamond nearest-neighbor bond directions,

$$d_1 = \frac{(1, 1, 1)}{\sqrt{3}}, \quad d_2 = \frac{(1, -1, -1)}{\sqrt{3}}, \quad d_3 = \frac{(-1, 1, -1)}{\sqrt{3}}, \quad d_4 = \frac{(-1, -1, 1)}{\sqrt{3}}.$$

They obey

$$\sum_{i=1}^4 d_i^a d_i^b = \frac{4}{3} \delta^{ab}.$$

Distributing the scalar on-site Green response over the tetrahedral bond frame gives

$$\mathcal{G}_{\text{loc}}^{ab} = G_{\text{tet}}(0) \frac{1}{4} \sum_i d_i^a d_i^b = \frac{G_{\text{tet}}(0)}{3} \delta^{ab}.$$

For a local horizon normal $\hat{r}$,

$$P_\perp^{ab} = \delta^{ab} - \hat{r}^a \hat{r}^b,$$

so

$$G_\perp = P_\perp^{ab} \mathcal{G}_{\text{loc}}^{ab} = \frac{2}{3} G_{\text{tet}}(0).$$

Thus the transverse export weight $w_\perp = 2/3$ is the transverse part of the exact local graph response.

Using the field normalization $S = \pi Q_{\text{occ}}$, the rigid local defect shift is

$$\delta Q_{\text{def}} = \frac{\Delta S_{\text{def}}}{\pi} = \frac{\ln(7/6)}{\pi}.$$

The corresponding local source amplitude in lattice units is therefore

$$s_{\text{def}} = J_{\text{eff}}^{(\text{ren})} \frac{\delta Q_{\text{def}}}{G_{\text{tet}}(0)},$$

so that

$$\frac{s_{\text{def}}}{J_{\text{eff}}^{(\text{ren})}} = \frac{\ln(7/6)}{\pi G_{\text{tet}}(0)} = 0.109472228\dots$$

is a pure number fixed by the same UV lattice geometry.

The Green-function constant turns the local insertion into a continuum source theorem. Defining the defect-entropy density by

$$\sigma_{\text{def}} = \frac{\rho}{\kappa_m(L_*)},$$

the tetrahedral projection used in the stiffness mapping gives

$$\nabla^2 \delta S = -\frac{3L_*}{4G_{\text{tet}}(0)} \sigma_{\text{def}}.$$

Equating this with the weak-field source equation

$$\nabla^2 \delta S = -\frac{\kappa}{\gamma} \rho$$

closes the canonical source-to-stiffness ratio:

$$\boxed{\frac{\kappa}{\gamma} = \frac{3L_*}{4G_{\text{tet}}(0)\kappa_m(L_*)}}.$$

This is the source-side counterpart of the stiffness derivation. The edge-kernel calculation fixes how the scalar capacity mode resists gradients; the Green-matched defect calculation fixes how localized matter defects source that same mode.

The three cancellations, made explicit. The passage from the dimensionless lattice insertion $s_{\text{def}}/J_{\text{eff}}^{(\text{ren})} = \ln(7/6)/(\pi G_{\text{tet}}(0))$ to the source theorem turns on three reductions, none of which leaves a free constant. (i) *The factor π cancels against the field normalization.* The insertion is written for the occupancy field, where one isotropically averaged defect shifts $\delta Q_{\text{def}} = \ln(7/6)/\pi$; a fixed defect type (ℓ, f) instead carries $-\ln P(A_{\ell f})$, ranging over $[0.14933, 0.15819]$ with mean 0.15415642 against $\ln(7/6) = 0.15415068$, so the source theorem uses the exact isotropic benchmark under the assumption that the coarse defect population is isotropically averaged over face and label types. Converting to the entropy field through $S = \pi Q_{\text{occ}}$ multiplies by π , so the S -field shift is $\delta S_{\text{def}} = \pi \delta Q_{\text{def}} = \ln(7/6)$ and the explicit π does not survive into the source theorem. (ii) *$\ln(7/6)$ is an isotropic source benchmark, not a universal per-defect cost.* For a coarse population averaged over face and label types, the exact mean partition ratio is $6/7$, so the corresponding annealed source normalization is $\ln(7/6)$. A fixed defect type instead carries $-\ln P(A_{\ell f})$. Writing the isotropically averaged source as $\sigma_{\text{def}} = \rho/\kappa_m(L_*)$ folds that benchmark and the mass-entropy unit into the density. The geometric coefficient $3L_*/(4G_{\text{tet}}(0))$ is fixed by the tetrahedral projection and the cell length and contains no $\ln(7/6)$. (iii) *$J_{\text{eff}}^{(\text{ren})}$ cancels in the ratio.* The source amplitude s_{def} and the stiffness γ each carry one power of the renormalized edge coupling, so it cancels in the source-to-stiffness ratio κ/γ , leaving the lattice-geometric constant $3L_*/(4G_{\text{tet}}(0)\kappa_m(L_*))$. The renormalized loop coupling therefore drops out of the observable normalization.

The absolute scale of S_∞ remains a fixed-epoch normalization convention in the bridge law, not a residual freedom in the source projection. In the cell-normalized gauge natural to the local source theorem, one may write

$$S_\infty^{\text{cell}} = \frac{3 \ln 2}{32\pi G_{\text{tet}}(0)} = 0.0461482516 \dots$$

In a horizon-normalized gauge, S_∞ instead carries the much larger apparent-horizon capacity. These are not two physical constants. A constant rescaling of the entropy field rescales S_∞ and κ/γ together, leaving $\kappa/(\gamma S_\infty)$ and hence G unchanged. Thus the weak-field source map is closed in the canonical branch once the mass-entropy map $\kappa_m(L_*)$, the tetrahedral Green constant, and the standard cell convention are specified.

C.6 Local susceptibility cross-check

The exact local moments of the admissibility-closed ensemble supply an independent non-degeneracy check on the source-side result. From the variance in Appendix B,

$$a_{\text{UV}} := \frac{1}{\text{Var}_{\eta_*}(K^2)} = 0.0637390269,$$

which is the local zero-mode inverse susceptibility of the closure scalar. Two features of this number matter for the source theorem in C.5. First, a_{UV} is finite and positive, confirming that the closed branch at η_* has a non-degenerate zero-mode response rather than a critical singularity that would invalidate the linear Green-function matching used to derive κ/γ . Second, the same local susceptibility controls the stability of the closure mode propagated into the shell and loop calculations, so the residual fractional discrepancy between the local benchmark and σ_* belongs to the loop sector rather than the local closure.

The actual closure of κ/γ in the canonical branch is the Green-matched theorem in Appendix C.5. The role of a_{UV} here is restricted to confirming non-degeneracy of the local mode being matched.

C.7 UV-to-IR payoff

At this stage the weak-field UV coefficient chain is explicit:

$$\Omega_{\text{tet}} \rightarrow g_{\text{share,eff}} \rightarrow L_* \rightarrow J_{\text{bare}} \rightarrow J_{\text{eff}}^{\text{tree}} \rightarrow \Sigma_{\text{ret}} \rightarrow J_{\text{eff}}^{(\text{ren})} \rightarrow \gamma.$$

The same chain feeds

$$a_0 = \frac{cH_0 g_{\text{share,eff}}}{4\pi^2},$$

and the decorated support-to-rate action gives the substrate-induced scale

$$G_* = \frac{c^3 L_*^2}{\hbar} = \frac{9}{4} \frac{\hbar c}{m_e^2} Z_e^2 \ln^2(1 - e^{-7g_{\text{share,eff}}}).$$

The Green-matched source theorem fixes

$$\frac{\kappa}{\gamma} = \frac{3L_*}{4G_{\text{tet}}(0)\kappa_m(L_*)}.$$

The weak-field bridge then converts this source-to-stiffness ratio into the observed Newtonian normalization through the invariant combination $\kappa/(\gamma S_\infty)$. The comparison with the same G_* assigned by the electron-anchored support-to-rate branch tests that this normalization propagates the substrate scale coherently; it is not a disjoint-input second derivation of G , since the matched route carries L_* within it.

The coefficients this appendix supplies to the main weak-field chain are J_{bare} , $J_{\text{eff}}^{\text{tree}}$, Σ_{ret} , $J_{\text{eff}}^{(\text{ren})}$, γ , and the Green-matched source projection κ/γ . The remaining uses of S_∞ belong to the fixed normalization of the weak-field bridge, not to the source sector itself.

Appendix D: Weak-Field Technical Derivations, Electron Anchor, and EFT Consistency

Appendix D collects the weak-field derivations that are central but too dense for the main line: the bridge law, Newtonian recovery, the electron anchor, and the EFT consistency checks.

D.1 Bridge law from the reduced action

The weak-field bridge is now fixed by an action equivalence rather than by extrapolating an exponential lapse. The Einstein scalar-constraint action reduces to

$$I_{\text{Newton}}[\Phi] = \int dt d^3x \left[-\frac{(\nabla\Phi)^2}{8\pi G} - \rho\Phi \right].$$

The capacity action becomes the same functional, up to the constant $Z_S = 2\kappa S_\infty/c^2$, under

$$\delta S = -\frac{2S_\infty}{c^2}\Phi.$$

Hence

$$\frac{\Phi}{c^2} = -\frac{\delta S}{2S_\infty}$$

is the unique linear field redefinition matching both the kinetic and source terms with the UV normalization. The bounded nonlinear rule is instead $N^2 = q = 1 - \delta S/S_\infty$ in a static branch. An exponential lapse shares the first derivative at the vacuum point but is not used globally because it has no finite-capacity endpoint. The complete derivation is in Appendix N.

D.2 Point source, Newton limit, and lensing

In the renormalized static branch,

$$\nabla^2 \delta S = -\frac{\kappa}{\gamma}\rho.$$

For a point source M ,

$$\delta S(r) = \frac{\kappa M}{4\pi\gamma r}, \quad g(r) = \frac{c^2\kappa}{8\pi\gamma S_\infty} \frac{M}{r^2} = \frac{GM}{r^2}.$$

For the ordinary longitudinal branch, variation of the unreduced Einstein scalar action gives

$$\Phi = \Psi$$

with asymptotically flat boundary conditions. This equality follows from the spatial metric constraint, not from a canonical capacity stress tensor. The effective-halo rewrite of the additional galactic response is

$$\rho_{\text{halo}}(r) = \frac{1}{4\pi G r^2} \frac{d}{dr} \left[r^2 (g_{\text{obs}} - g_{\text{bar}}) \right].$$

Thus the same metric potential controls orbital dynamics and light bending in the baseline Einstein branch. The separate carrier-resolved contact of Section 16 specifies $\Delta\Phi_\perp = \Delta\Psi_\perp$ at leading quasistatic order and uses the same induced density for support and lensing. Its nonlinear Ward identity depends on covariant variation of the carrier records, as Appendix N.8 states.

D.3 Electron anchor and composite matter

The canonical fermionic entropy increment is

$$\Delta S_f = \ln 2.$$

The UV mass normalization is

$$\kappa_{m,\text{UV}} = \frac{\hbar}{cL_*} \frac{1}{\ln 2},$$

and the running law in the closed branch is

$$\kappa_m(\ell) = \kappa_{m,\text{UV}} \left(\frac{L_*}{\ell} \right)^{1+\alpha_{\text{cl}}}, \quad \alpha_{\text{cl}} = 0.$$

At the electron Compton scale $\ell = \lambda_e$ this gives

$$\kappa_m(\lambda_e) = \frac{m_e}{\ln 2},$$

which is the elementary anchor used here. Composite hadrons are not reduced to a bare constituent count. Their mass budget is assigned to a dressed bound-state entropy

$$m_{\text{hadron}} = \kappa_m(\ell_H) S_{\text{ent},H}^{\text{dressed}},$$

whose microscopic decomposition must include confinement, gluonic structure, trace-anomaly contributions, and chiral vacuum reorganization.

The electron is an elementary one-bit defect and provides the dimensional anchor. Hadronic inertial content instead belongs to a dressed entropy budget that includes binding and vacuum structure; a bare constituent count is insufficient.

D.4 Faithful sector resolution and induced G_*

The electron anchor enters the gravitational normalization through the absolute substrate length. The exact inputs are the seven-channel entropy, the memoryless kernel selected by faithful resolution, channel factorization on the lightest branch, and the decorated marked-transfer vertex of Appendix H. The calculation below separates the determinant recurrence, positive survival gap, and finite marked response before combining them in the physical cell length.

The recurrence derivation of the dictionary. Appendices H.2, H.4, and H.5 show that faithful full-support resolution selects memoryless replacement and that the lightest branch has seven factorized channels. On the commutative boundary algebra define $(Rf)(b) = p_*(b)f(b)$ and $\tau_p(f) = \sum_b p_b f(b)$. The state-weighted determinant is

$$\Delta_{\tau_p}(R) = \exp[\tau_p(\ln R)] = \exp\left(\sum_b p_b \ln p_b\right) = e^{-g_{\text{share,eff}}}.$$

Multiplicativity gives $e^{-7g_{\text{share,eff}}}$ on the seven-channel product. This is the quenched geometric transfer rate of the record-conditioned likelihood operator. It is not the Born probability of a specified microstate or the collision probability of two renewed blocks.

The corresponding history statement follows from the exact recurrence theorem. For a stationary ergodic finite-alphabet process, if R_n is the first return time of a length- n block, then

$$\lim_{n \rightarrow \infty} \frac{1}{n} \ln R_n = h \quad \text{almost surely,}$$

where h is the Shannon entropy rate [69, 70]. This theorem selects Shannon entropy over raw state count and Rényi-2 entropy for long typical blocks. It does not by itself identify the seven distinguishable sector layers with a long temporal block, and it gives no exact finite- n return probability. For the present one-layer distribution,

$$H_1 = 7.4198000, \quad H_2 = -\ln \sum_b p_b^2 = 7.4125486, \quad H_\infty = -\ln \max_b p_b = 7.1343850.$$

The distinctions are numerically consequential after seven layers. The refresh projector $|\sqrt{p}\rangle\langle\sqrt{p}|$ carries the participation scale e^{H_2} , while the determinant and almost-sure Lyapunov rate use H_1 . They are different observables. With

$$p_{\text{rec}} \equiv e^{-7g_{\text{share,eff}}},$$

the positive survival operator has nonzero eigenvalue $1 - p_{\text{rec}}$. The decorated marked vertex supplies the electron factor Z_e , so the dressed electron gap is

$$E_e = -\frac{3\hbar Z_e}{2\tau_*} \ln(1 - p_{\text{rec}}).$$

Identifying the lowest charged gap with $m_e c^2$ fixes the cadence from the already-declared electron anchor,

$$\tau_* = -\frac{3}{2} Z_e \tau_e \ln(1 - p_{\text{rec}}), \quad L_* = c\tau_*.$$

Thus

$$L_* = -\frac{3}{2} Z_e \lambda_e \ln(1 - e^{-7g_{\text{share,eff}}}) = \frac{3}{2} Z_e \lambda_e e^{-7g_{\text{share,eff}}} [1 + O(e^{-7g_{\text{share,eff}}})].$$

The logarithmic survival correction is $p_{\text{rec}}/2 \simeq 1.4 \times 10^{-23}$. The finite closure-response correction is $Z_e - 1 = 0.00530828$. No second dimensional input, maximum-throughput clock, or tetrahedral-diameter equality is introduced.

The transfer-support dual. A spatial version uses the same dressed support and assigns the leading step count

$$N_{\text{eff}} = \frac{2}{3Z_e} e^{7g_{\text{share,eff}}}.$$

The temporal grammar gives $\tau_e = N_{\text{eff}}\tau_*$, while the spatial grammar gives $\lambda_e = N_{\text{eff}}L_*$. Their common factor cancels and returns $L_* = c\tau_*$. This is a consistency identity of the same electron phase mode; the positive transfer spectrum supplies its frequency.

Internal locality and the native vertex. The selected replacement kernel reaches the full ensemble. The tested single-label move class does not: shifts preserving injectivity split each parity copy into $4! = 24$ ordering sectors of $\binom{7}{4} = 35$ states because two slots cannot exchange order without a collision. Its strength-weighted walk saturates at entropy 7.374, below $g_{\text{share,eff}} = 7.4198$. This excludes that implementation. Appendix H.8 gives the whole-tetrahedron replacement gate, and H.9 decorates it with the fresh-state amplitude and charged event. The native gate is carried by the separate capacity/record factor; a particular GFT condensate realization may implement it dynamically, while the ordinary geometric propagation is supplied by the controlled factorized EPRL-class branch.

The reduced electron Compton wavelength is

$$\lambda_e = \frac{\hbar}{m_e c},$$

and the admissibility-closed sharing entropy is

$$g_{\text{share,eff}} = 7.41980002357.$$

The closure fixed point also gives

$$\langle K^2 \rangle_{\eta_*} = \frac{3}{2\eta_*}, \quad \eta_* = 0.0298668443935,$$

so the closure-saturation factor is

$$C_{\text{cl}} := \eta_* \langle K^2 \rangle_{\eta_*} = \frac{3}{2}, \quad C_{\text{cl}}^{-1} = \frac{2}{3}.$$

Separating the baseline recurrence from the finite marked response gives

$$\ln\left(\frac{\lambda_e}{L_*}\right) = 7g_{\text{share,eff}} - \ln\left(\frac{3}{2}\right) - \ln Z_e + O(e^{-7g_{\text{share,eff}}}).$$

Equivalently,

$$L_*^{\text{leading}} = \frac{3}{2} Z_e \lambda_e e^{-7g_{\text{share,eff}}}.$$

The exact expression is $L_* = -(3/2)Z_e \lambda_e \ln(1 - e^{-7g_{\text{share,eff}}})$. It gives

$$L_* = 1.6162537014 \times 10^{-35} \text{ m},$$

which differs from the CODATA Planck length by about 8.2×10^{-7} in fractional terms, or 8.2×10^{-5} percent.

The induced gravitational scale follows from the same algebra used in the stiffness matching:

$$G_* := \frac{c^3 L_*^2}{\hbar}.$$

Substituting the exact marked-transfer relation gives the closed form

$$G_* = \frac{c^3}{\hbar} \left[-\frac{3}{2} Z_e \lambda_e \ln(1 - e^{-7g_{\text{share,eff}}}) \right]^2 = \frac{9}{4} \frac{\hbar c}{m_e^2} Z_e^2 \ln^2(1 - e^{-7g_{\text{share,eff}}}).$$

Numerically,

$$G_* = 6.6742890772 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2},$$

or -0.073σ relative to CODATA. The same ζ_* gives the muon and tau ratios at -0.535σ and $+0.481\sigma$. One finite action therefore replaces the three leading deficits with one correction whose coefficient and routing are fixed jointly rather than adjusted independently for the three observables. The discrepancies were already known while the vertex was being constructed, so these are high-precision postdictions. Their evidential content is the common action, the absence of a continuous fit, and the nearby kernels rejected by the finite audits.

The factor $3/2$ is fixed structurally by the tetrahedral transverse-export geometry. The tetrahedral identity gives the transverse export fraction

$$\frac{1}{4} \sum_{i=1}^4 (\hat{n}_i \cdot \hat{u})^2 = \frac{1}{3}, \quad f_{\perp} = 1 - \frac{1}{3} = \frac{2}{3},$$

so $f_{\perp}^{-1} = 3/2$. The admissibility fixed point returns the same value,

$$C_{\text{cl}}^{-1} = (\eta_* \langle K^2 \rangle_{\eta_*})^{-1} = \frac{2}{3},$$

but it is built in. The condition defining η_* is $\langle K^2 \rangle_{\eta} = 3/(2\eta)$, so $C_{\text{cl}} = \eta_* \langle K^2 \rangle_{\eta_*} = 3/2$ holds by construction. It is not an independent cross-check. The decorated scale action reuses its reciprocal $2/3$ as the transverse export and inserts it once as a global factor, not seven times.

A discrete sensitivity audit enumerates 576 nearby formulas obtained by varying the response dimension, pair count, direction factor, determinant power, statistics, and singlet overlap. Only

the decorated-vertex formula lies within one muon standard deviation. This count is not a probability distribution over theories. It shows that the landing is sensitive to the field content and graph, which is why the action and its adversarial alternatives are displayed explicitly. A shared edge mode, directed complex determinant, stable bosonic determinant, or six-state symmetric response misses the muon respectively by approximately $1.15 \times 10^4 \sigma$, 13.8σ , 56.7σ , and $7.64 \times 10^4 \sigma$. Appendix H derives the selected entries before performing the observable trace.

The electron anchor carries three related roles. Its reduced Compton wavelength λ_e is the non-gravitational length used in faithful sector resolution. Its status as the lightest one-bit fermionic defect identifies which elementary excitation calibrates the seven-channel dressing block. Its mass fixes the mass-entropy map through

$$\kappa_m(\lambda_e) = \frac{m_e}{\ln 2}.$$

The same elementary defect enters the two normalization channels in distinct roles: its Compton length sets the UV cell scale, while its mass per one-bit defect entropy sets the source normalization.

The seven-sector scale-setting relation is not a counting heuristic; it has a concrete finite-dimensional realization on a transfer-operator history space, which we now construct. The face label algebra is

$$\mathcal{A}_7 = \bigoplus_{m=-3}^3 \mathbb{C} E_m, \quad E_m E_n = \delta_{mn} E_m, \quad \sum_{m=-3}^3 E_m = I_7.$$

The admissibility-closed tetrahedral state space has 1680 oriented injective states, with stationary weight

$$p_{\eta_*}(b) = Z^{-1} e^{-\eta_* K^2(b)}.$$

For a single sector, the refresh kernel

$$P_{\eta_*}(b, b') = p_{\eta_*}(b')$$

has Perron stationary entropy $g_{\text{share,eff}}$. The electron does not simultaneously occupy seven mutually exclusive face labels in one tetrahedron; the labels are sector channels in the dressing history of the one-bit defect. The appropriate support object is therefore a history space, with one admissibility-closed sector layer for each $m = -3, \dots, 3$.

Let

$$\mathcal{H}_{\text{hist}} = \bigotimes_{m=-3}^3 \mathcal{H}_B^{(m)}, \quad \dim \mathcal{H}_B = 1680,$$

and let $\mathcal{T}_m$ act as P_{η_*} on the m th factor and as the identity on the others. A representative one-pass dressing operator is

$$\mathcal{D}_e^{(0)} = \mathcal{T}_{-3} \mathcal{T}_{-2} \cdots \mathcal{T}_3.$$

The displayed order is only a representative of the symmetrized one-pass class. It introduces no physical ordering or extra factor of $7!$. The one-pass prescription visits each simple sector once. A history with omitted sectors is unresolved, while additional visits belong to a multi-pass dressing history rather than to repeated fermionic occupation.

A sector-resolution step should not be identified with conditioning the 1680-state ensemble on boundary states containing a given label m . Such conditioning changes the entropy and does not reproduce $g_{\text{share,eff}}$. The sector label instead specifies which simple face-algebra channel is being resolved while the admissibility cloud sampled in that step remains the full closed boundary ensemble.

With the effective dimension defined by the stationary Shannon entropy of the positive history kernel, each sector contributes $g_{\text{share,eff}}$, so

$$\dim_{\text{eff}}(\mathcal{D}_e^{(0)}) = \exp(7g_{\text{share,eff}}) = 3.60286052 \times 10^{22}.$$

Only the transverse part of this support is exported into the weak-field scalar channel. The normalized export weight is

$$w_{\perp} = \frac{2}{3},$$

the same fraction fixed by the tetrahedral transverse projection and by the reciprocal closure-saturation factor. Hence

$$\dim_{\text{eff}}(\mathcal{D}_{e,\perp}) = w_{\perp} \dim_{\text{eff}}(\mathcal{D}_e^{(0)}) = \frac{2}{3} e^{7g_{\text{share,eff}}} = 2.40190701 \times 10^{22}.$$

The decorated marked-transfer action gives the exact support-scale relation

$$\boxed{\frac{\lambda_e}{L_*} = \frac{2}{3Z_e [-\ln(1 - e^{-7g_{\text{share,eff}}})]} = \frac{1}{Z_e} \dim_{\text{eff}}(\mathcal{D}_{e,\perp}) \frac{r}{-\ln(1 - r)}}.$$

Thus $\lambda_e/L_* = Z_e^{-1} \dim_{\text{eff}}(\mathcal{D}_{e,\perp})[1 - r/2 + O(r^2)]$. Appendix H derives the memoryless replacement, determinant recurrence, positive gap, and decorated charged vertex. Fermionic exclusion and subadditivity select $k = 7$ and $\Delta_7 = 0$, while the marked graph fixes Z_e . The capacity-decorated rigging lift is exact and the selected metric-Regge branch has the Einstein TT limit. The geometric gluing tensor and measure remain independent data not fixed by the capacity premises; durable history capacity and the full source-coupled transverse/environmental spectrum also remain open.

The relation is fixed inside the displayed finite transfer action and the electron anchor. Its remaining conditionality is the broader claim that this transfer action is the charged sector of the eventual geometric GFT, not an unfixed number in the finite calculation.

This is the microscopic content of the length formula above: the elementary one-bit fermionic defect is supported over one complete transverse-exported dressing block. The matched weak-field gravitational constant then follows from the gauge-invariant bridge

$$G = \frac{c^2}{8\pi} \frac{\kappa}{\gamma S_{\infty}},$$

with the entropy-unit normalization handled by the rescaling convention described in Appendix C.5.

D.5 EFT consistency checklist

The ordinary longitudinal branch is a rewriting of the Einstein constraint sector, so it introduces no independent scalar propagator to which a separate ghost or tachyon test could be applied. Its consistency checks and those of the optional transport completion must be stated separately.

- *No extra longitudinal degree of freedom.* The Einstein parent supplies the Cauchy data and propagating tensor modes; δS is the static scalar-constraint coordinate.
- *Correct-sign sourcing.* Positive mass produces a capacity deficit and attractive Newtonian response in the reduced action.
- *Positive static quadratic form.* The Euclidean capacity functional has positive stiffness $\gamma > 0$ after the overall gravitational-sign convention is fixed.

- *Conditional causal transport.* The phenomenological telegrapher completion has finite characteristic speed when $D/\tau_0 = c^2$ with $D, \tau_0 > 0$.

These statements do not establish a UV completion or quantize an additional scalar. They show that the controlled static representation is consistent with its Einstein parent and that the separately proposed transport equation is linearly stable in its stated parameter range.

The checklist is intentionally modest. It verifies the reduced Einstein representation and the signs of the separate transport model; it does not establish an independently quantized scalar EFT.

The one place where an explicit formula is worth recording is linear vacuum stability in the time-dependent sector. Writing a small perturbation δs about the vacuum branch, the linearized telegrapher equation is

$$\tau_0 \ddot{\delta s} + \dot{\delta s} - D \nabla^2 \delta s = 0.$$

For a plane-wave mode $e^{-i\omega t + i\mathbf{k} \cdot \mathbf{x}}$, this gives the dispersion relation

$$\tau_0 \omega^2 + i\omega - Dk^2 = 0.$$

With $\tau_0 > 0$ and $D > 0$, the corresponding mode frequencies have non-growing time dependence, so the phenomenological transport vacuum is linearly stable.

D.6 Quadratic fluctuations and weak-field stability

Before imposing the Einstein scalar constraint, a formal covariant quadratic representation is

$$I_{\text{formal}}^{(2)}[\delta S] = - \int d^4x \sqrt{-g} \frac{\gamma}{2} g^{\mu\nu} \partial_\mu \delta S \partial_\nu \delta S.$$

After constraint reduction only its spatial Poisson quadratic form remains. The absence of a mass term expresses the unscreened constraint and does not prove an additional propagating boson. The galactic excess of Section 15 belongs to the separate carrier-resolved contact of Appendix N.8.

Appendix D provides the technical support layer for the weak-field bridge, Newton limit, electron anchor, substrate length branch, and EFT consistency audit.

Appendix E: Transport, Cosmology, and Hubble-Tension Implementation

Appendix E collects phenomenological time-dependent and homogeneous extensions of the static capacity variable. These additions are not implied by the Einstein constraint reduction and must recover it in their static limit. The transport relation is fixed only in its preferred branch, while the homogeneous and perturbation sectors remain open.

E.1 Telegrapher equation and causal closure

The time-dependent deficit field obeys

$$\tau_0 \partial_t^2 \delta S + \partial_t \delta S = D \nabla^2 \delta S + A_\chi, \quad \frac{A}{D} = \frac{\kappa}{\gamma}.$$

Causality requires

$$\frac{D}{\tau_0} = c^2.$$

In the canonical no-new-IR-scale branch,

$$\tau_0^{-1} = H_0, \quad D = \frac{c^2}{H_0}.$$

The telegrapher form is the minimal causal completion of the static Poisson sector. It supplies propagation and relaxation while preserving the static weak-field law as its exact late-time limit.

E.2 Static-limit recovery for galaxies

For a Fourier mode k , the telegrapher characteristic equation

$$\tau_0 s^2 + s + Dk^2 = 0$$

has the roots

$$s = -\frac{1}{2\tau_0} \pm i\omega_k, \quad \omega_k \simeq ck$$

whenever $4\tau_0 Dk^2 \gg 1$. Galactic wavelengths are far below the critical scale

$$\lambda_c = \frac{4\pi c}{H_0} \approx 54 \text{ Gpc},$$

so galactic modes are deeply underdamped. Time-averaging the sourced solution over intervals large compared with $2\pi/\omega_k$ returns the static Poisson branch exactly, and the residual ponderomotive correction scales parametrically as

$$\frac{\delta F_{\text{pond}}}{F_{\text{static}}} \sim e^{-T/(2\tau_0)} \left(\frac{\omega_{\text{orb}}}{\omega_k} \right)^2 \lesssim 10^{-6}$$

for representative orbital speeds, with the precise value depending on the averaging interval and system scale. The estimate keeps the correction below the near-stationary weak-field branch but does not supply a universal 10^{-8} bound.

E.3 Homogeneous mode and cosmological sourcing

The cosmological split is

$$S(x, t) = \bar{S}(t) + s(x, t),$$

with $\bar{S}(t)$ the homogeneous mode and $s(x, t)$ the inhomogeneous weak-field sector. The background capacity is normalized by the apparent horizon,

$$S_\infty(t) = \pi \frac{R_A(t)^2}{L_*^2}, \quad R_A(t) = \frac{c}{\sqrt{H^2 + kc^2/a^2}}.$$

Because the field couples to the trace of the stress-energy tensor, the homogeneous mode is suppressed during radiation domination and turns on near matter–radiation equality.

This timing is the central cosmological virtue of the mechanism. The homogeneous mode is quiet when it must be quiet, then becomes relevant close to the epoch where a sound-horizon shift is most useful.

E.4 Sound-horizon shift and shear lock

In the conditional cosmological proposal, the trace-sourced homogeneous mode acts as a transient early-energy contribution. It is intended to reduce the sound horizon without rewriting the local static Poisson law. Demonstrating that separation together with the committed component requires the open joint Boltzmann calculation.

The homogeneous mode can alter the cosmological background while leaving the coefficients of the local weak-field branch unchanged. A quantitative perturbation calculation remains open.

The transport relation and preferred branch are closed; the cosmological sector remains structurally supported but not yet Boltzmann-closed.

Appendix F: Strong-Field Spherical Reduction

Appendix F records the strongest action-level statement currently available in the black-hole sector. The bounded capacity rule is realized invariantly in spherical symmetry by the areal-radius gradient $q_{\text{geo}} = (\nabla R)^2$. The Schwarzschild exterior and the location $q_{\text{geo}} = 0$ then follow from the reduced Einstein action. Treating that surface as the physical end of the substrate EFT, however, is an additional domain postulate whose boundary microphysics remains open.

F.1 Bounded capacity and the unique lapse map

The strong-field order parameter is the surviving-capacity fraction

$$q(x) = \frac{S_{\text{ent}}(x)}{S_{\infty}} \in [0, 1].$$

This bound follows directly from finite local channel capacity. If the vacuum channel count is finite and S_{ent} is the logarithmic coarse entropy of the surviving local ensemble, then no physical branch can have either negative capacity or more than the asymptotic vacuum capacity.

In a static exterior, the lapse associated with the asymptotic Killing time is determined by the local surviving capacity. Let

$$N = f(q).$$

The conditions are:

$$f(1) = 1, \quad \lim_{q \rightarrow 0^+} f(q) = 0.$$

The substrate-level composition axiom is that independent serial capacity losses compose multiplicatively on the lapse:

$$f(q_1 q_2) = f(q_1) f(q_2).$$

This is the assumption that extends the linear weak-field match to a nonlinear lapse map. With continuity, the positive solutions on $(0, 1]$ are $f(q) = q^{\alpha}$. Expanding near $q = 1 - \epsilon$ gives

$$N = q^{\alpha} = 1 - \alpha\epsilon + O(\epsilon^2).$$

The weak-field bridge gives

$$N = 1 - \frac{\epsilon}{2} + O(\epsilon^2),$$

so $\alpha = 1/2$ and therefore

$$N = \sqrt{q}, \quad N^2 = q.$$

Equivalently, if one writes $N^2 = F(q)$, the unique continuous multiplicative completion is $F(q) = q$. The nonlinear static lapse rule is therefore fixed by capacity composition and weak-field matching; it is not a freely chosen black-hole ansatz. Because a lapse depends on foliation, this is a static constitutive statement. Its covariant spherical content is derived next.

F.2 Einstein action reduced to the capacity-adapted invariant

Take the most general spherically symmetric line element

$$ds^2 = h_{ab}(x)dx^a dx^b + R^2(x)d\Omega^2.$$

Here $x^0 = ct$, so the two-dimensional measure is $d^2x = dx^0 dr$. The same action written as $dt dr$ acquires one additional factor of c . The four-dimensional curvature decomposes as

$${}^{(4)}R = {}^{(2)}R + \frac{2}{R^2} [1 - (\nabla R)^2 - 2R\Box R].$$

After the angular integral, the Einstein–Hilbert plus GHY action becomes, up to the asymptotic and corner terms retained in $I_{\partial}^{(2)}$,

$$I_{\text{sph}} = \frac{c^3}{4G} \int d^2x \sqrt{-h} [R^2 {}^{(2)}R + 2(\nabla R)^2 + 2] + I_{\text{matter}}^{(2)} + I_{\partial}^{(2)}.$$

The integration by parts that converts $-4R\Box R$ into $+4(\nabla R)^2$ is legitimate only together with the reduced GHY contribution. The boundary term is therefore part of the statement. For $\partial\mathcal{M}_4 = \partial\mathcal{M}_2 \times S^2$ one has

$$K^{(4)} = K^{(1)} + \frac{2}{R} n^a \nabla_a R.$$

The second term cancels the surface term from the integration by parts, leaving

$$I_{\partial}^{(2)} = \varepsilon \frac{c^3}{2G} \int_{\partial\mathcal{M}_2} dy \sqrt{|\gamma_{(1)}|} R^2 K^{(1)} + I_{\text{joint}} + I_{\text{ref}},$$

with ε the standard orientation sign.

Define

$$q_{\text{geo}} \equiv (\nabla R)^2 = h^{ab} \partial_a R \partial_b R.$$

Varying R and h^{ab} in vacuum gives

$$R {}^{(2)}R - 2\Box R = 0,$$

$$2R(h_{ab}\Box R - \nabla_a \nabla_b R) + h_{ab}(q_{\text{geo}} - 1) = 0.$$

These equations imply

$$\nabla_a M_{\text{MS}} = 0, \quad M_{\text{MS}} = \frac{c^2 R}{2G} (1 - q_{\text{geo}}).$$

Therefore

$$q_{\text{geo}} = 1 - \frac{2GM_{\text{MS}}}{c^2 R}.$$

The variational status is now unambiguous: q_{geo} is a composite of the two-dimensional metric and the areal-radius dilaton, and its vacuum profile is a first integral. No scalar has been added to the Einstein degrees of freedom.

Audit of the discarded multiplier form. If instead one writes an ADM term $\sqrt{h} \lambda (N^2 - q)$ while giving q no other bulk dependence, the q equation sets $\lambda = 0$. The constraint then only identifies an arbitrary scalar with a foliation-dependent lapse. It does not reproduce the capacity source equation and it has no invariant content away from a specified static slicing. The spherical reduction above supplies the invariant result that construction was trying to capture and supersedes it.

F.3 Variational status of the $q_{\text{geo}} = 0$ boundary

For $\epsilon > 0$, let $\mathcal{B}_\epsilon$ be the timelike level surface $q_{\text{geo}} = \epsilon$. The exterior Einstein problem is well posed with the standard term

$$I_{\text{GHY}}[\mathcal{B}_\epsilon] = \epsilon \frac{c^3}{8\pi G} \int_{\mathcal{B}_\epsilon} d^3y \sqrt{|\gamma|} K$$

and fixed induced metric on $\mathcal{B}_\epsilon$, together with the reference term at infinity and any required joints. The limit $\epsilon \rightarrow 0^+$ is null and must be taken using the corresponding null-boundary and joint prescription; a bare timelike GHY expression cannot simply be evaluated at the null surface.

This construction fixes the universal gravitational variation. It does not make

$$q_{\text{geo}}|_{\partial\mathcal{M}_q} = 0$$

a new Euler–Lagrange boundary condition. The zero is the level set at which the spherical invariant becomes marginal. Declaring that level set to be the end of the physical substrate domain,

$$\mathcal{M}_q = \{q_{\text{geo}} > 0\},$$

is the bounded-capacity postulate.

Any additional functional

$$\Gamma_{\partial q}[\sigma_{AB}, \text{boundary channels}]$$

would describe genuine substrate physics: absorption, partial reflection, relaxation, entropy, or a moving-boundary stress. Its variation may contain a boundary stress and a response conjugate to the limiting capacity, but neither its form nor its spectrum follows from Einstein–Hilbert reduction. The static exterior can be solved without inventing this term; claims about excision, infalling evolution, or finite reflectivity cannot.

F.4 Static spherical vacuum exterior

In static spherical vacuum, the result follows immediately. On $\mathcal{M}_q$, the bulk equations are the vacuum Einstein equations. The unique asymptotically flat static spherical solution is the Schwarzschild exterior,

$$ds^2 = - \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2,$$

so the spherical geometric representative of the capacity variable is

$$q_{\text{geo}}(r) = N^2(r) = 1 - \frac{2GM}{c^2 r}, \quad r > r_h,$$

with

$$r_h = \frac{2GM}{c^2}.$$

This also agrees with the weak-field capacity deficit:

$$\frac{\delta S(r)}{S_\infty} = \frac{2GM}{c^2 r}, \quad q(r) = 1 - \frac{\delta S(r)}{S_\infty}.$$

The exterior $r > r_h$ is exactly the standard Schwarzschild exterior, and $q_{\text{geo}} = 0$ at $r = r_h$. The geometric invariant becomes negative in the trapped region. Interpreting that sign change

as exhaustion of a nonnegative substrate capacity motivates restricting the capacity EFT to $q_{\text{geo}} \geq 0$, but this restriction is not implied by the Einstein equations. The absence of a physical classical interior is therefore a conditional substrate claim, not an action-level theorem.

The exterior-domain result does not decide what an infalling observer experiences at the $q_{\text{geo}} = 0$ surface. Whether a capacity-exhaustion boundary is smooth, dissipative, anomalous, or absent requires the open functional $\Gamma_{\partial q}$, not merely the exterior Schwarzschild solution.

Geometric identification of the capacity variable. The spherical reduction supplies a normalization-independent gradient invariant on the orbit space,

$$q_{\text{geo}} = h^{ab} \partial_a R \partial_b R = |\nabla R|^2,$$

which in Schwarzschild gives

$$q_{\text{geo}} = 1 - \frac{2GM}{c^2 r} = N^2,$$

which can be identified with the substrate fraction $q = S_{\text{ent}}/S_{\infty}$ on the exterior. In spherical dynamical collapse with Misner–Sharp mass $M_{\text{MS}}(R, t)$,

$$q_{\text{geo}}(R, t) = 1 - \frac{2GM_{\text{MS}}(R, t)}{c^2 R}.$$

The marginal-trapped-surface condition $\theta_+ \theta_- = 0$ coincides with $q_{\text{geo}} = 0$ independently of null normalization: positive, zero, and negative q_{geo} label untrapped, marginal, and trapped spherical regions. Standard collapse in the metric parent can therefore produce the geometric zero. Equating that zero with substrate saturation and refusing the negative branch are the additional bounded-capacity interpretation to be tested by the transport and boundary theory.

Rotating and charged stationary exteriors. The baseline metric parent also admits Kerr, Reissner–Nordström, and Kerr–Newman exteriors when the appropriate conserved gauge sector is present. Their standard horizon entropy

$$S = \frac{k_B A}{4L_*^2}$$

retains its form with A the appropriate horizon area, and the Hawking temperature follows from the surface gravity as usual,

$$T_H = \frac{\hbar \kappa_{\text{sg}}}{2\pi k_B c}.$$

A nonspherical covariant capacity scalar whose zero selects the outer horizon and whose sign defines a physical domain has not yet been derived. The absence of inner or trapped regions therefore cannot be inferred from the spherical invariant alone.

F.5 Horizon thermodynamics, channel area, and the Immirzi normalization

Because the exterior geometry is unchanged, semiclassical quantities depending only on the exterior near-horizon saddle are unchanged. The Euclidean continuation is used here as an exterior-saddle calculation: the resulting periodicity depends on regularity of the near-horizon exterior geometry, not on adopting the bounded-domain interpretation past $q_{\text{geo}} = 0$. The Euclidean regularity argument therefore gives the standard Hawking temperature [10, 11],

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}.$$

For the GR exterior saddle, the same calculation gives the Bekenstein–Hawking area law [9, 10],

$$S_{\text{BH}} = \frac{k_B A}{4L_P^2}.$$

On the matched branch $L_P(G_*) = L_*$. The factorized $j = 3$ completion supplies a definite map between one capacity cut and physical geometric area. The semiclassical area law provides the continuum normalization for that map.

One cut carries one capacity bit while the geometric boundary face remains. If the bounded-capacity domain terminates at $q_{\text{geo}} = 0$, the exterior cell has no capacity partner across the outward face. The missing capacity pairing is the same elementary fermionic exclusion used by the mass–entropy postulate, so the canonical cut increment is

$$\boxed{\Delta S_f = \ln 2.}$$

The factorized construction of Appendix Q changes an important older reading of this statement. The absent partner removes the *capacity* pairing; it does not erase the geometric boundary datum. The open boundary still carries the geometric V_3^{geom} representation required for spin-network/spin-foam gluing, while the corresponding capacity V_3^{cap} has lost its partner across the cut. Thus a horizon puncture is a geometric $j = 3$ face decorated by one binary capacity-cut defect, not an additional sevenfold thermodynamic degeneracy.

Accordingly the entropy used here is conditional on the macroscopic boundary geometry,

$$S_{\text{hor}} = S_{\text{cap}}(\text{cut} \mid g_{\partial}),$$

so the geometric spin/intertwiner labels specify the area sector and are not multiplied as an independent entropy factor. Multiplying a separate isolated-horizon spin degeneracy by the capacity degeneracy would double count the geometry–capacity degree of freedom in this framework; such a construction would be a different microscopic theory, not an extra factor in the present one.

Microscopic channel-to-area theorem within the selected completion. For a generic LQG surface puncture the kinematical area spectrum is [89]

$$A_{\text{LQG}}(j) = 8\pi\gamma_{\text{BI}}L_*^2\sqrt{j(j+1)}.$$

The present branch, however, is built from spin coherent faces and maximal coherent fusion. The area variable that enters the associated Regge geometry is the coherent flux magnitude, linear in j ; for a coarse face made from parallel small facets, the large-face area is the sum of the small j_f and the coarse spin is $J = \sum_f j_f$ [90]. Appendix B therefore fixes the physical coarse area operator on the maximal-fusion ray to

$$A_{\text{phys}}(J) = 8\pi\gamma_{\text{BI}}L_*^2J,$$

or equivalently

$$A_{\text{phys}}(J) = Z_A(J)A_{\text{LQG}}(J), \quad \boxed{Z_A(J) = \frac{J}{\sqrt{J(J+1)}}.}$$

For N primitive transmitted faces, $J = 3N$ and hence

$$Z_A(N) = \sqrt{\frac{3N}{3N+1}}, \quad A_N = 24\pi\gamma_{\text{BI}}NL_*^2.$$

This operator renormalization is fixed by additive coherent area; it is not a new coupling. At one primitive face $Z_A(1) = \sqrt{3}/2$, and $Z_A \rightarrow 1$ as the block enters the ordinary large-spin Regge regime.

A patch of N cut capacity channels carries

$$\frac{S_{\text{cap}}}{k_B} = N \ln 2.$$

Requiring this same geometric patch to obey the already-established Einstein/Bekenstein–Hawking normalization gives

$$N \ln 2 = \frac{A_N}{4L_*^2} = 6\pi\gamma_{\text{BI}}N.$$

Therefore

$$\gamma_{\text{BI}} = \frac{\ln 2}{6\pi} = 0.0367726000254\dots$$

and the physical area carried by one microscopic capacity cut is

$$a_{\text{ch}} = \frac{A_N}{N} = 4 \ln 2 L_*^2, \quad \rho_{\text{ch}} = \frac{1}{4 \ln 2 L_*^2}.$$

The result is exact conditional on the selected factorized $j = 3$ embedding, the one-cell relational lock, the coherent/Regge additive-area prescription, and the existing one-bit cut rule. It fixes the embedding normalization. The Bekenstein–Hawking law and $L_P(G_*) = L_*$ supply the continuum normalization, so this is not independent evidence for that law. Within the embedding, the normalization fixes the Immirzi factor.

Bare and coarse area operators. If one equated $\ln 2$ directly to the unrenormalized $j = 3$ Casimir area, one would obtain $\gamma = \ln 2/(4\pi\sqrt{3})$. That assignment treats a single bare puncture eigenvalue as the macroscopic area variable and fails to preserve area under the exact maximal-fusion map. The coherent/Regge branch instead carries the renormalized operator above. Indeed, for every N ,

$$Z_A(N) 8\pi\gamma_{\text{BI}} L_*^2 \sqrt{3N(3N+1)} = 4N \ln 2 L_*^2,$$

so fine-channel additivity, maximal fusion, the spin-foam Regge area, and the horizon entropy all use one normalization. The two numerical values are therefore not competing Immirzi predictions: the smaller value belongs to a bare operator convention that is not the coarse observable selected by this branch.

Independent bulk-response identity. The graph response supplies a separate normalization check and should not be interpreted as the puncture count. Since

$$\mathcal{G}_{\text{loc}}^{ab} = \frac{G_{\text{tet}}(0)}{3} \delta^{ab}, \quad G_{\perp} = \frac{2}{3} G_{\text{tet}}(0),$$

the angular response number is

$$n_{\text{hor}} = 4\pi \frac{G_{\perp}}{\ln 2} = \frac{8\pi G_{\text{tet}}(0)}{3 \ln 2}.$$

It is dimensionless and is *not* the literal number of $j = 3$ punctures. Using the cell-normalized capacity baseline

$$S_{\infty}^{\text{cell}} = \frac{3 \ln 2}{32\pi G_{\text{tet}}(0)}$$

still gives the exact response identity

$$n_{\text{hor}} S_{\infty}^{\text{cell}} = \frac{1}{4}.$$

The microscopic puncture theorem above and this bulk Green-response identity are two descriptions of the same normalized continuum coefficient, but they count different objects and must not be multiplied together.

Status. Within the selected factorized $j = 3$ geometric completion and coherent/Regge area prescription, the channel-to-area normalization is therefore closed:

$$\Delta S_f = \ln 2, \quad j_{\text{geom}} = 3, \quad Z_A(N) = \sqrt{\frac{3N}{3N+1}}, \quad \gamma_{\text{BI}} = \frac{\ln 2}{6\pi}, \quad a_{\text{ch}} = 4 \ln 2 L_*^2.$$

What remains in the strong-field sector is dynamical rather than kinematical: the saturation/front evolution, boundary Hamiltonian, relaxation spectrum, reflectivity, stretched-layer corrections, and nonspherical capacity map.

F.6 Absorption, ringdown, and echoes

The exterior Regge–Wheeler/Zerilli operators are unchanged. With $q_{\text{geo}} = 0$ at the marginal surface, the tortoise coordinate $r_* \sim r_h \ln q_{\text{geo}}$ sends the surface to $r_* \rightarrow -\infty$, and the near-horizon wave equation reduces to $(c^{-2} \partial_t^2 - \partial_{r_*}^2) \psi \simeq 0$. If the usual GR future-horizon regularity condition is retained, it selects

$$\psi \sim e^{-i\omega(t+r_*/c)},$$

and therefore

$$\mathcal{R} = 0.$$

The standard greybody factors and quasinormal spectrum then follow [12, 13]. But if the substrate EFT truly terminates at the marginal surface, future-horizon regularity is a boundary choice, not a consequence of the exterior differential operator alone. The open functional $\Gamma_{\partial q}$ must determine whether that choice is correct.

If the microscopic boundary has finite reflectivity or lies on a stretched layer

$$q = \epsilon > 0,$$

the region between the exterior potential barrier and that layer behaves as a cavity. A typical echo delay then scales as

$$\Delta t_{\text{echo}} \sim \frac{2r_h}{c} |\ln \epsilon| + \tau_{\text{ch}},$$

where τ_{ch} is a boundary-channel relaxation time. Thus $\mathcal{R} = 0$ is the GR-matching boundary condition, while $\mathcal{R}(\omega)$ is a boundary observable to be calculated rather than assumed.

F.7 Dynamical formation as a free-boundary problem

Standard collapse in the metric parent can form a marginal surface $q_{\text{geo}} = 0$. It does not prove that a substrate capacity field saturates there or that the physical evolution terminates. That identification requires a bounded causal transport law for a capacity variable q_{cap} , together with a constitutive relation showing $q_{\text{cap}} = q_{\text{geo}}$ in the regime of overlap. A candidate test system is

$$\begin{aligned} \partial_t q_{\text{cap}} + D_i J^i &= -\Gamma(q_{\text{cap}}) \Sigma[T_{\mu\nu}], \\ \tau_J (\partial_t + \mathcal{L}_v) J^i + J^i &= -D(q_{\text{cap}}) D^i q_{\text{cap}}. \end{aligned}$$

Here J^i is the capacity flux, $\Sigma[T_{\mu\nu}]$ is a positive depletion source built from the collapsing stress-energy, $D(q_{\text{cap}})$ is a bounded mobility, $\Gamma(q_{\text{cap}})$ is a bounded depletion rate, and $\tau_J > 0$ is a relaxation time. Eliminating J^i gives a telegrapher-type equation with finite characteristic speed

$$v_{\text{cap}} \sim \sqrt{\frac{D_0}{\tau_J}}.$$

Choosing trial constitutive functions such as

$$D(q_{\text{cap}}) = D_0 q_{\text{cap}}(1 - q_{\text{cap}}), \quad \Gamma(q_{\text{cap}}) = \Gamma_0 q_{\text{cap}}$$

makes $q_{\text{cap}} = 0$ and $q_{\text{cap}} = 1$ invariant sets, preventing overshoot. These functions are examples, not a derived action.

If q_{cap} first reaches zero on a two-surface and the microscopic theory licenses domain termination, that surface becomes a moving boundary

$$\partial\mathcal{M}_q(t) = \{x \mid q_{\text{cap}}(t, x) = 0\}.$$

The level-set kinematics are fixed by differentiating $q_{\text{cap}}(t, X(t)) = 0$ along the moving surface:

$$V_n = - \frac{\partial_t q_{\text{cap}}}{|\nabla q_{\text{cap}}|} \Big|_{q_{\text{cap}} \rightarrow 0+}.$$

In spherical symmetry this becomes

$$\frac{dr_f}{dt} = - \frac{\partial_t q_{\text{cap}}}{\partial_r q_{\text{cap}}} \Big|_{r=r_f(t)}.$$

During continued infall the exterior should be Vaidya-like with a slowly varying mass parameter, settling to the Schwarzschild exterior after the front stabilizes. This is a well-posed program, but not yet a closed derivation: the transport coefficients, boundary action, and channel relaxation spectrum must be computed from the graph ensemble or constrained by simulation.

The dynamical system is presumed to preserve the usual covariant conservation of the combined matter-plus-capacity stress-energy, with any local matter depletion balanced by flux, boundary work, or capacity-sector stress. Showing that this conservation structure follows from a graph-derived transport action, rather than imposing it as a constitutive condition, is part of the dynamical closure work.

The concrete closure tests are correspondingly specific. A spherical collapse simulation should show formation of the first $q = 0$ surface without overshoot into $q < 0$. A coupled matter-plus-capacity run should approach a Vaidya exterior during accretion and a Schwarzschild exterior after settling while satisfying the combined conservation law. Exterior perturbation simulations with an absorbing boundary should reproduce standard Schwarzschild greybody factors and ringdown, while partial-reflectivity runs should produce controlled echo delays. Finally, a microscopic boundary-action calculation or a graph-ensemble Monte Carlo of saturated boundary channels should reproduce the channel-counting rule that yields $n_{\text{hor}} S_{\infty}^{\text{cell}} = 1/4$. These are not new fit knobs; they are the numerical and microscopic tests that would close the dynamical and boundary sectors.

F.8 Weak-field boundary and closure statement

In the weak-field Solar-System regime, the metric-only parent yields

$$\gamma_{\text{PPN}} = \beta_{\text{PPN}} = 1,$$

with the remaining standard PPN coefficients vanishing under the usual assumptions. This follows because the ordinary longitudinal capacity functional is a reduced representation of Einstein gravity. In the carrier-resolved galactic EFT, localized Solar-System sources are absent from the transverse source map, so the leading local solution remains on this Einstein branch. Nonlinear preferred-frame terms still require the covariant projector audit of Appendix N.8.

The weak-field capacity coordinate ceases to be adequate when

$$\frac{|\Phi|}{c^2} = O(1), \quad \frac{\delta S}{S_\infty} = O(1),$$

which is the regime where the spherical invariant q_{geo} provides the controlled nonlinear description.

Appendix F therefore closes two precise strong-field statements. The spherically reduced Einstein action makes $q_{\text{geo}} = (\nabla R)^2 = 1 - 2GM_{\text{MS}}/(c^2 R)$ a composite first integral and returns the Schwarzschild exterior. Separately, the factorized coherent $j = 3$ boundary construction fixes the microscopic channel area and Immirzi normalization, while the bulk Green-response product remains an exact 1/4 cross-check. The bounded capacity interpretation of q_{geo} , exclusion of the negative branch, and physical boundary dynamics remain additional hypotheses. Rotating and charged solutions belong to the baseline metric parent, but their capacity-variable completion is also open.

Appendix G: Many-Pasts, Operational Closure, Branch Realization, and the Arrow of Time

Appendix G separates the three probability objects used by Many-Pasts: amplitudes for unresolved alternatives, probabilities for decoherent record histories, and conditional probabilities for histories compatible with one present record. This repairs the ambiguity in the earlier notation $P(H | P) \propto e^{-D(H,P)}$. The operational construction is standard decoherent-histories quantum mechanics [71]; Many-Pasts supplies its record-conditioned ontology.

G.1 What is a history of the entanglement network?

A coarse projective history $h = (\alpha_1, \dots, \alpha_n)$ is a sequence of alternatives at ordered substrate times. The alternative α_k is represented by a projector $\Pi_{\alpha_k}^{(k)}$. With unitary evolution $U_{k,k-1}$ between times, its class operator is

$$C_h = \Pi_{\alpha_n}^{(n)} U_{n,n-1} \Pi_{\alpha_{n-1}}^{(n-1)} \dots U_{2,1} \Pi_{\alpha_1}^{(1)} U_{1,0}.$$

This operator retains amplitudes. It is defined before any classical probability is assigned to the individual history.

Given an initial state ρ_0 , the decoherence functional is

$$\mathcal{D}(h, h') = \text{Tr} \left(C_h \rho_0 C_{h'}^\dagger \right).$$

A family admits ordinary probabilities when its off-diagonal terms are negligible at the required accuracy,

$$\mathcal{D}(h, h') \simeq 0 \quad (h \neq h').$$

The diagonal entries $p(h) = \mathcal{D}(h, h)$ are then nonnegative and additive under coarse-graining. When alternatives do not decohere, their class operators must be added before the probability is evaluated. For $C_A = \sum_{h \in A} C_h$,

$$p(A) = \text{Tr} \left(C_A \rho_0 C_A^\dagger \right),$$

which retains the interference terms. Many-Pasts places no classical distribution over unresolved fine-grained paths.

G.2 What is the present coarse configuration P ?

The present P is a macroscopic record represented by a final projector Π_P . Let $\mathcal{H}_P$ be a decoherent family of histories whose final alternatives refine that record. Exhaustiveness and decoherence give

$$p(P) = \sum_{h \in \mathcal{H}_P} p(h) = \text{Tr}(\Pi_P \rho_{\text{now}}).$$

The conditional Many-Pasts measure is

$$p(h \mid P) = \frac{p(h)}{p(P)}, \quad h \in \mathcal{H}_P.$$

The common measure over all possible records is normalized first; conditioning on the realized record comes afterwards. Normalizing a new set of histories separately for each already-selected present would leave the probabilities of the alternative presents undefined.

G.3 What is the distance $D(H, P)$?

For a decoherent history ending in P , define

$$D(h, P) = -\ln p(h),$$

with $D = +\infty$ when $p(h) = 0$. Then

$$p(h \mid P) = \frac{e^{-D(h, P)}}{\sum_{h' \in \mathcal{H}_P} e^{-D(h', P)}}.$$

The distance notation is shorthand for the diagonal decoherence-functional weight, not a second probability law. The earlier expression $-\ln \text{Tr}(\Pi_P \rho_{H \rightarrow \text{now}})$ is recovered when H already denotes a decohered preparation history and only the final record remains unresolved.

G.4 The operational Born branch

A laboratory setting x is represented by a quantum instrument $\{\mathcal{M}_{a|x}\}_a$. Each map is completely positive and trace non-increasing, while $\sum_a \mathcal{M}_{a|x}$ is trace preserving. For an initial state ρ ,

$$p(a \mid x) = \text{Tr}[\mathcal{M}_{a|x}(\rho)].$$

If $C_{h,a|x}$ refines the histories ending in record a , then

$$p(h \mid a, x) = \frac{\text{Tr}(C_{h,a|x} \rho C_{h,a|x}^\dagger)}{p(a \mid x)}$$

for a decoherent refinement. The record marginal is the Born probability by construction. Its form is also unique under the hypotheses stated in Section 3.3: normalized noncontextual additivity on a sufficiently rich projector lattice gives $p(R) = \text{Tr}(\rho R)$ by Gleason's theorem, and medium decoherence supplies generalized record projectors for pure branch states [65, 66]. The uniqueness belongs to the measure once quantum kinematics and record completeness are granted. It does not derive the Hilbert space, the state, or the decoherence functional from the substrate. A future substrate theory must reproduce that structure rather than merely fit its projective limit.

G.5 No-signaling

Let Alice and Bob act locally on ρ_{AB} with instruments $\{\mathcal{M}_{a|x}^A\}_a$ and $\{\mathcal{N}_{b|y}^B\}_b$. Their joint record probability is

$$p(a, b \mid x, y) = \text{Tr} \left[(\mathcal{M}_{a|x}^A \otimes \mathcal{N}_{b|y}^B)(\rho_{AB}) \right].$$

Summing over Bob's record gives

$$p(a \mid x, y) = \sum_b p(a, b \mid x, y) \quad (25)$$

$$= \text{Tr} \left[(\mathcal{M}_{a|x}^A \otimes \mathcal{N}_y^B)(\rho_{AB}) \right] \quad (26)$$

$$= \text{Tr} \left[\mathcal{M}_{a|x}^A(\rho_A) \right] = p(a \mid x), \quad (27)$$

where $\mathcal{N}_y^B = \sum_b \mathcal{N}_{b|y}^B$ is trace preserving. Alice's marginal is independent of y , and the same calculation applies to Bob. Global conditioning on a joint present record does not create a controllable signaling channel because the unconditioned local marginals remain those of standard quantum mechanics.

G.6 Branch realization

The postulate contains one realized present with its records. Its multiplicity lies among the admissible *pasts* of that present, with no forward branching into co-real macroscopic worlds and no collapse event selecting among them. An outcome is specified by the records that obtain in the present, and the Born weights give their statistics. This account of branch realization adds neither many worlds nor a collapse dynamics.

The present record algebra fixes the resolution at which past alternatives are physically distinguished. Let $\{R_i\}_{i \in I}$ be mutually orthogonal fine records that a later physical channel maps to one coarse record $\bar{R} = \sum_{i \in I} R_i$. For a decoherent joint refinement,

$$p(h, \bar{R}) = \sum_{i \in I} p(h, R_i), \quad p(h \mid \bar{R}) = \frac{\sum_{i \in I} p(h, R_i)}{\sum_{h'} \sum_{i \in I} p(h', R_i)}.$$

This is the push-forward of the same joint quantum measure. Record loss needs no additional rule and does not alter the earlier unitary dynamics. It reduces the distinctions represented in the realized present. The history register of H.8 is one microscopic carrier of such distinctions; environmental and macroscopic records are others. The paper does not assume an inaccessible second archive after every physical carrier of a distinction has been erased.

The conditioning also has no forward dynamical role. The class operator C_h , the state, and the quantum instruments determine $p(h, P)$ before the ratio $p(h \mid P)$ is formed. A later record permits retrodictive conditioning once it exists; the conditional probability never appears as a force in the preceding evolution. Many-Pasts therefore preserves the causal and no-signaling content of the imported quantum channel structure.

This record-retention reading removes one apparent cosmological alternative under a precise condition. If a state has a trivial macroscopic record algebra with respect to an earlier era, two descriptions that differ only by whether that era occurred before the record-free state are represented by the same present state. A physically meaningful cycle count would require a record surviving the interval. The framework neither supplies such a meta-register nor derives that a record-free cosmological state is reached. The result is an ontological equivalence and makes no claim about cosmic recurrence.

G.7 Arrow of time from conditional typicality

The proposed arrow-of-time extension requires a further typicality statement. Let $h = \{M_t\}_{t_i \leq t \leq t_0}$ be a decoherent macrohistory conditioned on present records M_{t_0} . A coarse Markov description would assign

$$P(h \mid M_{t_0}) \propto \mu_i(M_{t_i}) \prod_{t_i \leq t < t_0} T(M_{t+\Delta t} \mid M_t),$$

where μ_i is a boundary measure and T is the conditional macro-transition probability obtained only after summing the microscopic transitions compatible with each pair of macrostates. Multiplying a separate factor $e^{S(M_t)}$ at every time would generally count the same microscopic multiplicity twice. A neutral transition law is also insufficient: conditional counting without a low-entropy boundary condition is dominated by high-entropy pasts and Boltzmann-fluctuation histories. The required theorem must show that μ_i and T exponentially suppress those histories strongly enough for ordinary entropy-increasing histories to dominate. The boundary measure now has a forced support. Many-Pasts posits no meta-time and no unrecorded archive: absent a surviving record, “a prior era occurred and all traces vanished” and “no prior era occurred” are not distinct physical states, so the fundamental boundary is the minimal-record state, and a heavily recorded state cannot be initial because its record content constitutes a past (Appendix P.4). μ_i is therefore defined on the first physically distinguishable record-bearing state, and the arrow is conditional typicality from that anchor; what the quotient removes is the freedom to place the boundary at an arbitrary high-entropy state. The needed substrate transition law has still not been derived here. This equation defines the target without claiming a completed thermodynamic arrow.

The minimal missing boundary condition can be stated directly. A *Substrate Past Hypothesis* would require

$$\text{supp } \rho_0 \subseteq \mathcal{H}_{M_{\text{low}}}, \quad t_0 - t_i \ll \min(t_{\text{relax}}, t_{\text{rec}}),$$

where $\mathcal{H}_{M_{\text{low}}}$ is a low-entropy macro-subspace and the elapsed time is short compared with equilibration and recurrence. Under a mixing substrate dynamics, standard large-deviation counting would then favor entropy growth away from that boundary. The paper has not derived this hypothesis or the required mixing estimate. The saturated phase of Section 20 supplies the capacity-sector half of the hypothesis by construction — the pinned state is a near-zero-entropy configuration of the capacity sector — while the matter-sector clause is supplied, for the canonical defect class, by the visibility–source contrapositive of Appendix P.3: a state with no elementary massive defects carries no defect records, so matter-sector simplicity at the anchor is a named conditional rather than a free assumption. The quantitative companion is the well-foundedness bound of Appendix P.4, which prices a false durable record of content K at $e^{\ln N_{\text{trials}} - K}$ with $\ln N_{\text{trials}} \sim 5.6 \times 10^2$ nats, at the conditional grade it inherits from Section 20. Naming these premises isolates the remaining arrow-of-time problem from the already-closed operational probability branch.

The faithful-history-resolution theorem of Appendix H does not supply the missing boundary. Its stationary kernel satisfies

$$p(b)K_*(b, b') = p(b)p(b') = p(b')K_*(b', b),$$

so it is exactly detailed-balanced and time-reversal symmetric. The quantum dilation is globally reversible as well. Local export distinguishes a present register from a history register within the chosen update description, but the thermodynamic orientation still comes from the boundary condition and finite-time typicality theorem above.

G.8 Memoryless dressing and the connection to L_*

The dressing kernel's temporal reading has a spatial twin. For the canonical elementary defect, the rate at which the exported history becomes distinguishable from vacuum history and the scalar source insertion of Appendix C.5 are the same likelihood ratio, with the one-bit defect statistic sufficient; the identity, its isotropy benchmark $\ln(7/6)$, and its exact decomposition are established in Appendix P.3. Postulate III and the weak-field source theorem, separable modules until that identity, consume one shared constant.

The history measure does not imply memorylessness merely because it is written as an exponential. The foundational faithful full-support principle selects it when applied to paths at the fixed admissibility marginal; maximum caliber is only the conventional name for that application. For any stationary dressing kernel with one-time marginal p_{η_*} ,

$$H(B_{t+1} \mid B_t) = g_{\text{share,eff}} - I(B_t; B_{t+1}) \leq g_{\text{share,eff}}.$$

Equality holds only when consecutive configurations are independent, which fixes $K(b, b') = p_{\eta_*}(b')$ on the support. More generally, the entropy rate of any stationary process with this one-time marginal is bounded by $g_{\text{share,eff}}$, with equality only when the present is independent of its complete past. The quantum replacement channel transfers the old local information into a dilation register rather than destroying it. Many-Pasts gives that register its history-space interpretation. The finite marked event is a separate statement from this renewal theorem and is supplied by the decorated vertex in H.9.

G.9 Relation to many-worlds, collapse, hidden variables, and decoherence

Many-Pasts is a form of history-space realism built on the decoherent-histories formalism. It posits one realized record-bearing present and no stochastic collapse term. Its probabilities are the diagonal entries of the standard decoherence functional, conditioned on that present only after the alternative records have been normalized. The distinctive claim lies in the ontology assigned to this conditional measure and in its proposed substrate realization. Born statistics and no-signaling are imported with the operational quantum structure; the substrate does not yet derive them.

G.10 Familiar quantum examples: double-slit, EPR/Bell, and measurement

In a double-slit experiment the two path alternatives remain combined in one class operator while no which-path record exists. The probability of a detection event therefore includes their interference term. A durable which-path record defines a decoherent refinement, after which the path histories admit separate conditional probabilities and the fringe term is suppressed.

In an EPR or Bell experiment the record is joint, so its history measure carries the standard nonclassical correlations. Summing over the remote record invokes a trace-preserving local map and returns a marginal independent of the remote setting, as shown in G.5. Conditioning on the observed joint record does not alter that prior no-signaling marginal.

Measurement creates a durable record and identifies a decoherent family. Operationally the calculation is ordinary quantum mechanics. The additional claim is that the realized present is supported by the conditional ensemble of compatible past histories.

These examples introduce no new laboratory predictions. They show where positive history probabilities are legitimate and where amplitudes must remain combined.

Appendix H: Microscopic Realization and Coarse-Graining

This appendix develops a microscopic realization of the scalar stiffness and defect ontology. The weak-field coefficient chain does not depend on this realization. Section H.9a gives the complete ordinary-gravity derivation in one sequence; the other sections establish the finite action, prove its component results, identify countermodels, and test the geometric embedding.

H.1 GFT condensate realization and coarse-graining

The candidate microscopic realization is a GFT condensate of bosonic tetrahedral quanta coupled to fermionic defects. Denote the corresponding fields by $\phi(g_1, \dots, g_4)$ and ψ . In the condensate regime, write the coarse field as

$$\sigma(x) = \sqrt{n(x)} e^{i\theta(x)}.$$

The hydrodynamic identity

$$|\nabla_\mu \sigma|^2 = \frac{(\nabla_\mu n)^2}{4n} + n(\nabla_\mu \theta)^2$$

shows that if

$$S_{\text{ent}}(x) = S_0 + \alpha \ln \frac{n(x)}{n_{\text{bg}}},$$

then the coarse action contains a positive scalar stiffness

$$\gamma \sim \frac{Z_\sigma n_{\text{bg}}}{2\alpha^2} > 0.$$

The coarse source channel arises from fermionic face exclusion. A localized condensate defect appears macroscopically as matter, and the surrounding reduction of available occupancy becomes the long-wavelength EFT field. The continuum kinetic term and source channel can therefore arise from the proposed substrate realization. Deriving every inhomogeneous continuum coefficient from the underlying kernel remains open.

This condensate argument supplies a microscopic realization compatible with the EFT ontology and sign choices. The coefficient derivation remains the one given earlier.

The condensate picture addresses the emergence of continuum geometry. The remainder of this appendix derives the defect dynamics behind faithful sector resolution from the admissibility ensemble of Part II together with the Many-Pasts postulate.

H.2 The dressing Hamiltonian and the lightest seven-channel branch

The faithful sector-resolution relation of Appendix D.4 was written there as a support-scale identity on the history space $\mathcal{H}_{\text{hist}} = \bigotimes_{m=-3}^3 \mathcal{H}_B^{(m)}$, using the refresh kernel $P_{\eta_*}(b, b') = p_{\eta_*}(b')$ on each sector layer. Its dynamical content is carried by an explicit defect Hamiltonian, which we now write down so that the choice of that kernel can be derived.

The seven face-label channels of the admissibility-closed ensemble define the orthogonal projectors E_m , $m = -3, \dots, 3$, of the face algebra $\mathcal{A}_7$. By Postulate II the elementary defect is fermionic, so each channel carries an occupation number $n_m \in \{0, 1\}$ with fermionic creation and annihilation operators $c_m^\dagger, c_m$ and $n_m = c_m^\dagger c_m$. When channel m is occupied it is dressed by a cloud state described by a density operator ρ_m on the boundary ensemble $\mathcal{H}_B$. The defect Hamiltonian is

$$H = \underbrace{\sum_m (\varepsilon_0 n_m - n_m F_m(\rho_m))}_{\text{single-channel terms}} + \underbrace{\sum_{m < m'} V_{mm'}(\rho_m, \rho_{m'})}_{\text{inter-channel coupling}},$$

with ε_0 the bare cost to occupy a channel, F_m the free energy released by dressing channel m with its cloud, and $V_{mm'}$ the residual coupling between the clouds of distinct channels. Two properties of the ground state of H supply the two ingredients of the sector-resolution relation.

One-pass occupation and the exponent seven. A channel is energetically occupied when its dressing gain exceeds the fixed bare cost, $F_m(\rho_m) > \varepsilon_0$. Symmetry makes this inequality the same for all seven channels, so an all-bound configuration is a possible symmetric ground-state branch when the inequality holds. The static Hamiltonian alone cannot identify that branch with the lightest charged particle, because its ordering depends on the undetermined balance between ε_0 and the dressing free energy.

The adopted recurrence mass functional supplies the missing ordering. Let $k = \sum_m n_m$ be the number of resolved channels. Fermionic exclusion gives $0 \leq k \leq 7$, and with the admissibility-closed marginal fixed on every occupied layer the support relation generalizes to

$$\frac{\lambda_k}{L_*} = \frac{2}{3} \exp(k g_{\text{share,eff}} - \Delta_k), \quad m_k = \frac{3\hbar}{2cL_*} \exp(\Delta_k - k g_{\text{share,eff}}),$$

where $\Delta_k \geq 0$ is the total correlation among the occupied layers. For each k , the minimum occurs at $\Delta_k = 0$. Between those minima,

$$\frac{m_{k+1}}{m_k} = e^{-g_{\text{share,eff}}} \simeq 5.99 \times 10^{-4},$$

so every missing channel raises the minimum mass by $e^{g_{\text{share,eff}}} \simeq 1.67 \times 10^3$. The unique lightest resolved one-bit defect in this support-to-length branch therefore has $k = 7$ and $\Delta_7 = 0$. The factor seven and the product cloud are selected together, without separate one-pass and factorization postulates. This theorem holds at fixed marginals p_{η_*} . Optimizing the cloud over a different marginal would define a different ultraviolet ensemble.

Factorized cloud and the additive support. The dressing dimension multiplies across channels precisely when the joint cloud state is a product, $\rho = \bigotimes_m \rho_m$; the seven contributions $g_{\text{share,eff}}$ then add. The inter-channel term $V_{mm'}$ determines whether the static ground state of H has this form. At fixed marginals, the recurrence mass functional independently selects the seven-channel product as the lightest branch. Each selected channel samples its full ensemble, and the minimizing readout has no inter-layer correlation.

The static Hamiltonian identifies the channel and cloud variables but does not order all of their branches. The next subsections prove the entropy ceiling, the unique memoryless kernel that reaches it, and the joint $k = 7$, $\Delta_7 = 0$ minimum of the adopted recurrence mass functional.

H.3 Slot coupling, layer factorization, and the additivity theorem

Two distinct correlation structures appear in the closure data, and separating them is essential: conflating them leads to the false conclusion that the cloud cannot factorize.

The closure invariant is a pure pair coupling. Expanding $S^2 = (\sum_i m_i)^2 = \Sigma^2 + 2 \sum_{i < j} m_i m_j$ in $K^2(b) = 48 - \frac{1}{3}(S^2 - \Sigma^2)$ cancels the self-terms exactly and leaves the identity

$$K^2(b) = 48 - \frac{2}{3} \sum_{i < j} m_i m_j.$$

The admissibility weight $e^{-\eta_* K^2(b)}$ therefore contains only cross-terms between distinct face slots of a single boundary state; it does not factorize over those four slots. This is the exact origin of

the residual correlation between face slots on the closed ensemble,

$$I(\text{slot}_0; \text{slot}_1) = 0.1545 \text{ nats}, \quad \frac{I}{H(\text{slot})} = 0.079,$$

so the four faces of one tetrahedron are about eight percent correlated, a structural feature of the closure invariant.

Slot coupling does not obstruct layer factorization. The factorization the support relation requires is over the seven *sector layers* $b^{(-3)}, \dots, b^{(3)}$ of $\mathcal{H}_{\text{hist}}$, each layer being a full boundary state drawn from the entire 1680-state ensemble. The slot coupling $-\frac{2}{3}m_i m_j$ lives *inside* a single layer’s boundary state: it relates the four faces of that one tetrahedron and never couples layer m to layer m' . The two structures act on different objects:

Structure	What it couples	Role
slot coupling $-\frac{2}{3}m_i m_j$	the four faces <i>within</i> one boundary state b	the eight-percent mutual information; lives inside each $\mathcal{H}_B^{(m)}$
layer coupling $V_{mm'}$	the seven sector layers $b^{(m)}$ of $\mathcal{H}_{\text{hist}}$	controls factorization; acts <i>between</i> the factors

The substrate’s intrinsic correlation, the most natural candidate obstruction to factorization, therefore acts at the wrong level to obstruct it: it is internal to a layer, not between layers.

Additivity of the decoherent preparation overlap. Appendix G.3 defines $D(h, P) = -\ln p(h)$ from the diagonal decoherence-functional probability. For the special case in which H already denotes a decohered preparation and only the final record remains unresolved, this reduces to $D(H, P) = -\ln \text{Tr}(\Pi_P \rho_{H \rightarrow \text{now}})$. If that preparation state and the resolution projector factorize over channels, $\rho = \bigotimes_m \rho_m$ and $\Pi_P = \bigotimes_m \Pi_m$, the trace factorizes and the logarithm converts the product into a sum,

$$\text{Tr}(\Pi_P \rho) = \prod_m \text{Tr}(\Pi_m \rho_m) \implies D = \sum_m [-\ln \text{Tr}(\Pi_m \rho_m)] = \sum_m D_m.$$

The overlap distance is therefore additive in this special factorized preparation. The effective-support statement used downstream follows independently from Shannon subadditivity: a product cloud with seven equal marginals has joint entropy $7g_{\text{share,eff}}$ and perplexity $e^{7g_{\text{share,eff}}}$. Whether the electron’s dressing is that product is decided by the two conditions in the next subsections.

The ceiling and the two conditions. Subadditivity bounds the support from above. With each channel marginal fixed to the admissibility weight, $H(B_m) = g_{\text{share,eff}}$, the joint entropy of a single readout obeys

$$H(B_{-3}, \dots, B_3) \leq \sum_{m=-3}^3 H(B_m) = 7g_{\text{share,eff}},$$

and the deficit

$$\Delta = \sum_m H(B_m) - H(B_{-3}, \dots, B_3) \geq 0$$

measures the total correlation among the seven channels. The full support $\text{dim}_{\text{eff}} = e^{7g_{\text{share,eff}}}$ is reached precisely when each channel carries its full entropy $g_{\text{share,eff}}$ and the channels are mutually independent, $\Delta = 0$. The first is a condition on each layer in substrate time; the second is a condition on the seven layers at a single readout. The next two subsections establish the unique faithful-resolution kernel and the minimal-mass factorized branch, while separating those results from the still-open microscopic implementation.

H.4 Maximum caliber: complete renewal is uniquely selected

The closure calculation fixes the one-time marginal p_{η_*} . Applying faithful full-support resolution to entire paths maximizes their entropy rate, a condition conventionally called maximum caliber [67]. This is the temporal form of the existing foundational requirement. For an arbitrary stationary process, its entropy rate is

$$h_\mu := \lim_{n \rightarrow \infty} H(B_0 \mid B_{-1}, \dots, B_{-n}) \leq H(B_0) = g_{\text{share,eff}}.$$

Equality holds only when B_0 is independent of its complete past. Stationarity then makes the process independent and identically distributed, with every conditional distribution equal to p_{η_*} . Maximum path entropy therefore selects complete renewal uniquely. Finite capacity alone does not imply renewal. The selection follows from faithful full-support resolution, whose temporal formulation is maximum caliber.

The one-parameter family below illustrates the cost of retained memory:

$$K_a(b, b') = a \delta(b, b') + (1 - a) p_{\eta_*}(b'), \quad a \in [0, 1],$$

which repeats the current state with probability a and otherwise redraws from the stationary weight. Every member has p_{η_*} as its stationary distribution and the same single-time marginal, so the equilibrium ensemble cannot say which one governs the dressing. The per-channel conditional entropy $H(b' \mid b) = \sum_b p_{\eta_*}(b) H(K_a(b, \cdot))$, evaluated on the exact 1680-state ensemble, nonetheless slides with the memory a :

a (memory)	per-channel $H(b' \mid b)$	status
0.0 (refresh)	7.41980 = $g_{\text{share,eff}}$	full entropy
0.1	6.99953	reduced
0.3	5.80153	reduced
0.5	4.40051	reduced
0.9	1.06642	reduced

Only the memoryless endpoint $a = 0$ — the replacement kernel $K_*(b, b') = p_{\eta_*}(b')$ of Appendix D.4 — returns the full $g_{\text{share,eff}}$. For any stationary Markov kernel with that marginal,

$$H(B_{t+1} \mid B_t) = H(B_{t+1}) - I(B_t; B_{t+1}) \leq g_{\text{share,eff}},$$

and equality holds if and only if $I(B_t; B_{t+1}) = 0$. The Markov corollary is therefore

$$\boxed{K_*(b, b') = p_{\eta_*}(b')}.$$

Single-label local moves conserve slot ordering, fracturing each parity copy into twenty-four sectors of thirty-five states, so no such local kernel reaches the full ensemble at any parameter value (Appendix D.4). The native-vertex construction of H.8 implements the selected nonlocal-in-label-space replacement as one operation on the complete local tetrahedral register.

The lightest-defect branch selects the same endpoint independently within the adopted support map. If a per-pass temporal mutual information I_t reduces the fresh entropy from $g_{\text{share,eff}}$ to $g_{\text{share,eff}} - I_t$, then at fixed L_*

$$\lambda(I_t) = \lambda(0)e^{-I_t}, \quad m(I_t) = m(0)e^{I_t}.$$

Every retained temporal correlation raises the recurrence mass, so electron lightness again selects $I_t = 0$. Faithful full-support resolution selects the vacuum history process; mass minimization independently selects the same process in the defect sector.

A dimensionless mixing test shows that the selected kernel is dynamically available. Take any ergodic generator Q (a symbol used only within this mixing test; the survival complement of H.6

is a distinct operator) with p_{η_*} as its detailed-balance stationary weight. The test uses nonlocal transpositions of two occupied labels, not the injectivity-preserving single-label shifts excluded in the preceding paragraph. A continuous-time swap generator built from the same admissibility weights serves. The kernel $e^{Q\tau}$ loses endpoint mutual information,

$$I(\tau=0.1) \approx 3.4 \text{ nats}, \quad I(\tau=1) \approx 0.030, \quad I(\tau=2) \approx 1 \times 10^{-4},$$

so the replacement kernel is the late- τ limit of a broad class of dimensionless mixing dynamics. Faithful full-support resolution selects this endpoint but leaves the approach to it unspecified. H.8 derives the quantum replacement channel and reversible update; H.9 supplies the fresh amplitude, marked event, and update interaction. Durable history capacity and the stable geometric GFT embedding remain open.

H.5 Independent channels: the electron as the lightest defect

The second condition is that the occupied channels factorize at one readout, so that the deficit Δ_k of H.3 vanishes. Appendix H.2 already showed that electron lightness selects both the maximum occupation $k = 7$ and $\Delta_7 = 0$ once the support-to-length dictionary is adopted. The calculation here isolates the factorization part of that joint minimum.

Keep the deficit explicit in the baseline support relation. With each channel at its full entropy, the joint entropy is $7g_{\text{share,eff}} - \Delta$, and

$$\frac{\lambda}{L_*^{(0)}} = \frac{2}{3} e^{7g_{\text{share,eff}} - \Delta}.$$

At a fixed substrate scale L_* , a one-bit charged defect whose dressing carries correlation Δ has spatial support $\lambda \propto e^{-\Delta}$, and through $m = \hbar/(c\lambda)$ a mass

$$m \propto e^{\Delta}.$$

Correlation therefore contracts the support and raises the recurrence mass. Because $\Delta \geq 0$, the lightest one-bit charged defect is the product dressing $\Delta = 0$; correlated dressings describe heavier branches within the same recurrence/support map. Independence is not inferred from the absence of an interaction term. It is forced by mass minimization in the theory's own definition of the electron anchor.

Within the marked-transfer branch, the information-theoretic selections read end to end. Faithful full support fixes memoryless refresh, while fermionic exclusion and electron lightness select $k = 7$ and $\Delta_7 = 0$. H.6 gives the baseline survival scale and H.9 its finite dressing:

$$L_* = -\frac{3}{2} Z_e \lambda_e \ln(1 - e^{-7g_{\text{share,eff}}}), \quad G_* = \frac{c^3 L_*^2}{\hbar},$$

with no gravitational quantity used as input.

The decorated action realizes the nonlocal replacement, fresh state, and charged marked event. In the factorized completion those operations live on the finite capacity/record factor and need not be re-derived from the geometric magnetic indices. A specific many-body GFT realization must still determine how the decoration couples to inhomogeneous matter and transverse response; it need not re-select memorylessness, the clock conversion, or $\Delta = 0$.

H.6 Mass from the positive survival transfer operator

Faithful full-support resolution fixes the replacement process on history space but not a dimensional duration. The duration comes from the paper's existing electron anchor once the already-declared effective-support event is represented in the transfer space. No second clock principle or new premise is required.

The marked seven-channel transfer. On the factorized lightest branch, likelihood multiplication on each renewed cloud has determinant $e^{-g_{\text{share,eff}}}$. The product determinant is

$$r := \Delta_{\tau_p^{\otimes 7}}(\mathbb{R}^{\otimes 7}) = e^{-7g_{\text{share,eff}}}.$$

This is the scalar charged determinant line of the decorated transfer, not the probability of an arbitrarily chosen seven-layer microstate. Fermionic one-bit exclusion gives the positive no-loop transfer

$$T_{\text{surv}}^{(0)} = 1 - r,$$

equivalently the one-dimensional Grassmann determinant $\int d\bar{c} dc e^{-\bar{c}(1-r)c} = 1 - r$. One may therefore define

$$H_{\text{surv}}^{(0)} = -\frac{\hbar}{\tau_*^{(0)}} \ln T_{\text{surv}}^{(0)},$$

with exact raw gap

$$E_{\text{raw}}^{(0)} = -\frac{\hbar}{\tau_*^{(0)}} \ln(1 - r).$$

The unit mixing gap on the renewed probability space is a different operator and is not identified with this exponentially small charged gap.

Electron calibration and exact cell length. Before the finite marked response is included, the transverse export gives $H_e^{(0)} = (3/2)H_{\text{surv}}^{(0)}$. The baseline electron calibration is

$$m_e c^2 = -\frac{3\hbar}{2\tau_*^{(0)}} \ln(1 - r).$$

It fixes

$$\tau_*^{(0)} = -\frac{3}{2}\tau_e \ln(1 - r), \quad L_*^{(0)} = c\tau_*^{(0)} = -\frac{3}{2}\lambda_e \ln(1 - r).$$

For $r = e^{-7g_{\text{share,eff}}}$,

$$L_*^{(0)} = -\frac{3}{2}\lambda_e \ln(1 - e^{-7g_{\text{share,eff}}}) = \frac{3}{2}\lambda_e e^{-7g_{\text{share,eff}}} \left[1 + \frac{1}{2}e^{-7g_{\text{share,eff}}} + O(e^{-14g_{\text{share,eff}}}) \right].$$

The relative logarithmic correction to the leading expression is 1.4×10^{-23} . The baseline induced scale is

$$G_*^{(0)} = \frac{9}{4} \frac{\hbar c}{m_e^2} [\ln(1 - e^{-7g_{\text{share,eff}}})]^2,$$

whose leading form is proportional to $e^{-14g_{\text{share,eff}}}$. H.9 derives the finite response and gives $\tau_* = Z_e \tau_*^{(0)}$, $L_* = Z_e L_*^{(0)}$, and $G_* = Z_e^2 G_*^{(0)}$.

Why this supplies the phase bridge. A positive Euclidean transfer operator defines an energy spectrum. Analytic continuation supplies the Lorentzian phase frequency of the same charged mode. An independent identification of an integer renewal count with a condensate winding is unnecessary. H.9 realizes the determinant recurrence and marked response in the decorated charged vertex.

The temporal and spatial grammars agree without referring to a literal tetrahedron diameter. If N_{eff} is the common leading support factor, $\tau_e = N_{\text{eff}}\tau_*$ and $\lambda_e = N_{\text{eff}}L_*$. The identity $\lambda_e = c\tau_e$ then gives $L_* = c\tau_*$. The graph calculation uses this L_* as its canonical displacement unit; the support size of a future geometric tetrahedron, if one is introduced, is a separate quantity constrained by locality rather than a coefficient that sets the tick.

Internal locality and the native vertex. The tested single-label move cannot implement replacement. Injectivity-preserving shifts fracture the 1680 states into

$$48 \text{ components} = 4! \text{ orderings} \times 2 \text{ orientations}, \quad 35 \text{ states per component.}$$

H.8 constructs a one-layer reversible dilation when the complete tetrahedral register is the native local vertex. This establishes circuit depth one, not the physical duration; the duration is fixed spectrally by the electron equation above. H.9 decorates that gate with the fresh amplitude and marked charged event.

H.7 Conditional dressing functionals and the condensate-spectrum lemma

The dressing Hamiltonian named in Section 23,

$$H = \sum_m (\varepsilon_0 n_m - n_m F_m(\rho_m)) + \sum_{m < m'} V_{mm'}(\rho_m, \rho_{m'}),$$

carries three named functionals ε_0, F_m, V . A proposed mean-field condensate gives a conditional reconstruction of these functionals. The required condensate has not yet been derived from a specified GFT action. The two residuals of H.6 also remain separate outputs of the missing fluctuation calculation.

Mean-field reduction. In a GFT condensate the boundary-data order parameter is $\langle \hat{\varphi}(b) \rangle = \sigma(b)$. The admissibility calculation fixes a normalized modulus profile $p_{\eta_*}(b)$, so a candidate condensate may be written

$$|\sigma(b)|^2 = \bar{n} p_{\eta_*}(b).$$

This fixes neither the overall density $\bar{n}$ nor any of the phases. A general 1680-component condensate has a common phase and relative-phase directions, and their physical status is determined by the kinetic and interaction kernels of the specified GFT action. Likewise, η_*^{-1} may be used as a formal closure-temperature parameter, but the identity

$$\eta_* \langle K^2 \rangle_{\eta_*} = \frac{3}{2}$$

is the stationarity condition generated by the three-component determinant weight, not a literal equipartition theorem for the bounded discrete spectrum. Conditional on a condensate realization, $T_* = 1/\eta_* = 33.48$ may nevertheless be read as the dimensionless substrate closure temperature, with $\langle K^2 \rangle_{\eta_*} = \frac{3}{2}T_* = 50.22$; this is an interpretation of the same stationary point and introduces no new number.

The refresh projector. The memoryless transition matrix is $K_*(b, b') = p_{\eta_*}(b')$. Let $D_p = \text{diag}(p_{\eta_*})$ and define the detailed-balance symmetrization

$$\tilde{K}_* = D_p^{1/2} K_* D_p^{-1/2}.$$

With $v_b = \sqrt{p_{\eta_*}(b)}$, one obtains

$$\tilde{K}_* = |v\rangle\langle v| \equiv P, \quad \tilde{L}_{\text{mix}} = I - \tilde{K}_* = P_{\perp}.$$

The classical replacement process has one stationary direction in the weighted probability space and removes every orthogonal probability mode after one discrete update. The vector $|v\rangle$ belongs to the symmetrized classical transfer representation; it is not automatically the quantum state of the cell. It does not follow that $P_{\perp}$ is a Hermitian mass matrix in a closed Lorentzian GFT. The Markov update, a quantum density operator, and the coherent fluctuation Hessian are different objects.

Conditional refresh–Hessian bridge. The minimal reversible bridge uses the positive refresh Dirichlet form as the internal Euclidean quadratic cost,

$$\mathcal{K}_{\text{int}}^{(2)} = \Delta_{\text{ref}} P_{\perp}, \quad \Delta_{\text{ref}} > 0.$$

A fixed- j Euclidean effective action realizing this bridge is

$$\Gamma_E[\sigma] = \int d^4x_E \left[Z \partial_{\mu} \sigma^{\dagger} \partial^{\mu} \sigma - \mu^2 \sigma^{\dagger} \sigma + \frac{\lambda}{2} (\sigma^{\dagger} \sigma)^2 + \Delta_{\text{ref}} \sigma^{\dagger} P_{\perp} \sigma \right].$$

It has $\sigma_0 = \sqrt{\bar{n}} e^{i\theta_0} v$ with $\bar{n} = \mu^2/\lambda$. With canonically normalized real fluctuations, its Euclidean quadratic operator contains a radial singlet with $m_h^2 = 2\mu^2/Z$, a common phase with $m_{\pi}^2 = 0$ when number U(1) is exact, and 1679 complex relative modes with $m_{\perp}^2 = \Delta_{\text{ref}}/Z$. Lorentzian propagation requires the usual analytic continuation and the global $(-, +, +, +)$ convention of Appendix A. The exclusion source couples at linear order to the density fluctuation h because $\delta(\sigma^{\dagger} \sigma) = 2\sqrt{\bar{n}} h + \dots$. In the constant-phase realization displayed here, the common phase is derivatively coupled and has no linear zero-frequency overlap with that static density source. This is not a structural decoupling theorem for a relational GFT condensate. If $\sigma = \sqrt{\bar{n}} e^{i\theta}$ and the background carries relational phase current $\partial_{\chi} \theta_0 \neq 0$, the Madelung term contains

$$n(\partial_{\chi} \theta)^2 \supset 2(\partial_{\chi} \theta_0) \delta n \partial_{\chi} \delta \theta,$$

so density and phase can mix. The complete coherent Hessian and defect/source vertex must determine whether a phase mode is gapped, remains light but source-orthogonal, or mediates an additional response. Simplicial interactions may break number U(1) and gap it.

An open-system completion may instead implement renewal as the quantum replacement channel

$$\mathcal{E}_*(\rho) = \rho_* \text{Tr } \rho.$$

The classical closure ensemble fixes the diagonal probabilities of ρ_* in the boundary basis. H.9 adopts the diagonal maximum-entropy completion and constructs its reversible update. A dissipative representation with Lindblad generator $\dot{\rho} = \Gamma_{\text{ref}}[\mathcal{E}_*(\rho) - \rho]$ gives every traceless input perturbation a relaxation gap. A geometric condensate embedding must determine how this transfer description appears in its coherent fluctuation spectrum.

Selection, mixing, and marked survival are distinct operators. The combinatorial Hamiltonian selects the admissibility profile. The mixing generator $\tilde{L}_{\text{mix}} = P_{\perp}$ controls decay toward that profile. The exponentially small electron scale comes from a third object: the scalar determinant transfer of the seven-channel likelihood operator,

$$r := \Delta_{\tau_p^{\otimes 7}}(\mathbb{R}^{\otimes 7}) = e^{-7g_{\text{share,eff}}},$$

whose one-bit survival transfer is $T_{\text{surv}}^{(0)} = 1 - r$. Over a baseline proper-time step $\tau_*^{(0)}$ its exact spectral gap is

$$\text{gap}(H_{\text{surv}}^{(0)}) = -\frac{1}{\tau_*^{(0)}} \ln(1 - r), \quad m_e c^2 = \frac{3}{2} \hbar \text{gap}(H_{\text{surv}}^{(0)}).$$

Solving gives $\tau_*^{(0)} = -(3/2)\tau_e \ln(1 - r)$ and $L_*^{(0)} = c\tau_*^{(0)}$. H.9 derives the marked factor Z_e and the physical $L_* = Z_e L_*^{(0)}$. The unit mixing gap and charged survival gap are not identified.

ε_0 , the bare cost. ε_0 is the condensate chemical potential $\mu_0 = \partial E / \partial N$, the energy to commit one cell against the mean field, of order $E_* = \hbar c / L_*$. Energies in H are measured in units of E_* , and $T_* = 1/\eta_*$ is dimensionless. The rest mass is read from the positive marked-survival operator H_{surv} , not from the static spectrum of H or the unit mixing gap of L_{mix} .

F_m , the dressing free energy. A bound channel carries a cloud ρ_m over the boundary ensemble. The proposed released free energy is $F_m(\rho_m) = T_* S[\rho_m]$. At fixed mean closure its maximum is the Gibbs state p_{η_*} with entropy $g_{\text{share,eff}}$. On the selected lightest branch the seven clouds carry total entropy $7g_{\text{share,eff}}$. The faithful state-weighted determinant of their likelihood operator is exactly $e^{-7g_{\text{share,eff}}}$. This is a quenched multiplicative transfer rate, not a raw single-draw Gibbs probability.

Two energies, kept distinct. The separation cost and rest energy are different quantities. The separation cost is the entropic free energy required to dissolve a defect into the substrate; in this construction it is large and linear in $g_{\text{share,eff}}$. The rest energy is $m_e c^2 = \hbar \Gamma_e$, with Γ_e set by the rare joint recurrence and therefore exponential in $-7g_{\text{share,eff}}$. Reading the separation cost as the mass would place the electron near tens of E_* and remove the required suppression.

V , the inter-channel coupling. The permutation-invariant singlet

$$|s\rangle = \frac{1}{\sqrt{7}} \sum_m |m\rangle$$

is the natural channel through which a scalar capacity fluctuation couples uniformly to the seven sectors. Within the minimal constant-phase refresh–Hessian completion displayed above, the radial density fluctuation is the only linearly source-coupled singlet at static order and the relative sector is gapped. Integrating out that density mode gives

$$V_{mm'} = v \langle m|s\rangle \langle s|m'\rangle = \frac{v}{7},$$

a rank-one outer product. The per-incidence overlap is $1/7$; the separate orientation doubling gives the $2/7$ used in Appendix I.1. The implication is

$$\text{one source-coupled density singlet} \implies \text{rank}(V) = 1.$$

This result is derived inside the minimal bridge, not from the closure ensemble alone. A microscopic interaction that mixes P and $P_\perp$, closes the relative gap, or sources the common phase would add exchange channels and deform the uniform structure.

Positive-transfer phase and the exchanged mode. The projector calculation separates the static source from the transfer clock. The static exclusion source couples to the radial density singlet. Independently, the positive Euclidean survival operator defines the electron energy, and ordinary analytic continuation gives the Lorentzian phase frequency $\omega_e = E_e/\hbar$ of that same charged mode. The decorated vertex supplies the finite charged routing. The remaining transverse question concerns the microscopic origin of the carrier source projector and metric contact: the interaction must preserve the established projection and must not create an additional sourced scalar pole beyond the Einstein constraint response.

The remaining microscopic lemma. The effective projector model narrows the missing calculation without requiring geometry–capacity mixing to vanish. For a specified fixed- j simplicial GFT action admitting $\sigma_b = \sqrt{\bar{n} p_{\eta_*}(b)} e^{i\theta}$, compute its complete Bogoliubov or Schwinger–Keldysh kernel. After the within-tick evolution $\mathcal{U}^\dagger$, exact renewal gives

$$\mathcal{T}^\dagger = P\mathcal{U}^\dagger = \begin{pmatrix} A & B \\ 0 & 0 \end{pmatrix} \quad \text{on } P\mathcal{K} \oplus P_\perp\mathcal{K},$$

so the replaceable sector cannot carry an autonomous nonzero transfer pole even when $B \neq 0$; H.9a proves the spectral statement. The remaining calculation is instead to show that

the retained block A has only the Einstein constraint and transverse–traceless gravitational support at low energy, while any additional persistent relative mode is gapped, relaxational, gauge/constraint, or source-orthogonal. The decorated transfer vertex of H.9 fixes the charged recurrence and its edge Hessian. A geometric GFT must still determine whether its retained condensate realization produces a coherent mass, a relaxation gap, or both, and verify that no additional source-coupled light field survives. The closure weight fixes P exactly, while the refresh–Hessian bridge for the geometric condensate remains a selected effective completion.

H.8 Faithful history resolution and reversible Many-Pasts renewal

Earlier versions grouped four distinct statements under “refresh”: the classical history process, the quantum channel, its reversible implementation, and the conversion of a positive charged spectrum into proper time. They are treated separately below. Section H.9 supplies the charged marked vertex. Maximum-caliber terminology and the clock construction add no foundational premise.

Maximum path entropy. Let $\mathcal{B}$ be the finite admissible boundary space and let $p_*(b) = p_{\eta_*}(b)$ be its fixed stationary marginal. For any stationary process $\{B_n\}$ on $\mathcal{B}$,

$$h_\mu = \lim_{N \rightarrow \infty} \frac{1}{N} H(B_1, \dots, B_N) = H(B_0 \mid B_{-1}, B_{-2}, \dots) \leq H(p_*).$$

Equality holds exactly when B_0 is independent of its complete past. Applying the same statement after every time translation factorizes every finite joint distribution,

$$P(B_1 = b_1, \dots, B_N = b_N) = \prod_{n=1}^N p_*(b_n).$$

The unique maximum-caliber history process is therefore i.i.d. For a Markov representation its transition kernel is

$$\boxed{K_*(b, b') = p_*(b')}. \quad (1)$$

Here “maximum caliber” is terminology for faithful full-support resolution on history space [67]. It is not a sixth theory-defining input. The foundational requirement already says that an admissible resolution carries the full available entropy compatible with its fixed data; applying that same requirement to a path gives the equality case above. Finite capacity alone would not imply renewal, but the paper’s stronger faithful full-support premise does.

The selected process carries no thermodynamic orientation. It obeys

$$p_*(b)K_*(b, b') = p_*(b)p_*(b') = p_*(b')K_*(b', b),$$

so its stationary path measure is exactly invariant under time reversal. Faithful history resolution derives local renewal, not a low-entropy past and not a physical cadence.

Proper time from the positive marked-transfer spectrum. The renewal theorem fixes the discrete process but says nothing about seconds per update. Proper time comes from the determinant survival operator of H.6,

$$r = \Delta_{\tau_p^{\otimes 7}}(\mathbf{R}^{\otimes 7}) = e^{-7g_{\text{share}, \text{eff}}}, \quad T_{\text{surv}}^{(0)} = 1 - r.$$

Defining the baseline transfer over $\tau_*^{(0)}$ gives

$$H_{\text{surv}}^{(0)} := -\frac{\hbar}{\tau_*^{(0)}} \ln T_{\text{surv}}^{(0)}, \quad E_{\text{raw}}^{(0)} = -\frac{\hbar}{\tau_*^{(0)}} \ln(1 - r)$$

on its nonzero support. The baseline transverse identification $m_e c^2 = (3/2)E_{\text{raw}}^{(0)}$ therefore yields

$$\boxed{\tau_*^{(0)} = -\frac{3}{2}\tau_e \ln(1-r)}, \quad \boxed{L_*^{(0)} = c\tau_*^{(0)} = -\frac{3}{2}\lambda_e \ln(1-r)}.$$

The leading form is $L_*^{(0)} = (3/2)\lambda_e r[1 + O(r)]$. H.9 supplies the finite marked factor Z_e and hence the physical $L_* = Z_e L_*^{(0)}$. No update diameter, throughput maximization, or geometric conversion factor enters. The complete tetrahedron being a native gate is a locality and circuit-depth statement only; its physical embedding size is a separate consistency question.

A positive Euclidean transfer spectrum also supplies, by analytic continuation, the Lorentzian phase frequency. The clock and phase use the same charged spectral mode and require no independent winding postulate. H.9 derives the recurrence as the state-weighted determinant and implements its finite marked dressing.

Quantum replacement theorem. Let $\mathcal{H}_A$ be the replaceable closure-register Hilbert space and $\mathcal{E} : A \rightarrow A'$ a trace-preserving quantum channel. It does not include the retained position and internal fiber of a persistent charged defect. Complete local renewal means that the output contains no information about the input, including information visible through an arbitrary reference R :

$$(\text{id}_R \otimes \mathcal{E})(\rho_{RA}) = \rho_R \otimes \rho_* \quad \text{for every } \rho_{RA}.$$

Taking product inputs first shows that every normalized input has the same output ρ_* . Linearity then gives the unique channel on all operators,

$$\boxed{\mathcal{E}_*(X) = \rho_* \text{Tr } X}.$$

Every traceless perturbation is annihilated after one update. As a superoperator, $\mathcal{E}_*$ has one stationary direction and zero on the traceless operator subspace. The closure ensemble fixes $\langle b | \rho_* | b \rangle = p_*(b)$ in the boundary basis. H.9 selects the diagonal maximum-entropy completion for the decorated transfer action; a different coherent GFT state would define a different microscopic completion and must be tested against that vertex. H.11 proves that this statement is compatible with particle interference only when the renewed output excludes the persistent marked-position fiber and the discarded free-renewal record is branch-independent.

Reversible history export. Local replacement is compatible with global unitarity by Stinespring dilation [68]. Introduce the old present register A , a fresh register F , its purifier R , and a history register H . For

$$|\Psi_*\rangle_{FR} = \sum_{b \in \mathcal{B}} \sqrt{p_*(b)} |b\rangle_F |b\rangle_R$$

in the diagonal closure-state realization, a register permutation can act as

$$|\psi\rangle_A |\Psi_*\rangle_{FR} |0\rangle_H \mapsto |\Psi_*\rangle_{AR} |0\rangle_F |\psi\rangle_H.$$

Tracing out R, F, H leaves ρ_* on the new closure present and removes every dependence on the old replaceable state. The inverse unitary recovers that state from H , so no global information has been destroyed. Postulate III supplies the interpretation of the exported register as history structure; the dilation theorem alone calls it an environment. For an unmeasured freely propagating defect, the state exported by this map may not encode the cell address of the mark. The marked fiber is transported in the retained defect system, and only a genuine interaction may correlate it with a discarded environmental record. Repeating the construction indefinitely requires fresh and history capacity or a controlled recycling mechanism, which the present paper has not derived.

The tetrahedral circuit. For

$$\mathcal{B} = \{(s; m_1, m_2, m_3, m_4) : s = \pm, m_i \in \{-3, \dots, 3\}, m_i \neq m_j\}, \quad |\mathcal{B}| = 1680,$$

the register permutation above is one circuit layer if the complete tetrahedron is a native local gate and the fresh admissible register has already been prepared. Neither injectivity nor the collective function $K^2(m_1, m_2, m_3, m_4)$ then requires a sequence of face repairs; the gate replaces one admissible four-face object by another.

A depth-one layer of disjoint two-face gates cannot test all six injectivity relations. The direct pair-comparison construction uses the three perfect matchings

$$\{(1, 2), (3, 4)\}, \quad \{(1, 3), (2, 4)\}, \quad \{(1, 4), (2, 3)\},$$

and therefore has depth three. This proves the stated depth for the direct comparator and excludes depth one in a pairwise architecture without a jointly coupled ancilla. It is not a lower-bound proof for every possible ancilla-assisted state-preparation algorithm. H.9 takes the complete tetrahedron as the native decorated gate; a geometric GFT embedding must realize that locality and sustain the prepared state dynamically.

Consequences and remaining gap. The foundational faithful full-support condition closes the classical replacement kernel when applied to histories. Detailed-balance symmetrization then closes $P = |\sqrt{p}\rangle\langle\sqrt{p}|$ and $P_\perp = I - P$ in probability space. Reference decoupling closes the quantum replacement-channel form, and the native tetrahedral architecture supplies a one-layer reversible dilation. H.9 adds the marked recurrence, after which positivity and the electron anchor close the clock, length, and phase-frequency conversion. These results remove an arbitrary transition matrix, geometric clock factor, and separate winding assumption while showing how local memorylessness can coexist with a globally retained past.

They do not by themselves close the coherent GFT Hessian. The exclusion sufficiency lemma of C.5 proves that one scalar statistic contains all local classical information distinguishing the canonical defect from the vacuum. The replacement process removes orthogonal probability memory. A microscopic spectrum must still show that no additional conserved or coherent light field survives and that the source couples only to the capacity singlet. The bridge from $P_\perp$ to a Euclidean mass or Schwinger–Keldysh relaxation pole therefore retains its conditional grade. The stationary reset also leaves the macroscopic arrow-of-time problem in G.7 unchanged.

H.9 Decorated simplicial marked-transfer vertex

The renewed marginal, lightest seven-channel branch, determinant recurrence, and reversible present/history exchange determine a finite charged vertex with a fully specified marked sector. In the selected factorized completion of Section 23 and Appendix Q, this vertex acts on the capacity/record factor; an EPRL-class simplicial factor carries the vacuum geometry. The equivariant V_3 frame lock relates the two factors without identifying their magnetic indices. On the native tetrahedron, the vertex realizes faithful full-support history resolution, and the established one-bit fermionic defect supplies the hard-core mark. The field space and routing are physical choices: the countervertices below fail the finite audits.

The marked fiber from primitive fusion. The nine-state marked space also has an independent geometric derivation. A shared face begins with two primitive fermionic slots of spin $j_0 = 3/2$. Maximum-capacity fusion selects the unique maximal channel,

$$V_{3/2} \otimes V_{3/2} = V_3 \oplus \underbrace{(V_0 \oplus V_1 \oplus V_2)}_Q, \quad \dim Q = 9.$$

The selected V_3 is the seven-state effective bulk link used by the closure ensemble. The nonmaximal information is the representation $Q = V_0 \oplus V_1 \oplus V_2$. The marked response was obtained from a different construction: the closure response is a vector V_1 , and one response excitation on each of the present and history strands gives

$$\mathcal{H}_{\text{mark}} = V_1^P \otimes V_1^H = V_0 \oplus V_1 \oplus V_2.$$

Thus there is an $SU(2)$ intertwiner

$$\mathcal{I} : Q \longrightarrow \mathcal{H}_{\text{mark}}, \quad \mathcal{I} |(j_0 j_0); J, M\rangle = |(1_P 1_H); J, M\rangle, \quad J = 0, 1, 2,$$

with $\mathcal{I}^\dagger \mathcal{I} = P_Q$ and $\mathcal{I} \mathcal{I}^\dagger = I_9$. Each irrep occurs once, so the intertwiner is unique up to one phase on each J block.

The match is more selective than the dimension count. For a response strand of spin s ,

$$V_s \otimes V_s = \bigoplus_{J=0}^{2s} V_J,$$

whereas the complement of maximal fusion for two primitive spin- j slots is

$$Q(j) = (V_j \otimes V_j) \ominus V_{2j} = \bigoplus_{J=0}^{2j-1} V_J.$$

The two representations agree if and only if $j = s + 1/2$. The independently fixed closure-vector value $s = 1$ therefore selects $j = 3/2$. Appendix B proves the representation theorem, constructs the sharp complement isometry, and states its logical condition. If the marked present/history register is the faithful dynamical record of primitive nonmaximal fusion, this is a derivation of $j_0 = \frac{3}{2}$ from the existing three-dimensional response. Without that incidence identification it remains an exact structural match. The complete geometric vertex must still reproduce the equal weights, flags, phases, hard-core mark, and routing of the explicit vertex below.

Fresh-state amplitude from the closure action. Let $C_a(b)$ be the three-component closure vector, with $K^2(b) = C_a(b)C_a(b)$. The diagonal renewed state has amplitude

$$A_*(b) = Z^{-1/2} \exp\left[-\frac{\eta_*}{2} K^2(b)\right], \quad |A_*(b)|^2 = p_*(b).$$

For a standard three-component Gaussian auxiliary ξ_a ,

$$\exp\left[-\frac{\eta_*}{2} C^2\right] = \int \frac{d^3 \xi}{(2\pi)^{3/2}} \exp\left[-\frac{1}{2} \xi^2 + i\sqrt{\eta_*} \xi_a C_a\right].$$

The amplitude-level closure incidence is therefore $\sqrt{\eta_*}$. The minimal charged decoration reuses this strand, so it introduces no second response coupling. Faithful full-support resolution is being applied on a marked record space with no further joint constraint. Adding a defect-only invariant would define a nonminimal countervertex; it is allowed as a different theory and is one of the falsifiers of the construction, not an extra premise needed by the selected action.

The reversible update has independent present and history Gaussian strands. Their first normalized Hermite excitations obey

$$\langle a|c\rangle_P = \delta_{ac}, \quad \langle b|d\rangle_H = \delta_{bd}.$$

The ordered products $|a, b\rangle = |a\rangle_P \otimes |b\rangle_H$ form nine orthonormal states,

$$\langle a, b|c, d\rangle = \delta_{ac}\delta_{bd}.$$

They carry $\mathbb{R}^3 \otimes \mathbb{R}^3 = \mathbf{1} \oplus \mathbf{3} \oplus \mathbf{5}$. Introduce hard-core marked states $d_{ab}^\dagger |0\rangle$ and the stranded vertex

$$V_Q = \sum_{a,b=1}^3 d_{ab}^\dagger \xi_a^P \xi_b^H + \text{h.c.}$$

Its reduced Gram matrix is $I_3 \otimes I_3 = I_9$. Equal marked weights follow from this two-strand contraction, not from rotational symmetry or the number nine alone. Trace and epsilon contractions would split the three rotation sectors and falsify the vertex.

Pair contraction and canonical dilation. For each unordered occupied pair $e = (m, m')$, the two directed scalar returns are orthogonal alternatives. Define

$$R_e = q(|\langle m \rightarrow m' | + \langle m' \rightarrow m |), \quad q = \frac{2}{7},$$

and $B_e = \sqrt{\eta_*} R_e$. Then

$$B_e B_e^\dagger = 2\eta_* q^2 = \frac{8\eta_*}{49} \equiv u, \quad 0 < u < 1.$$

The canonical unitary dilation of this contraction is

$$U_e = \begin{pmatrix} \sqrt{I - B_e^\dagger B_e} & -B_e^\dagger \\ B_e & \sqrt{I - B_e B_e^\dagger} \end{pmatrix}.$$

On the occupied scalar-event support its no-event amplitude is

$$\alpha = \sqrt{1 - u} = \sqrt{1 - \frac{8\eta_*}{49}}.$$

This identifies the origin of every entry: η_* comes from the same closure amplitude that prepares the vacuum, $2q^2$ is the norm of the directed return pair, and the square root follows from reversible dilation.

For an auxiliary Grassmann representation, introduce $\Gamma_e = (\gamma_e^P, \gamma_e^H)^T$ and $J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. With $n_m = c_m^\dagger c_m$ and $n_d = \sum_{ab} d_{ab}^\dagger d_{ab}$, the marked action is

$$\begin{aligned} S_{\text{mark}} &= g_{\text{share,eff}} n_d + \Lambda_d n_d (n_d - 1) \\ &\quad + \frac{1}{2} \sum_{m < m'} \Gamma_{mm'}^T [1 + n_d n_m n_{m'} (\alpha - 1)] J \Gamma_{mm'}, \\ &\quad \Lambda_d \rightarrow +\infty. \end{aligned}$$

The quadratic coefficient is $\alpha = \sqrt{1 - u}$, not the probability $1 - u$. The Grassmann integral returns a Pfaffian amplitude, so inserting $1 - u$ in each 2×2 Majorana block would count the no-event probability twice and produce $(1 - u)^{21}$ instead of the required amplitude $(1 - u)^{21/2}$.

The Majorana variables represent the antisymmetric present/history amplitude; they are not additional propagating particles. When the marked fiber or either incident channel is absent, the reference block is J and has Pfaffian one. On the established $n_m = 1$ seven-channel branch, all $\binom{7}{2} = 21$ blocks become αJ , so

$$Z_{\text{edge}} = \alpha^{21} = \left(1 - \frac{8\eta_*}{49}\right)^{21/2}.$$

These event records do not belong to the physical composite space $\Lambda^2\mathbb{C}^7$. The natural rank-one singlet lift on that composite space has the exact $6 + 15$ split derived in the audit; it does not act on the separately stranded internal present/history response records used here.

This distinction is why all 21 pair records survive even though the scalar source couples through one rank-one singlet. The singlet projector acts on physical channel amplitudes. The 21 Majorana blocks label mutually distinguishable internal present/history incidences, so quotienting them by the physical singlet would identify different incidences and violate faithful resolution. Their edge labels are internal to the marked response and carry no spatial cell address under the free vertex.

Complete decorated vertex. Let $\mathcal{G}_v$ denote the geometric fixed-spin gluing tensor and W_* the native update that prepares $A_*(b)$ and exports the old replaceable closure register. The position and nine-state marked fiber belong to the retained defect system, not to a cell-addressed environment chain. Under free transport W_* must therefore be translation-covariant and must leave the discarded renewal output independent of the defect's spatial branch. This is the no-which-path condition derived in H.11; it restricts the realization of the already-required quantum vertex and adds no foundational postulate. The incidence V_Q above identifies the nine marked basis vectors with the ordered Gaussian response products. The transfer space also contains the unmarked vacuum. Its projector is

$$\Pi_Q = |0\rangle\langle 0| + \sum_{a,b=1}^3 |a,b\rangle\langle a,b|.$$

For $e = (m, m')$, define

$$P_e = n_d n_m n_{m'}, \quad \tilde{U}_e = (I - P_e) + P_e U_e.$$

The complete one-step decorated transfer is

$$\mathcal{A}_v = \mathcal{G}_v W_* \Pi_Q \exp[-g_{\text{share,eff}} n_d - \Lambda_d n_d (n_d - 1)] \prod_{m < m'} \tilde{U}_{mm'}.$$

The empty state therefore has unit weight, while each of the nine one-mark states has fugacity $e^{-g_{\text{share,eff}}}$. The factors $\tilde{U}_e$ act on distinct edge-history strands and commute. The Grassmann action above is the Pfaffian trace representation of these controlled pair factors; it is not an additional operator multiplied into $\mathcal{A}_v$. Faithful full-support resolution on this internal response space, which has no further joint edge constraint, selects the tensor product of its one-edge marginals. Taking its finite internal trace at fixed position branch gives

$$\zeta_* = 9e^{-g_{\text{share,eff}}} \left(1 - \frac{8\eta_*}{49}\right)^{21/2} = 0.005123584484947.$$

Complete edge-Hessian audit. On the 21-dimensional edge-label space, two distinct edges either share one endpoint or are disjoint. Let A_1 and A_0 be their adjacency matrices. The Johnson edge algebra has spectra

$$\begin{aligned} \text{spec}(A_1) &= \{-2^{\times 14}, 3^{\times 6}, 10^{\times 1}\}, \\ \text{spec}(A_0) &= \{-4^{\times 6}, 1^{\times 14}, 10^{\times 1}\}. \end{aligned}$$

A generic permutation-covariant edge kernel can therefore carry three eigenvalues. The conditioned vertex instead gives

$$\mathcal{K}_{\text{edge}}^{(2)} = I_{21} \otimes \alpha J, \quad \text{spec}_{\text{edge}} = \{\alpha^{\times 21}\},$$

with zero off-diagonal edge norm. Coherently averaging the induced edge permutation matrix over all $7! = 5040$ channel relabelings gives $\mathbf{1}\mathbf{1}^T/21$, of rank one. That operation would collapse the result to a shared singlet. It is not the charged gluing used here: $m = -3, \dots, 3$ are physical spin weights, K^2 is not invariant under arbitrary relabeling, and the defect Hamiltonian contains diagonal occupations and density interactions rather than hopping terms $c_m^\dagger c_{m'}$.

Primitive routing enumeration. Write $w = e^{-g_{\text{share,eff}}} \alpha^{21}$, so $\zeta_* = 9w$. The one-step transfer block contains three finite state spaces. The universal sector has one empty state and nine marked states, hence

$$Z_\mu = 1 + 9w = 1 + \zeta_*.$$

The electron feedback sector has one empty state and states $|m; I_L, I_R\rangle$, with seven persistent charged labels and two independently renewed nine-state endpoints. Its dimension is

$$1 + 7 \times 9 \times 9 = 568,$$

and its trace is

$$Z_{e,\text{return}} = 1 + 7 \times 81w^2 = 1 + 7\zeta_*^2.$$

The second-shell sector has one empty state and nine marked states with one scalar passage $q = 2/7$, giving

$$Z_{\tau,2} = 1 + 9qw = 1 + \frac{2}{7}\zeta_*.$$

The direct product contains $10 \times 568 \times 10 = 56,800$ configurations. Exhaustive enumeration reproduces the analytic product exactly. The finite routing factors are therefore

$$Z_\mu = 1 + \zeta_*, \quad Z_e = (1 + \zeta_*)(1 + 7\zeta_*^2), \quad Z_{\tau,2} = 1 + \frac{2}{7}\zeta_*.$$

One native update applies each controlled gate once. Hard-core unit charge restricts the marked return to vacuum plus one return motif; the charged label persists diagonally while the response endpoints renew; and the shell grading terminates after $N = 2$. These restrictions implement structures already present elsewhere in the paper: one native renewal pass, the one-bit fermionic anchor, the physical spin-weight labels, and the three-shell closure ladder. They add no separate foundational premise. Relaxing them gives explicit controls. A shared endpoint polarization changes the electron factor to $1 + 7\zeta_*^2/9$ and moves G to -14.6σ ; off-diagonal charged propagation gives $1 + 49\zeta_*^2$ and moves it to $+98.1\sigma$.

Predictions and scope. The physical scale and charged-lepton ratios are

$$L_* = Z_e \left[-\frac{3}{2} \lambda_e \ln(1 - e^{-7g_{\text{share,eff}}}) \right], \quad G_* = \frac{c^3 L_*^2}{\hbar},$$

$$\frac{m_\mu}{m_e} = 720 \frac{2}{7} Z_\mu, \quad \frac{m_\tau}{m_e} = 720^2 \left(\frac{2}{7} \right)^4 Z_\mu Z_{\tau,2}.$$

They evaluate to $G_* = 6.6742890772 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$, $m_\mu/m_e = 206.768280237$, and $m_\tau/m_e = 3477.343310$. The residuals are -0.073σ , -0.535σ , and $+0.481\sigma$.

The finite marked-transfer action is specified and audited. Its countermodels remain useful falsifiers: a collective edge mode, a physical pair-composite interpretation, persistent endpoint polarization, additional defect-only coupling, or off-diagonal charged propagator changes the answer. The geometric branch is also more specific than a “stable condensate” requirement. Appendix B supplies exact $J = 3$ phase selection and maximal-fusion blocking; Appendix Q supplies a separate gauge-covariant capacity factor with an equivariant frame lock; and EPRL/Regge

constructions supply controlled two-helicity block witnesses. H.9a gives the full ordinary-gravity chain. H.10 proves the massless-TT infrared theorem on the selected metric-Regge branch, constructs an exact fixed- $j = 3$ perfect-action realization, closes the factorized capacity audits, and proves the clock-test theorem. It also shows that the capacity premises do not fix the geometric gluing tensor or measure. That geometric selection problem, durable history storage, the thermodynamic arrow, and the microscopic origin of the carrier-resolved transverse contact remain open.

Reproduction. The supplementary scripts reproduce the ensemble invariants, state-weighted determinant, 48×35 locality fracture, Gaussian closure amplitude, unitary pair dilation, nine-state Gram matrix, 21-block edge Hessian, and 56,800-state routing trace. They also verify the dressed scale and corrected charged-lepton ratios of Appendix I.1. Appendix R contains the self-contained numerical-spine script; the complete Hessian and graph enumerations remain in the accompanying audit files.

H.9a From the finite cell to the Einstein action

Purpose and logical structure. The ordinary-gravity claim is not that the 1680-state capacity code secretly contains the Einstein action. Geometry and capacity are distinct operator factors in the selected realization, and each step below has its own logical grade. The derivation is

$$\begin{array}{c} \text{finite capacity cell} \longrightarrow \text{selected relational quantum geometry} \longrightarrow \text{metric Regge branch} \\ \longrightarrow \text{Einstein gravity in the infrared.} \end{array}$$

The representation theory and maximal-fusion ray are exact once the factorized geometric completion is selected. Metricity, block-scale Lorentz invariance, and membership in the appropriate semiclassical phase are the named geometric premises. The capacity action fixes the channel, record dynamics, source, and dimensional scale; it does not by itself choose the complete ultra-violet geometric propagator.

1. Relational frame geometry. In the selected spatial-frame completion, each tetrahedral cell c carries a local frame R_c , while only relative transport between adjacent cells is observable:

$$g_{cc'} = R_c^{-1} R_{c'} \in SU(2).$$

A change of local frame, $R_c \mapsto h_c R_c$, acts by

$$g_{cc'} \mapsto h_c g_{cc'} h_{c'}^{-1}.$$

Thus the selected relational description has the standard local $SU(2)$ gauge redundancy. If $\mathbf{J}_{cf}$ is the flux through face f in the frame of cell c , gauge invariance imposes the Gauss constraint

$$\mathbf{G}_c = \sum_{f \subset \partial c} \mathbf{J}_{cf} = 0.$$

The geometric state space of a primitive tetrahedron with four spin-three faces is therefore

$$\mathcal{H}_c^{\text{geom}} = \text{Inv}_{SU(2)}(V_3^{\otimes 4}).$$

Since

$$V_3 \otimes V_3 = \bigoplus_{J=0}^6 V_J,$$

each intermediate $J = 0, \dots, 6$ can be paired with the same J in the other pair to form a singlet. Hence

$$\dim \text{Inv}_{SU(2)}(V_3^{\otimes 4}) = 7.$$

This seven-dimensional intertwiner space is geometric. It must not be identified with the independent seven-state capacity carrier. The cell space remains

$$\mathcal{H}_{\text{cell}} = \mathcal{H}_{\text{geom}} \otimes \mathcal{H}_{\text{cap}} \otimes \mathbb{C}_{\text{orient}}^2,$$

with the precise factorization and gauge action given in Appendix Q.4. The equality of two dimensions does no physical work.

Classically one writes the face flux as

$$\mathbf{E}_f = A_f \mathbf{n}_f, \quad \sum_f A_f \mathbf{n}_f = 0.$$

For nondegenerate outward normals, the closure data reconstruct a convex tetrahedron up to translation. The dictionary is therefore

$$j_f \leftrightarrow A_f, qquad l_c \leftrightarrow \text{tetrahedral shape}, qquad g_{cc'} \leftrightarrow \text{relative connection}.$$

This is the first gravity step: the selected geometric factor carries the usual holonomy–flux data of relational three-geometry. It is a selected completion of the finite substrate, not a consequence of the 1680 count alone.

2. Carrier selection and the exact coarse ray. The primitive positive mismatch operator acts in the capacity fusion factor:

$$Q_f = 6P_0 + 5P_1 + 3P_2, \quad \ker Q_f = V_3^{\text{cap}}.$$

Consequently

$$F_3 e^{-sQ_f} F_3^\dagger = I_{V_3^{\text{cap}}} \quad (s > 0).$$

The equivariant lock of Appendix Q.5 then pairs this selected carrier with a geometric copy,

$$L_3 : V_3^{\text{cap}} \longrightarrow V_3^{\text{geom}}, \quad L_3 U_{\text{cap}}(g) = U_{\text{geom}}(g) L_3.$$

Thus Q_f selects the capacity carrier and the lock selects the matching geometric representation. Neither operation supplies curvature:

$$\begin{array}{c} Q_f \text{ and } L_3 \text{ select and relate the face data;} \\ \text{the geometric host supplies their propagation and curvature.} \end{array}$$

For a block of N aligned primitive faces, the highest-spin irrep occurs once in $V_3^{\otimes N}$. There is therefore a unique normalized equivariant fusion map up to phase,

$$F_N : V_3^{\otimes N} \longrightarrow V_{3N}, \quad J_N = 3N.$$

On spin coherent states,

$$F_N |3, \mathbf{n}\rangle^{\otimes N} = |3N, \mathbf{n}\rangle,$$

and coherent transport fuses exactly:

$$\langle 3, \mathbf{n} | D^3(u) | 3, \mathbf{n}' \rangle^N = \langle 3N, \mathbf{n} | D^{3N}(u) | 3N, \mathbf{n}' \rangle.$$

The semiclassical parameter is therefore the block size, not a running microscopic spin:

$$j_{\text{primitive}} = 3 \text{ fixed}, \quad N \rightarrow \infty, \quad J = 3N \rightarrow \infty.$$

The fusion identity is exact on the maximal coherent ray. It is not a proof that every interacting refinement trajectory flows onto that ray; the inter-cell dynamical refinement remains a host-theory test.

2a. The locked ray has an exact collective phase space. The number and phase of a pre-glued spin-three face mode form the collective phase space of the coarse spin. For coherent amplitude

$$\alpha = \sqrt{\bar{N}} e^{-i\theta}$$

and transported endpoint normals $\mathbf{n}_L, \mathbf{n}_R$, the Berry one-form is

$$\vartheta_B = \bar{N} d\theta + 3\bar{N} [\mathcal{A}(\mathbf{n}_L) - \mathcal{A}(\mathbf{n}_R)].$$

With

$$J = 3\bar{N}, \quad \xi = \frac{\theta}{3},$$

this becomes

$$\boxed{\vartheta_B = J d\xi + J [\mathcal{A}(\mathbf{n}_L) - \mathcal{A}(\mathbf{n}_R)],}$$

the twisted-geometry symplectic potential on $T^*SU(2)$. The identification is exact on the locked ray. Because every allowed spin is $J = 3N$, the twist fiber obeys

$$\boxed{(J, \xi) \in 3\mathbb{N} \times \mathbb{R}/(2\pi/3)\mathbb{Z} \simeq (N, \theta) \in \mathbb{N} \times S^1.}$$

The resulting $\mathbb{Z}_3$ quotient is structural, not a coordinate artifact.

The same ray has an exact overcomplete family. With

$$c_N(\alpha) = e^{-|\alpha|^2/2} \frac{\alpha^N}{\sqrt{N!}}, \quad |\alpha, \mathbf{n}\rangle := \sum_{N=0}^{\infty} c_N(\alpha) \sqrt{6N+1} |3N, \mathbf{n}\rangle,$$

Glauber orthogonality and the spin-coherent resolution give

$$\boxed{\int \frac{d^2\alpha}{\pi} \int \frac{d\Omega(\mathbf{n})}{4\pi} |\alpha, \mathbf{n}\rangle \langle\langle \alpha, \mathbf{n}| = \bigoplus_{N=0}^{\infty} I_{V_{3N}}.}$$

Thus the collective number, twist, and normals provide a genuine coherent-state path-integral chart, with relative uncertainties $\Delta J/\bar{J} = O(\bar{J}^{-1/2})$. This construction requires a physical phase: the selected GFT field must be real or its global $U(1)$ phase must be broken. It is stated for a pre-glued link because the twist is relational between its two ends.

2b. Lorentzianization is applied after fusion. The representation fixes the ordering. On the lowest $SU(2)$ block V_j of the principal-series representation $(\rho, k) = (\gamma j, j)$, rotational covariance and $\mathbf{K} \cdot \mathbf{L} = \rho k$ give

$$\boxed{P_j K_i P_j = \frac{\gamma j}{j+1} L_i.}$$

Applying the Lorentzian Y map separately to primitive $j = 3$ facets therefore gives the coefficient $3\gamma/4$. Fusing first to $J = 3N$ and applying Y only at block scale gives instead

$$\frac{\gamma J}{J+1} = \gamma \frac{3N}{3N+1} \rightarrow \gamma.$$

The two orderings differ by 25% at primitive spin. The area normalization below was fixed from the large- J law with $J = 3N$, so its consistent implementation is

$$\boxed{\text{exact } SU(2) \text{ fusion first, block-scale } Y_{\gamma*} \text{ second.}}$$

Facet-first Lorentzianization would require a different Immirzi normalization and would define a different theory.

2c. The orientation doublet is renewed and geometrically inert; its fermion audit remains open. The binary orientation factor cannot be a hidden persistent register. If a uniformly populated lift label $\sigma \in \{+, -\}$ survived from one renewal step to the next, then the renewed boundary data would obey

$$I(B_t; B_{t+1}) \geq H(\sigma) = \ln 2,$$

contradicting the full-support memoryless kernel proved in H.4. The doublet is therefore refreshed rather than transported as an additional geometric degree of freedom.

On the selected pure-gravity branch this causes no loss of geometric information. The two lifts $(\nu, \omega) = (+, +)$ and $(-, -)$ have the same Regge-sector label $\mu = \nu\omega = +1$, determine the same metric, and carry the same e^{+iS_R} asymptotic phase. The proper vertex may consequently be extended as

$$\mathcal{G}_v = A_{v, \mu=+}^{\text{prop}} \otimes I_{\text{orient}},$$

so that gravity depends only on the metric-equivalence class and the refreshed bit is inert.

That conclusion does not automatically extend to fermions, which couple to a tetrad rather than to the metric alone. A matter completion must verify

$$\mathcal{A}_{\text{matter}}[e, \psi] = \mathcal{A}_{\text{matter}}[e', \psi'] \quad \text{under the simultaneous lift flip at fixed } g_{\mu\nu},$$

with the corresponding spinor transformation included. In practice this means auditing the Dirac term and any parity-odd Holst– or Nieh–Yan-type fermion coupling for residual lift dependence. If the equality failed, preserving a lift would conflict with the renewal result above. The tetrad-sign check is therefore an open consistency audit on the matter completion; none of the pure-geometric results below depends on its outcome.

3. One channel fixes one area increment on the matched branch. The physical area on the coherent maximal-fusion ray is linear in the additive flux,

$$A(J) = 8\pi\gamma_* L_*^2 J.$$

A boundary cut removes one capacity partner and contributes

$$S_N/k_B = N \ln 2.$$

Matching this microscopic cut to the Bekenstein–Hawking normalization on the Einstein branch gives

$$N \ln 2 = \frac{A_N}{4L_*^2} = \frac{8\pi\gamma_* L_*^2 (3N)}{4L_*^2},$$

and hence

$$\gamma_* = \frac{\ln 2}{6\pi}, \quad A_N = 4 \ln 2 L_*^2 N, \quad a_{\text{ch}} = 4 \ln 2 L_*^2.$$

This is a normalization match, not an independent derivation of the black-hole area law; Appendix Q.11 gives the finite- N area-operator statement and that caveat in full.

The dimensional scale is not left free here. Appendix D.4 and Sections H.6–H.9 use the electron recurrence and the marked-transfer factor to obtain

$$L_* = -\frac{3}{2} Z_e \lambda_e \ln(1 - e^{-7g_{\text{share, eff}}}), \quad G_* = \frac{c^3 L_*^2}{\hbar}.$$

Thus the area relation fixes the geometric normalization in units of L_* , while the independent charged-transfer calculation fixes the numerical length and therefore the numerical Newton scale. The two statements should not be conflated.

4. Metricity is a selected physical subspace. Gauge-closed tetrahedra alone span a twisted-geometry space: adjacent cells can agree on a face area while assigning different intrinsic shapes to that face. The ordinary one-metric branch additionally imposes shape matching,

$$q_{ab}^{(c)}|_f = q_{ab}^{(c')}|_f.$$

On this support, neighboring tetrahedra glue into a single piecewise-flat metric geometry. Therefore

$$\boxed{\text{Gauss closure} + \text{shape matching} \implies \text{Regge geometry.}}$$

Shape matching is not generated by Q_f . It is part of the selected metric completion and must be preserved by the geometric vertex and by its coupling to the capacity decoration.

4a. Equal face areas force disphenoid cells; the face-wise lock forces regular cells.

Two facts narrow the selected subspace. First, every primitive face carries $j = 3$, so on the coherent ray every face has the same area $a_{\text{ch}} = 4 \ln 2 L_*^2$ (paragraph 3). A tetrahedron whose four faces have equal area is a disphenoid: opposite edges are equal and the four faces are congruent. Primitive cells are therefore disphenoids exactly, with no selection, and the only residual shape freedom is the shape of the congruent face triangle, two parameters per cell. Second, the capacity-closure polynomial of Appendix Q.7, $|\sum_i b_i v_i|^2 = 4 \sum_i b_i^2 - (\sum_i b_i)^2$, is written on the vertex directions v_i of the regular tetrahedron, and Appendix Q.4 implements Postulate I by the equivariant lock on the two relational frames. If that lock is read face-wise, so that capacity port i and geometric face f_i are one face of one substrate and $\mathbf{n}_{f_i} = R_c v_i$ for a single rotation R_c , then the four normals form a regular tetrahedron and the cell is regular. Every face is then an equilateral triangle of area a_{ch} , adjacent cells agree on face shape identically, and

$$\boxed{\text{face-wise lock} \implies \text{shape matching holds kinematically.}}$$

Twisted configurations are excluded rather than selected, and the native spatial geometry is the equilateral-triangulation class with fixed edge

$$\boxed{a^2 = \frac{16 \ln 2}{\sqrt{3}} L_*^2 = 6.403 L_*^2, \quad a = 2.530 L_*.$$

With time-like edges set by the renewal cadence and equal by homogeneity, the spacetime building blocks are the two causal-dynamical-triangulation simplex types with one asymmetry $\tilde{\alpha} = \ell_t^2/a^2$: the native geometric host is the CDT class with Regge weight $(\ln 2/2\pi) \sum_h N_h \Theta_h$ and $G = G_*$. The first fact is a theorem. The second is a reading of Postulate I that the present text does not derive; without it the cells are disphenoids with a two-parameter shape freedom and metricity returns to the selected status of paragraph 4. Appendix J.10 draws the consequences: the CDT bare couplings become predictions rather than tuned inputs, and the phase of the native point becomes a definite question.

Capacity transport cannot replace this reading. Every capacity observable has the form $I_{\text{geom}} \otimes O$ (Appendix Q.4), while shape mismatch is a geometric datum, so no condition built from bandwidth transport, face labels, or face holonomies can enforce shape matching.

5. Local composition fixes the Regge form, not its coefficient. In a piecewise-flat metric geometry, curvature is concentrated on triangular hinges h with area A_h and deficit or boost angle Θ_h . Let the leading local hinge amplitude be a continuous phase $W(A, \Theta) \in U(1)$. Invariance under subdivision of a hinge and composition of successive curvature wedges requires

$$W(A_1 + A_2, \Theta) = W(A_1, \Theta)W(A_2, \Theta),$$

$$W(A, \Theta_1 + \Theta_2) = W(A, \Theta_1)W(A, \Theta_2).$$

The two continuous Cauchy equations imply

$$W(A, \Theta) = e^{i\alpha A\Theta}$$

for a real constant α . Local composition therefore fixes the bilinear Regge form,

$$S_{\text{geom}} = \hbar\alpha \sum_h A_h \Theta_h,$$

but it does not determine α . Matching this branch to the already-fixed Newton normalization sets

$$\alpha = \frac{c^3}{8\pi\hbar G_*} = \frac{1}{8\pi L_*^2}.$$

The resulting action and its channel form are

$$S_R = \frac{c^3}{8\pi G_*} \sum_h A_h \Theta_h,$$

$$\frac{S_R}{\hbar} = \frac{\ln 2}{2\pi} \sum_h N_h \Theta_h = \gamma_* \sum_h J_h \Theta_h.$$

The last equation is the central microscopic normalization identity. Each cut channel supplies $\ln 2$ of boundary entropy, and bending its associated area through Θ supplies the phase $(\ln 2/2\pi)\Theta$ on the matched Einstein branch. Proper-EPRL asymptotics provide a controlled witness of the same Regge phase; they do not turn the composition argument into an independent derivation of its coefficient.

6. Smooth metric limit. For a sequence of Regge geometries approaching a smooth metric,

$$2 \sum_h A_h \Theta_h = \int d^4x \sqrt{-g} R + O\left(a^2 \int d^4x \sqrt{-g} \mathcal{R}^2\right),$$

where a is the coarse lattice spacing and $\mathcal{R}^2$ denotes the appropriate curvature-squared combination. Hence

$$S_R = \frac{c^3}{16\pi G_*} \int d^4x \sqrt{-g} R + S_{\text{higher}}.$$

Allowing the volume term gives

$$S_{\text{grav}} = \frac{c^3}{16\pi G_*} \int d^4x \sqrt{-g} (R - 2\Lambda) + S_{\text{higher}}.$$

The fixed-measure argument of Appendix O determines how Λ enters the ordinary branch; it is logically separate from the local curvature derivation above.

The stronger infrared uniqueness statement is conditional. If the selected phase has one metric, locality at leading derivative order, diffeomorphism covariance, block-scale local Lorentz invariance, and only two massless helicity-2 modes, then the two-derivative nonlinear action is Einstein–Hilbert plus a cosmological term. The cited Han-type and proper-EPRL results witness the Regge/Einstein and two-helicity sectors in their controlled regimes. They are not one theorem for the full decorated action. In particular,

$$\begin{array}{c} \text{local Lorentz invariance at block scale is selected and witnessed,} \\ \text{not derived from the primitive } SU(2) \text{ cell.} \end{array}$$

The ordinary infrared logic and the capacity-side compatibility theorem do not require a proof of every spin-foam refinement. A claim that a particular proper or causal EPRL vertex is the faithful block image does, however, retain the open inter-cell fusion and host-convergence tests of H.10.

The uniqueness statement stops at two derivatives. Gauge-invariant, parity-even quasilocal operators suppressed by powers of kL_{block} leave the infrared coefficient chain unchanged, and the finite 1680 count does not fix their Wilson coefficients. The corresponding TT kernel has the schematic expansion

$$K_{\text{eff}}(k) = Z_G k^2 P_{\text{TT}} + c_4 L_{\text{block}}^2 k^4 + \dots,$$

so the controlled Einstein regime is $kL_{\text{block}} \ll 1$ and the first ordinary-branch ultraviolet deviations are expected at the block scale defined in H.10, not automatically at one primitive length. In four dimensions, Lovelock uniqueness and massless spin-two self-coupling select Einstein–Hilbert only after locality, one-metric covariance, and the two-helicity Lorentzian phase are supplied [105, 106]. Therefore

$$S_{\text{higher}} = \sum_i c_i \mathcal{O}_i, \quad c_i \text{ not fixed by the present postulates.}$$

Pure-gravity curvature-squared terms are field-redefinition or topological at this order, while matter moves the corresponding freedom into contact operators. Predictions at the holonomy-turnover or other Planckian scales are consequently conditional on the stated minimal completion $c_i = 0$ beyond the lock-generated tower; the infrared Einstein universality class is not.

7. The transfer defect becomes a metric source. Let a stable localized defect have transfer energy

$$E_d = m_d c^2.$$

Over a proper-time interval $\Delta\tau$, its Lorentzian transfer phase is

$$\mathcal{A}_d(\Delta\tau) = \exp\left(-\frac{iE_d\Delta\tau}{\hbar}\right).$$

Composition along a worldline therefore gives

$$S_d[z, g] = -m_d c^2 \int d\tau = -m_d c \int ds.$$

At leading derivative order, this is also the unique local reparametrization-invariant scalar action built from one metric and the worldline tangent. Its metric variation produces

$$T_d^{\mu\nu}(x) = \frac{m_d c}{\sqrt{-g}} \int d\tau u^\mu u^\nu \delta^{(4)}(x - z(\tau)).$$

The ordinary action is consequently

$$S_{\text{ord}} = \frac{c^3}{16\pi G_*} \int d^4x \sqrt{-g} (R - 2\Lambda) - \sum_d m_d c^2 \int d\tau_d + \dots,$$

and variation of the same metric gives

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G_*}{c^4} T_{\mu\nu}.$$

Appendix O's fixed covariant event measure supplies a conditional microscopic reading of the proper-time composition. For a defect worldtube of invariant four-volume $\mathcal{V}_4 = A_\perp L$, the premise $\mathcal{V}_4 = v_* N_{\text{evt}}$ gives, at fixed transverse profile,

$$N_{\text{evt}} \propto L = c\tau.$$

Equal transfer phase per event then produces

$$\prod_{n=1}^{N_{\text{evt}}} e^{-i\delta\varphi_d} = \exp\left(-\frac{i}{\hbar} m_d c^2 \tau\right).$$

For a Poisson covariant count,

$$\mathbb{E}[N_{\text{evt}}] = \frac{A_\perp L}{v_*}, \quad \text{Var } N_{\text{evt}} = \mathbb{E}[N_{\text{evt}}], \quad \frac{\delta\tau}{\tau} = O(N_{\text{evt}}^{-1/2}).$$

This explains how microscopic event cadence can count invariant proper time, conditional on the fixed-measure premise. Covariance and one-metric coupling fix the worldline form; the event measure is its proposed microscopic realization, not an additional proof of Lorentz invariance. Variation with respect to the path gives $u^\nu \nabla_\nu u^\mu = 0$, while variation of the same functional with respect to the metric gives $T_d^{\mu\nu}$. One functional therefore fixes both inertial and gravitational mass.

8. Newtonian limit and the weak equivalence principle. For

$$ds^2 = -\left(1 + \frac{2\Phi}{c^2}\right) c^2 dt^2 + \left(1 - \frac{2\Psi}{c^2}\right) d\mathbf{x}^2,$$

a slowly moving defect obeys

$$d\tau = dt \left[1 + \frac{\Phi}{c^2} - \frac{v^2}{2c^2} + O(c^{-4}) \right].$$

Therefore

$$S_d = \int dt \left[-m_d c^2 + \frac{1}{2} m_d v^2 - m_d \Phi + \dots \right].$$

The same transfer coefficient multiplies the inertial and gravitational terms:

$$\boxed{m_{\text{inertial}} = m_{\text{gravitational}} = E_d/c^2.}$$

There is no independent gravitational charge to tune. The linearized 00 equation yields

$$\boxed{\nabla^2 \Phi = 4\pi G_* \rho,}$$

and ordinary matter with negligible anisotropic stress gives

$$\boxed{\Phi = \Psi}$$

at leading order. These are the Newtonian force law and baseline no-slip lensing derived earlier in Appendix D, now displayed as consequences of the common metric action.

The three-dimensional commutant $\text{End}_{SU(2)}(V_0 \oplus V_1 \oplus V_2) \simeq \mathbb{C}^3$ does not introduce three gravitational charges. That space describes the internal marked record fiber; the source tensor structure is fixed instead by the covariant worldline functional. Its $V_0 \oplus V_1 \oplus V_2$ channel content can dress the response, but cannot replace the single coefficient m_d in $T_d^{\mu\nu}$.

9. Renewal cannot add a second ordinary gravitational pole. Let Π_* be the stationary projector on the renewed capacity operator space and define

$$P = I_{\text{geom}} \otimes \Pi_*, \quad Q = I - P.$$

For an arbitrary bounded within-tick geometry–capacity evolution $\mathcal{U}^\dagger$, the renewal step is

$$\mathcal{T}^\dagger = P\mathcal{U}^\dagger.$$

On $PK \oplus QK$ it has the exact block form

$$\boxed{\mathcal{T}^\dagger = \begin{pmatrix} A & B \\ 0 & 0 \end{pmatrix}}, \quad (\mathcal{T}^\dagger)^n = \begin{pmatrix} A^n & A^{n-1}B \\ 0 & 0 \end{pmatrix}.$$

Consequently

$$\boxed{\text{spec}(\mathcal{T}^\dagger) \setminus \{0\} = \text{spec}(A) \setminus \{0\}}.$$

Within-tick capacity fluctuations may dress the retained evolution through B , but a replaceable capacity mode has no autonomous persistence after renewal. This is stronger and cleaner than requiring geometry–capacity mixing to vanish. A conservation-protected collective variable already contained in A is not excluded; that is why the infrared galactic response is graded separately from ordinary vacuum gravity.

10. What is closed. The derivation can now be read in one line:

$$\begin{aligned} &\text{selected relational cells} \Rightarrow SU(2) \text{ flux geometry,} \\ &\quad j_{\text{primitive}} = 3 \Rightarrow J = 3N, \\ &\quad N \ln 2 \Rightarrow A_N = 4 \ln 2 L_*^2 N, \\ &\quad \text{shape matching} \Rightarrow \text{piecewise-flat metric,} \\ &\quad \text{local composition} + G_* \Rightarrow S_R = \frac{c^3}{8\pi G_*} \sum_h A_h \Theta_h, \\ &\text{smooth Lorentzian two-helicity phase} \Rightarrow S_{\text{EH}}, \\ &\quad E_d + \text{proper-time composition} \Rightarrow S_d = -m_d c^2 \int d\tau, \\ &\quad \delta(S_{\text{EH}} + S_d) = 0 \Rightarrow G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G_*}{c^4} T_{\mu\nu}, \\ &\quad \text{renewal triangularity} \Rightarrow \text{no additional replaceable capacity pole.} \end{aligned}$$

Exact results in this chain include the representation decompositions, the selected equivariant lock, maximal-channel fusion, the locked coherent phase space, the channel–area algebra after continuum normalization, the transfer scale, and renewal triangularity. The selected completion supplies the geometric factor, shape-matched one-metric support, and ultraviolet propagator. The one-metric requirement and local composition fix the leading Regge/Einstein universality class; the channel area and G_* fix its normalization. Proper-EPRL, Han-type, and area-Regge results independently witness that the selected coarse data realize this class. H.10 then proves the TT infrared limit and gives an exact fixed- $j = 3$ perfect-action pullback. The unresolved question is which additional geometric principle selects a unique native gluing tensor and measure from the many hosts compatible with the capacity premises.

H.10 Gaussian GFT–geometric-relative-entropy bridge and reduced embedding audits

At any finite regulator, positive dressed and reference Hessians on a common gauge-reduced support determine a unique normalized Gaussian determinant and positive relative-covariance functional. This is an algebraic result. In the factorized completion, the finite capacity sector fixes the equilibrium projector, records, defects, and coefficient chain. The geometric spin-foam/GFT sector supplies the gauge-reduced spin-2 geometry. The selected maximal-fusion ray and the Han-type refinement results of Appendix B and Section 23 provide a controlled semiclassical witness. Proper-EPRL one-sector asymptotics supply a separate compatibility check. The exact operator identification for the relative-information functional, the coupled geometry–capacity audits below, and the all-state nonperturbative Lorentzian continuation remain conditional.

A. Physical Hessians and relative covariance. Let $\mathcal{H}'_{\text{phys}}$ be a finite-dimensional regulated real fluctuation space after gauge fixing and removal of the same exact zero modes in the equilibrium and dressed backgrounds. For a complex GFT field this means its real Nambu form, with the conjugate components included in the physical trace. On that common support, take

$$S_0^{(2)}[\varphi] = \frac{1}{2}\langle\varphi, \mathbb{K}_0\varphi\rangle, \quad S^{(2)}[\varphi] = \frac{1}{2}\langle\varphi, \mathbb{K}\varphi\rangle, \quad \mathbb{K}_0 > 0, \quad \mathbb{K} > 0.$$

Whitening by the equilibrium Hessian defines

$$\Theta = \mathbb{K}_0^{-1/2} \mathbb{K} \mathbb{K}_0^{-1/2} > 0, \quad \mathcal{G} = \Theta^{-1} = \mathbb{K}_0^{1/2} \mathbb{K}^{-1} \mathbb{K}_0^{1/2} > 0.$$

Equivalently, with $C_0 = \mathbb{K}_0^{-1}$ and $C = \mathbb{K}^{-1}$,

$$\mathcal{G} = C_0^{-1/2} C C_0^{-1/2}.$$

Every prime below means this identical gauge and zero-mode prescription. Equality of the two physical supports is a hypothesis to be tested in the microscopic spectrum; it is not supplied by the determinant identity.

B. Finite-regulator determinant theorem. The equilibrium-normalized Gaussian functional is

$$\widehat{Z}_G = \frac{\int_{\mathcal{H}'_{\text{phys}}} D\varphi e^{-\frac{1}{2}\langle\varphi, \mathbb{K}\varphi\rangle}}{\int_{\mathcal{H}'_{\text{phys}}} D\varphi e^{-\frac{1}{2}\langle\varphi, \mathbb{K}_0\varphi\rangle}} = (\det' \Theta)^{-1/2} = (\det' \mathcal{G})^{1/2}.$$

Consequently,

$$2 \ln \widehat{Z}_G = -\text{Tr}' \ln \Theta = \text{Tr}' \ln \mathcal{G}.$$

The proof is just the finite Gaussian integral

$$Z[\mathbb{K}] = (2\pi)^{N_{\text{phys}}/2} (\det' \mathbb{K})^{-1/2}.$$

Taking the ratio cancels the measure normalization, while multiplicativity of the determinant under the congruence defining Θ gives the displayed result. The proof requires neither $[\mathbb{K}_0, \mathbb{K}] = 0$ nor simultaneous diagonalization. Positivity is needed to define the real Gaussian measure and the principal operator logarithm. The real/Nambu convention fixes the exponent $-1/2$; a complex-coordinate calculation gives the same result only after its conjugate variables and Jacobian are counted consistently in N_{phys} and Tr' .

C. Exact quadratic-source Legendre transform and Gaussian KL divergence. Write the whitened dressed Hessian as

$$\Theta = I - U > 0, \quad U = U^\dagger,$$

and define the normalized connected functional

$$W[U] = \ln \hat{Z}_G[U] = -\frac{1}{2} \text{Tr}' \ln(I - U).$$

Its matrix derivative is

$$2 \frac{\delta W}{\delta U} = (I - U)^{-1} = \mathcal{G}.$$

The convex conjugate with the corresponding quadratic-source normalization is

$$\Gamma_{\text{rel}}[\mathcal{G}] = \sup_{\substack{U=U^\dagger \\ I-U>0}} \left\{ \frac{1}{2} \text{Tr}'(\mathcal{G}U) - W[U] \right\}.$$

Stationarity gives $U = I - \mathcal{G}^{-1}$. Substitution therefore yields the exact identity

$$\boxed{\Gamma_{\text{rel}}[\mathcal{G}] = \frac{1}{2} \text{Tr}' [\mathcal{G} - I - \ln \mathcal{G}].}$$

If $g_i > 0$ are the eigenvalues of $\mathcal{G}$, then $g_i - 1 - \ln g_i \geq 0$, with equality only at $g_i = 1$. Hence

$$\Gamma_{\text{rel}} \geq 0, \quad \Gamma_{\text{rel}} = 0 \iff \mathcal{G} = I \iff C = C_0$$

on the common support. In probabilistic language this is precisely

$$\Gamma_{\text{rel}} = D_{\text{KL}}(\mathcal{N}(0, C) \parallel \mathcal{N}(0, C_0)).$$

Thus the positive mismatch functional in Sections 25 and 29.9 is not an inserted potential. It is the convex conjugate of the normalized Gaussian trace logarithm.

D. Conditional relation to geometric relative entropy. Bianconi's geometric-relative-entropy construction uses a positive metric-response operator $G_B = \Theta_B^{-1}$ and the same operator-convex mismatch form [25]. If a specified decorated GFT gluing tensor derives the operator identification

$$\Theta_B = \Theta, \quad G_B = \mathcal{G},$$

then the Bianconi trace-log and Legendre functionals are the Gaussian effective functionals of the dressed GFT covariance relative to equilibrium. Under that still-conditional identification, G_B can be read in this theory as a composite covariance or susceptibility ratio, slaved to the condensate, metric, matter, and relational boundary data. The functional identity proved above is exact; the microscopic gluing and operator map are not. Bianconi's independent derivative dynamics are not imported into the metric-only ordinary branch, and this paper does not attribute the composite reading to her construction itself.

E. Euclidean sign and continuation. The Euclidean one-loop contribution obtained by integrating out the Gaussian fluctuation is

$$\boxed{\Delta \Gamma_{E, \text{eff}} = -\ln \hat{Z}_G = \frac{1}{2} \text{Tr}' \ln \Theta = -\frac{1}{2} \text{Tr}' \ln \mathcal{G}.}$$

This sign is not the sign of the positive Legendre/KL mismatch. The logarithm of a multiplicity or partition ratio and the Euclidean effective free-energy contribution carry opposite signs, while the convex conjugate measures the nonnegative cost of changing the covariance. A common Lorentzian parent must fix the analytic continuation and the sign of its retarded influence kernel. The determinant theorem and positive Legendre identity do not depend on that unresolved continuation.

F. Conditional local derivative expansion. Suppose the regulated operator is quasilocal in a geometric cell basis,

$$\Theta_{vv'} = \delta_{vv'} \Theta_v + \mathcal{R}_{vv'},$$

with $\mathcal{R}$ derivative-suppressed on fields varying slowly compared with L_* . Only under that assumption may the exact trace be organized as

$$2 \ln \hat{Z}_G = \nu_* \int d^4x \sqrt{|g|} \left[-\text{tr}_{\text{phys}} \ln \Theta(x) \right] + \mathcal{O}(L_* \nabla \Theta, L_*^2 \nabla^2 \Theta).$$

Here tr_{phys} is the trace over the local gauge-reduced geometric fiber and ν_* is its continuum block density. Neither is fixed by the 1680-state capacity probability distribution. In the factorized completion they follow from the geometric gluing and blocking calculation, while the equilibrium capacity factor contributes its rank-one stationary transfer projector in probability space, and matter/record insertions couple through the equivariant lock. The finite determinant is exact before locality is assumed; applying it as a local continuum density remains conditional on the quasilocal derivative expansion.

G. Low-curvature and static Einstein corollary. If the covariant operator supplied by the embedding has the small-response form

$$\Theta = I - \tilde{U}, \quad \tilde{U} = \beta \tilde{R} - \alpha \tilde{M} + \mathcal{O}(R^2, RM, M^2, \nabla^2),$$

where $\tilde{R}$ and $\tilde{M}$ are the dimensionless curvature and matter insertions on the physical fiber, then

$$-\text{Tr}' \ln(I - \tilde{U}) = \text{Tr}' \tilde{U} + \frac{1}{2} \text{Tr}' \tilde{U}^2 + \dots$$

For the controlled geometric branch the EPRL/Regge refinement results of Appendix B supply the Einstein–Hilbert spin-2 support independently of this trace-log expansion. The extra question here is whether the relative-covariance operator itself inherits the same covariant fiber trace, Ward identity, and normalization when capacity defects dress that geometry. The already-controlled longitudinal reduction fixes its static target:

$$I_{\text{cap}}^{\text{static}} = \int dt d^3x \left[-\frac{\gamma}{2} (\nabla \delta S)^2 + \kappa \rho \delta S \right], \quad \delta S = -\frac{2S_\infty}{c^2} \Phi,$$

$$G = \frac{c^2 \kappa}{8\pi \gamma S_\infty}, \quad \boxed{I_{\text{cap}}^{\text{static}} = Z_S I_{\text{Newton}}.}$$

The ordinary longitudinal capacity response is not a propagating scalar pole. Under the capacity–metric dictionary it is the Hamiltonian-constraint combination of the Einstein metric, so its static Green function may be unscreened without adding independent Cauchy data. A microscopic condensate realization therefore passes the ordinary-gravity audit when every scalar fluctuation beyond that constraint response is gapped, relaxational, pure gauge/constraint, or exactly source-orthogonal in the physical retarded kernel. The radiative gauge-reduced support must remain transverse-traceless. This spectrum statement follows from the geometric completion, not from the trace-log identity by itself.

H. Marked-record support-preservation lemma. The pair contraction and the seven-channel recurrence are different quantities. From H.9,

$$u = \frac{8\eta_*}{49} = 0.00487621949\dots, \quad \alpha = \sqrt{1-u} = 0.9975589108\dots,$$

$$\alpha^{21} = 0.9499693634\dots, \quad r = e^{-7g_{\text{share,eff}}} = 2.77557 \times 10^{-23}.$$

The positivity of the marked edge factor follows from $0 < u < 1$, not from the exponential smallness of r . On the established seven-channel one-mark branch, integrating out the 21 internal present/history Majorana blocks multiplies the remaining bosonic geometric transfer by the strictly positive scalar α^{21} . Taking the branch-independent internal trace of the nine-state hard-core marked fiber gives

$$\zeta_* = 9e^{-g_{\text{share,eff}}} \alpha^{21} = 0.00512358448 \dots > 0.$$

Therefore the integrated marked-record sector, by itself, neither removes nor creates a bosonic geometric mode. This is a decoration-side support lemma. On the factorized vacuum branch the geometric TT support is fixed by the controlled spin-foam continuum; for non-equilibrium defect backgrounds one must still verify that dressing does not introduce new gauge-reduced zero modes, alter the constraints, or create an additional sourced pole.

I. Exact internal-fiber identity and limited ADM reading. The primitive fusion identity of H.9 is

$$V_{3/2} \otimes V_{3/2} = V_3 \oplus V_0 \oplus V_1 \oplus V_2.$$

Removing the selected maximal channel gives

$$Q = (V_{3/2} \otimes V_{3/2}) \ominus V_3 = V_0 \oplus V_1 \oplus V_2, \quad \dim Q = 1 + 3 + 5 = 9.$$

Independently, the marked response fiber obeys

$$\mathcal{H}_{\text{mark}} = V_1^P \otimes V_1^H = V_0 \oplus V_1 \oplus V_2.$$

The intertwiner already constructed in H.9 therefore proves equivalence of the two internal fibers, with one copy of each irrep. Under spatial rotations, the ADM shift and symmetric spatial metric decompose as

$$h_{0i} \in V_1, \quad h_{ij} \in V_0 \oplus V_2.$$

The nine marked states have the spatial-rotation content of the three shift components and six spatial-metric components. The lapse h_{00} remains a separate scalar constraint multiplier. This $SU(2)$ identity does not make the marked fiber the metric. It supplies the blockwise representation target for the equivariant geometry–capacity lock. Inter-cell covariance, the lapse constraint, the diffeomorphism Ward identity, and TT propagation remain in the geometric factor; the marked factor can only dress those responses.

J. Equilibrium capacity normalization and physical support. The stationary distribution has two related but distinct representations. In the detailed-balance/symmetrized probability space,

$$|v\rangle = \sum_{b \in \mathcal{B}} \sqrt{p_{\eta_*}(b)} |b\rangle, \quad P_{\text{stat}} = |v\rangle\langle v|$$

is the rank-one stationary projector. The actual renewed quantum register is instead the diagonal density operator already fixed in H.8,

$$\rho_* = \sum_b p_{\eta_*}(b) |b\rangle\langle b|, \quad \text{Tr } \rho_* = 1,$$

and the replacement channel is $\mathcal{E}_*(X) = \rho_* \text{Tr } X$. Here P_{stat} is rank one in transfer/probability space, whereas ρ_* has the full support required by faithful renewal.

After the complete homogeneous capacity free energy is centered as in Section 25, the normalized homogeneous equilibrium reference contributes unit capacity weight. Thus for that reference state,

$$\mathcal{A}_{\text{full}}^{\text{ref}} = \mathcal{A}_{\text{geom}}^{\text{ref}}$$

up to the already-divided gauge/measure normalization. For a generic empty geometry T , however,

$$\mathcal{A}_{\text{full}}[T] = \mathcal{A}_{\text{geom}}[T] \hat{Z}_{\text{cap}}[T],$$

where centering removes the source-independent homogeneous extensive term. Geometry-dependent finite contributions need not equal unity. The capacity Hilbert space is also not one-dimensional. Instead, the renewed local closure register has one stationary superoperator direction: $\mathcal{E}_*$ annihilates every traceless perturbation of that replaceable register. The renewed register therefore cannot furnish an additional coherent pole. Any gapless capacity pole must lie in a retained or conservation-protected collective sector, such as the transport carrier tested in Section 24. Persistent marked fibers must satisfy the source-overlap conditions above.

Schematically, on the controlled linearized witness branch, the combined continuum results imply a physical Hessian of the form

$$\mathbb{K}_{\text{phys}}(k) \sim Z_G k^2 P_{\text{TT}} + O(a^2 k^4) + O(\Lambda_{\text{sf}}^{-1} k^2),$$

where the separate terms summarize the Einstein spin-2 limit and the known derivative/large-spin suppressions rather than quote one theorem for a single decorated proper-EPRL model. The first explicit length-metric correction in the area-Regge continuum is $O(a^2 C^2)$ [101]. The independent area-metric RG analysis of Ref. [107] supports the decoupling of nonmetric shape-mismatch modes in the infrared, but it is not used here as a theorem about the full EPRL flow. Its failure to make parity generic is instead handled by the exact microscopic orientation exchange imposed in Appendix Q.

K. The selected geometric hosts and why their specification matters. The capacity construction fixes the $j = 3$ carrier, its maximal-fusion ray, and the infrared normalization; it does not fix a unique ultraviolet geometric propagator. Two distinct selected completions are therefore kept separate.

Covariant fixed-facet host. Every microscopic geometric face is a spin-three facet with its own relational incidence identity and its own dual loop. A coarse face is a partition into N such facets. Refinement creates new interfaces; it never replaces one boundary cycle by many indistinguishable copies carrying a common holonomy variable. Schematically,

$$f_{\text{coarse}} = \bigsqcup_{a=1}^N f_a, \quad j(f_a) = 3, \quad g_{f_a} \text{ retained separately.}$$

This distinction answers the known stack-localization objection. Repeating the same face on one cycle produces

$$\omega_h \sim [\lambda \tau_k(g_h)]^{p_h}, \quad \text{Re } S_h = p_h \ln \frac{|\tau_k(g_h)|}{d_k^2} \leq 0,$$

with equality only at flat holonomy. It therefore localizes onto flat $SU(2)$ connections as $p_h \rightarrow \infty$, even at fixed k [91]. A coarse-label host does not evade the problem merely by restricting J to multiples of three:

$$\sum_{J \in 3\mathbb{N}} d_J \chi_J(\phi) = \frac{1}{3} \sum_{r=0}^2 \sum_J d_J \chi_J(\phi) e^{2\pi i r J/3},$$

which is supported on flat or $\mathbb{Z}_3$ -twisted-flat class angles. By contrast, a relational facet partition has weight $\prod_a w(g_{f_a})$ on distinct loops and contains no power of a single $\tau_k(g)$, so that specific mechanism does not directly apply. This is not a proof that every RG flow avoids a topological phase; the non-topological clause in the convergence condition below remains essential.

Han's many-small-spin construction supplies only the single-vertex half of the desired blocking statement: a generalized vertex with parallel fixed-spin facets has the same large total spin $J = \sum_a j_a = 3N$ and the corresponding Euclidean Regge critical data [90]. It does not show that many primitive four-simplices coarse-grain into one block vertex. The grades are therefore

$F_N 3, \mathbf{n}\rangle^{\otimes N} = 3N, \mathbf{n}\rangle$	exact kinematics,
parallel facets within one vertex	controlled Euclidean witness,
inter-cell vertex fusion	open dynamics.

The parallel-normal restriction is exactly the maximal channel $V_{3N} \subset V_3^{\otimes N}$. In the amplitude realization

$$\mathcal{A}_{\text{fine}} = \mathcal{A}_N F_N, \quad \|Q_N \psi\|_{\text{phys}} = 0,$$

so the nonmaximal geometric complement is null rather than a second set of propagating face modes. The marked $V_0 \oplus V_1 \oplus V_2$ records remain entirely in $\mathcal{H}_{\text{cap}}$, consistent with the factorization of Appendix Q. Proper and causal EPRL amplitudes are then block-scale witnesses on V_{3N} , not primitive definitions; their face dimension $6N + 1$ is the fused image of the primitive $d_3 = 7$ carrier. Their grade remains conditional on the open inter-cell line above.

Canonical compression freezes the area spectrum. Let $q(\Psi) = \sum_I \|C_I \Psi\|^2$ be a closable positive master form and let $\mathcal{H}_*$ be the closure of spin networks whose edge spins lie in $3\mathbb{N}$. Restricting and closing the form defines a positive self-adjoint compression $\mathbf{M}_*$ with

$$\langle \Psi, \mathbf{M}_* \Psi \rangle = \sum_I \|C_I \Psi\|^2, \quad \ker \mathbf{M}_* = \ker \mathbf{M} \cap \mathcal{H}_*.$$

This is compression in form sense, not the sum of separately projected constraints:

$$\langle \Psi, PMP\Psi \rangle - \sum_I \|PC_I P\Psi\|^2 = \sum_I \|(I - P)C_I \Psi\|^2 \geq 0.$$

For the standard fundamental-holonomy regularization, $C_I^\dagger C_I$ changes any edge spin by at most two units. Projection back to $3\mathbb{N}$ therefore retains only the zero shift. Hence

$$[\mathbf{M}_*, \widehat{A}_e] = 0 \quad \text{for every edge } e,$$

and its symbol on the locked coherent ray is independent of the conjugate twist ξ . Areas are frozen: the compressed standard master constraint is excluded as a geometric host, rather than listed as an admissible candidate.

The lock itself fixes the repair. Replace the fundamental trace by the normalized spin-three trace

$$\text{Tr}_{1/2}(h[h^{-1}, V]) \mapsto \frac{c_3}{3 \cdot 4 \cdot 7} \text{Tr}_3(h[h^{-1}, V]),$$

with c_3 fixed by the same classical limit. Locked transitions $0, \pm 3$ then survive, and $\mathbf{M}_*^{(3)}$ carries $\Delta J = 0, \pm 3, \pm 6$ with the nontrivial symbol

$$\mathbf{M}_*^{(3)} \supset \frac{2}{9}(1 - \cos 3\xi) = \xi^2 - \frac{3}{4}\xi^4 + O(\xi^6).$$

Thus

$$\langle \mathbf{M}_*^{(3)} \rangle = \mathbf{M}[J, \xi, \mathbf{n}] + O(\xi^4) + O(\bar{N}^{-1}),$$

and the canonical alternative is specifically the spin-three-regularized master constraint, not the compression of the standard one. Its fixed holonomy scale and $2\pi/3$ periodicity are the structural $\mathbb{Z}_3$ of the locked phase space. Apparent higher-representation zeros at $\xi + 2\pi/3$ are the same locked state, not additional kernels. This canonical completion and the covariant fixed-facet completion are both admissible, but their equivalence is not assumed.

L. Exact gapped embedding and perfect-action pullback. A one-scale nearest-neighbor alignment does not justify discarding nonmaximal block channels. On a connected primitive graph G ,

$$H_G = \kappa \sum_{(ab) \in E(G)} (9 - \mathbf{J}_a \cdot \mathbf{J}_b)$$

has the maximal-spin ray as its kernel, but on the one-magnon sector

$$H_G = 3\kappa L_G, \quad \Delta_G = 3\kappa \lambda_2(G) \longrightarrow 0$$

for growing local blocks. Nearest-neighbor alignment therefore leaves a gapless spin-wave complement.

The tree fusion map supplies a uniformly gapped parent instead. At each node v joining spins j_L, j_R , define

$$Q_v = j_L j_R - \mathbf{J}_L \cdot \mathbf{J}_R, \quad q_J = \frac{1}{2} [(j_L + j_R)(j_L + j_R + 1) - J(J + 1)].$$

Nested-block Casimirs commute, so

$$H_T = \kappa \sum_{v \in T} Q_v, \quad \ker H_T = V_{3N}, \quad \text{gap } H_T \geq 6\kappa.$$

The kernel is independent of the fusion tree by multiplicity one; only the decoupled excited spectrum remembers the tree.

Let $F_N : V_3^{\otimes N} \rightarrow V_{3N}$ be the normalized coisometry, $F_N F_N^\dagger = I$, and $Q_N = I - F_N^\dagger F_N$. For any positive self-adjoint coarse operator K_N^{coarse} , define

$$K_N^{\text{prim}} = F_N^\dagger K_N^{\text{coarse}} F_N + \Delta_N Q_N, \quad \Delta_N \geq 6\kappa.$$

On the physical ray,

$$F_N K_N^{\text{prim}} F_N^\dagger = K_N^{\text{coarse}}, \quad F_N (K_N^{\text{prim}} - z)^{-1} F_N^\dagger = (K_N^{\text{coarse}} - z)^{-1}.$$

Moreover,

$$\text{spec } K_N^{\text{prim}} = \text{spec } K_N^{\text{coarse}} \cup \{\Delta_N\}_{Q_N}.$$

The flat complement $\Delta_N Q_N$ may be replaced by the hierarchical H_T above; its excited eigenvalues then spread while its kernel and uniform lower gap are unchanged. Poles, residues, the $i0$ prescription, and the zero fiber therefore pull back exactly. Associativity of maximal fusion,

$$F_{N+M} = F_{(N,M)} \circ (F_N \otimes F_M) \quad \text{up to phase,}$$

makes $I_N := F_N^\dagger$ a projective family. For $\mathcal{A}_{\text{fine}} := \mathcal{A}_N F_N$ one has $\mathcal{A}_{\text{fine}} \circ I_N = \mathcal{A}_N$, which is cylindrical consistency at the amplitude level. This construction proves that a positive primitive operator can carry a chosen healthy coarse Einstein sector. It does not show that the strictly local primitive weights dynamically generate that perfect operator. In four dimensions the corresponding perfect action is generically nonlocal [104]; here the nonlocality is confined to the first reliable semiclassical block,

$$L_{\text{block}} := \sqrt{A_0} = \sqrt{a_{\text{ch}} N_0} = \sqrt{4 \ln 2 N_0} L_*,$$

where N_0 is the smallest facet number for which $J_0 = 3N_0$ supports the required proper/causal Lorentzian asymptotics at the stated numerical tolerance. This definition follows from the

physical coarse area $A_0 = a_{\text{ch}} N_0$; $\sqrt{3N_0} L_*$ would instead confuse the spin label with an area in units of L_*^2 .

The present calculation does not derive N_0 . For numerical planning it uses the provisional working bracket $J_0 \sim 10\text{--}30$, hence $N_0 \sim 4\text{--}10$ and

$$L_{\text{block}} \sim 3.3\text{--}5.3 L_*.$$

Existing analytic and single-vertex numerical asymptotic studies motivate this test range [100, 110], but do not establish it for the decorated vertex. A direct calculation must replace the bracket with a measured tolerance bound. Provisionally, $kL_* \ll 0.2\text{--}0.3$ is sufficient for $kL_{\text{block}} \ll 1$, so ordinary-branch ultraviolet corrections should appear at a few L_* rather than exactly at L_* . This is an estimated locality range, not a closed prediction.

Selected metric-Regge host theorem. The perfect-action construction above permits a stronger statement than an existence witness. Choose the covariant host to be a regular, shape-matched metric-Regge block sequence on the ordinary branch. Four-dimensional linearized Regge calculus has propagating gauge-invariant lattice curvature degrees of freedom [102], and its weak-field limit recovers the standard continuum theory [103]. On the regular centrally subdivided lattice, the modern area-Regge calculation gives the gauge-reduced length-metric symbol [101]

$$\mathbb{K}_n^{\text{TT}}(k) = Z_G k^2 P_{\text{TT}} + O(a_n^2 k^4), \quad Z_G > 0,$$

where the metric/shape-matched branch is the length-Regge subspace and does not retain the additional area-metric modes. Here $a_n = L_{\text{block}}/L_{\text{obs},n} \rightarrow 0$ is an infrared scaling parameter; the physical microscopic cutoff L_* is not sent to zero.

This symbol also completes the operator statement. For $z \notin \mathbb{R}$, identify the regular-lattice TT spaces with their Fourier interpolation in the continuum TT Hilbert space and write

$$R_n(k; z) = [\mathbb{K}_n^{\text{TT}}(k) - z]^{-1}.$$

For every fixed physical momentum,

$$R_n(k; z) \longrightarrow [Z_G k^2 P_{\text{TT}} - z]^{-1}, \quad \|R_n(z)\| \leq \frac{1}{|\text{Im } z|}.$$

Dominated convergence first gives resolvent convergence on TT Schwartz wave packets of compact Fourier support, and the uniform bound extends it by density:

$$\mathbb{K}_n^{\text{TT}} \xrightarrow{\text{s.r.}} Z_G(-\Delta) P_{\text{TT}}.$$

Equivalently, positive functional calculus gives for the deparametrized Hamiltonians

$$H_n = c \sqrt{\mathbb{K}_n^{\text{TT}}/Z_G} \xrightarrow{\text{s.r.}} c|\mathbf{k}| P_{\text{TT}}.$$

Thus $\inf \sigma(H_{\text{TT}}) = 0$, genuine TT states occur arbitrarily close to zero energy, and the retarded kernel has two massless helicity-2 poles with positive residue. This is the relevant masslessness criterion; a nonzero pointwise density at the spectral threshold is not required.

The exact primitive pullback now carries the result to fixed microscopic spin:

$$\mathbb{K}_n^{\text{prim}} = F_n^\dagger \mathbb{K}_n^{\text{TT}} F_n + H_{T,n}, \quad H_{T,n} F_n^\dagger = 0, \quad H_{T,n}|_{Q_n} \geq 6\kappa,$$

$$F_n(\mathbb{K}_n^{\text{prim}} - z)^{-1} F_n^\dagger = (\mathbb{K}_n^{\text{TT}} - z)^{-1}.$$

The physical maximal-fusion ray therefore inherits the Einstein resolvent exactly, while every nonmaximal fusion mode remains uniformly gapped. Denote this closed branch theorem by

$$(H_{\text{TT}}) \quad \text{metric-Regge branch} \implies \text{massless two-helicity Einstein TT limit.}$$

Together with the perfect-action pullback, it also proves that a compatible fixed- $j = 3$ primitive realization exists.

This is not yet a phase theorem for the native theory. The Hessian result is restricted to the selected shape-matched branch, and the perfect operator is constructed from the healthy coarse operator. Neither controls which configurations dominate a nonperturbative sum. Once a definite local geometric tensor and measure have been supplied, the remaining covariant host condition would be

$$(H_{\text{phase}}) \quad \mu_{\text{native}} \xrightarrow{\text{IR}} \mu_{\text{metric-Regge/Einstein}},$$

meaning that the resulting measure must concentrate at long distance on shape-matched, non-topological metric configurations with a gapless positive-residue TT sector, rather than on a BF-like, degenerate, bifurcated, or otherwise non-Einstein phase. The underdetermination theorem below shows that the present premises do not yet define μ_{native} , so this condition is not presently a well-posed derived calculation. An unrestricted Lorentzian EPRL sum and uniqueness of the selected ultraviolet propagator are not claimed.

Capacity-side closure on the displayed factorized branch. The previously listed capacity audits are consequences of the finite decoration and the host identities above, not additional spectral assumptions. After the internal marked fiber is traced,

$$G_{\text{dec}} = \zeta_* G_{\text{geom+particle}}, \quad \zeta_* = 9e^{-g_{\text{share,eff}}} \alpha^{21} > 0.$$

Therefore the decoration changes no denominator: its poles are those of geometry plus the worldline particle, and the signs of all residues are preserved. It introduces neither a ghost nor a tachyon. Renewal triangularity separately removes autonomous replaceable-register poles, while the tensor lift below preserves the positive physical inner product and causal boundary value.

There are two equivalent bookkeeping pictures. In the dilated picture the complete update is unitary, with

$$V^\dagger V = P + \frac{1}{9} \sum_{a=1}^9 W_a^\dagger W_a Q = I,$$

so Hermiticity and positivity are explicit. In the traced Euclidean picture, the corresponding requirement is an Osterwalder–Schrader audit of the complete transfer kernel, including its marked subblocks. Vacuum renewal is counted per invariant event and the defect cadence is proper time. The capacity sector contains no vector n^μ from which to construct a leading preferred-frame curvature operator. Refresh-first ordering also gives exact trigger independence:

$$(\text{id}_R \otimes \mathcal{E}_*)(\rho_{R,\text{ren}}) = \rho_R \otimes \rho_*, \quad \langle K^2 \rangle_{\text{cond}} = \langle K^2 \rangle_{\eta_*}.$$

Thus the factorized ordinary branch closes the capacity pole, residue, positivity, causality, preferred-frame, and refresh-order audits. A different nonfactorized ultraviolet realization must reproduce these identities.

Local Einstein-phase stability under finite capacity dressing. The exact factorized theorem above is stronger than what is needed for a general gauge-compatible decoration. Suppose an undecorated host already has the gauge-reduced infrared kernel

$$K_g(k) = Z_G k^2 P_{\text{TT}} + O(k^4 L_*^2), \quad Z_G > 0,$$

with no additional massless gravitational pole. If the capacity sector has no conservation-protected vacuum pole, its static geometric correction is local or quasilocal, and its TT correction is bounded by the host stiffness, then the Ward identity forbids a TT mass term:

$$\delta K_{\text{TT}}(0) = 0, \quad \delta K_{\text{TT}}(k) = \delta Z_G k^2 P_{\text{TT}} + O(k^4 L_*^2).$$

Consequently, for $|\delta Z_G| < Z_G$,

$$K_{\text{full}}(k) = Z_G^{\text{eff}} k^2 P_{\text{TT}} + O(k^4 L_*^2), \quad Z_G^{\text{eff}} = Z_G + \delta Z_G > 0,$$

and no new gravitational zero mode appears. Thus a finite, pole-free, gauge-compatible, perturbatively bounded decoration is locally stable around an already established Einstein phase. This is not a claim that a positive $Z_{\text{cap}}[T]$ leaves every saddle unchanged, that it cannot renormalize a curvature coefficient, or that it is phase-neutral globally. The microscopic normalization G_* fixes the target renormalized coefficient; a coupled realization must still pass the normalization audit below. On the displayed factorized vacuum branch the stronger identity $G_{\text{dec}} = \zeta_* G_{\text{geom}+\text{particle}}$ establishes these hypotheses exactly. For a nonfactorized embedding they are explicit acceptance conditions.

Geometric-host underdetermination theorem. The preceding stability result removes the capacity factor from the host question; it does not make the host follow from the capacity premises. Write the complete factorized vertex as

$$\boxed{\mathcal{A}_v[\mathcal{G}] = \mathcal{G}_v \mathcal{D}_v}, \quad \mathcal{D}_v = W_* \Pi_Q e^{-g_{\text{share}, \text{eff}} n_d - \Lambda_d n_d (n_d - 1)} \prod_{m < m'} \tilde{U}_{mm'}.$$

The finite factor $\mathcal{D}_v$ fixes the V_3 carrier, stationary measure, renewal projector, marked transfer, and dimensional coefficient chain. The geometric gluing tensor $\mathcal{G}_v$ supplies propagation and curvature. If $\mathcal{G}_v$ is replaced by another gauge-covariant tensor $\mathcal{G}'_v$ with the same V_3 boundary representation and equivariant-lock interface, every closed capacity invariant is unchanged. This follows because the gauge action factorizes and all capacity operators commute with its geometric factor. More decisively, the exact vacuum lift gives

$$\boxed{\mathcal{H}_{\text{phys}}^{\text{full, vac}} \cong \mathcal{H}_{\text{phys}}^g},$$

so on the displayed branch the full vacuum phase is the phase of the chosen geometric tensor.

The manuscript already contains a concrete separation. The sharp primitive object

$$\mathcal{G}_{v, \#}^{(0)} = \mathcal{P}_v^{\text{inv}} \left[\bigotimes_{(ab) \in E(v)} P_3^{(ab)} \right] \mathcal{P}_v^{\text{inv}}$$

enforces the representation and intertwiner support but contains no curvature phase. The selected Einstein completion instead carries

$$\mathcal{G}_v^E \sim \exp \left[\frac{i}{\hbar} \frac{c^3}{8\pi G_*} \sum_h A_h \Theta_h \right].$$

Both admit the same finite capacity decoration. Requiring the geometric factor to admit an EPRL/Regge branch is therefore a host acceptance condition, not a consequence of the capacity construction.

Even the existence of the Regge saddle does not determine phase dominance. At finite regulator, a local positive gauge-invariant factor

$$\exp \left[\lambda \sum_h f(L_*^2 \mathcal{R}_h) \right], \quad f(x) = 0 \text{ near } x = 0,$$

can leave the smooth saddle, its perturbative Hessian, G_* , and the two-helicity branch unchanged while exponentially favoring rough configurations at large volume. Hence

$$\boxed{\text{existence of an Einstein saddle} \not\Rightarrow \text{Einstein-phase dominance.}}$$

It follows that the proposition

$$\boxed{\text{the present finite-capacity premises uniquely determine } \mathcal{G}_v}$$

is excluded. The perfect-action pullback proves that an Einstein-compatible fixed- $j = 3$ completion exists; it does not remove this selection freedom. A hosted minimal completion may select that operator and inherit (H_{TT}) exactly. A genuinely derived completion requires a new geometric principle that fixes a definite local gluing tensor and measure, followed by a proof that its nonperturbative measure lies in the shape-matched Einstein phase. Until the first step is supplied, (H_{phase}) is not a well-posed consequence of the current premises.

What a nonfactorized geometry–capacity embedding must preserve. The external geometric witness leaves the following interface checks for any alternative fully coupled vertex. They are already satisfied by construction on the displayed factorized branch, but a proposed nonfactorized realization is acceptable only if all conditions hold on the same gauge-reduced background:

1. *Spectrum/source overlap:* the capacity decoration introduces no additional source-coupled zero pole; beyond the Einstein scalar constraint, scalar fluctuations are gapped, relaxational, pure gauge/constraint, or exactly source-orthogonal. In condensate realizations with a nonzero relational-time phase gradient, density–phase mixing must be checked in the full Hessian rather than assumed to vanish from a zero-gradient truncation.
2. *Constraint and Ward structure:* coupling the capacity/record factor leaves the geometric constraint algebra and diffeomorphism Ward identity intact, including in defect backgrounds.
3. *Normalization:* the static source normalization continues to reproduce $G_* = c^3 L_*^2 / \hbar$ without an additional geometric-capacity coefficient.
4. *Transfer/physical-inner-product structure:* for a Euclidean/GFT transfer realization, the complete decorated transfer—not merely the finite marked subblocks—must satisfy the required positivity/reflection property; self-adjointness alone is not sufficient. For a directly Lorentzian covariant realization, the corresponding requirement is positivity of the rigging form together with consistent causal gluing, rather than Osterwalder–Schrader positivity.
5. *Lorentzian continuation:* the retarded poles have the correct causal prescription and the capacity dressing does not create ghosts, tachyons, or preferred-frame radiative modes.

These are interface-preservation tests, not a demand that the finite capacity factor rederive the Einstein graviton sector. Failure would reject the alternative realization, not reopen the exact factorized capacity theorem.

M. Exact rigging-map factorization and the Lorentzian endpoint. The controlled spin-foam result is stronger than a purely kinematic embedding and weaker than an exact all-state covariant projector. Refs. [97, 98] give a coupled large-spin/refinement regime in which Regge solutions approach the Einstein sector; in the controlled linearized construction the low-energy excitations give all smooth linearized Einstein solutions and hence exactly the two graviton helicities. This establishes the identity and existence of the ordinary semiclassical gravitational branch.

The unmodified EPRL edge map is not idempotent, so the formal sum over arbitrary foams does not define a projector as written; this obstruction is independent of the detailed spin-foam model [89, 92]. The Riemannian “Warsaw” modification repairs the edge projector, but no corresponding general Lorentzian theorem is available. Ref. [108] derives a rigging-map structure only after assuming the appropriate distributional continuum limit. The raw Lorentzian EPRL state sum is therefore not treated as an exact physical projector here.

Exact capacity-decorated lift theorem. The capacity contribution to the rigging problem can be evaluated exactly when the mixed renewal state is represented on its operator space. Let

$$\omega_*(A) = \text{Tr}(\rho_* A), \quad \rho_* = \sum_b p_{\eta_*}(b) |b\rangle\langle b|$$

be the faithful stationary state of the renewed quantum register and let $\mathcal{K}_{\text{cap}}$ be the GNS/Hilbert–Schmidt completion of the capacity observable algebra with inner product

$$\langle A, B \rangle_{\omega_*} = \text{Tr}(\rho_* A^\dagger B).$$

The Heisenberg renewal map is

$$\Pi_*(A) \equiv \mathcal{E}_*^\dagger(A) = \omega_*(A)I.$$

It obeys

$$\boxed{\Pi_*^2 = \Pi_*, \quad \Pi_*^\dagger = \Pi_*, \quad \text{Ran } \Pi_* = \mathbb{C}I}$$

with the adjoint taken in the ω_* inner product: indeed $\langle A, \Pi_* B \rangle_{\omega_*} = \overline{\omega_*(A)} \omega_*(B) = \langle \Pi_* A, B \rangle_{\omega_*}$. The rank-one projector $|\sqrt{p}\rangle\langle\sqrt{p}|$ used in the detailed-balance probability representation is the commutative-coordinate image of this same one-dimensional stationary transfer direction; neither object says that the quantum state ρ_* is pure.

Let

$$\eta_g : D_g \longrightarrow D_g^*$$

be any positive geometric rigging map. On the empty, unmarked vacuum sector define

$$\boxed{\eta_{\text{full}}(\phi \otimes A)[\psi \otimes B] = \eta_g(\phi)[\psi] \langle A, \Pi_* B \rangle_{\omega_*}.$$

Sesquilinearity, the constraint-annihilation property, reality, and positivity are inherited from η_g and the orthogonal projector Π_* . Quotienting its null space and completing gives

$$\boxed{\mathcal{H}_{\text{phys}}^{\text{full, vac}} \cong \mathcal{H}_{\text{phys}}^g \otimes \text{Ran } \Pi_* \cong \mathcal{H}_{\text{phys}}^g.}$$

This theorem is exact and regulator independent for the equilibrium capacity decoration. It says that renewal adds no vacuum constraint solution or physical pole once a geometric rigging map exists. Persistent marked/defect sectors are not projected by Π_* and require their own coupled matter dynamics; that is a different problem from the vacuum geometric rigging map considered here.

Exact finite-regulator group averaging. There is also an exact regulated construction. Parametrized GFT makes the reparametrization constraint explicit; for a self-adjoint regulated geometric Hamiltonian H_Λ , introduce an auxiliary clock pair (χ, p_χ) and

$$C_\Lambda = p_\chi + H_\Lambda.$$

Then refined algebraic quantization gives

$$\eta_\Lambda = 2\pi\delta(C_\Lambda) = \int_{\mathbb{R}} d\tau e^{i\tau C_\Lambda}$$

as a distributional group average, with the physical inner product obtained by inserting $\delta(C_\Lambda)$ [93]. At any finite spin/mode/particle-number regulator a Hermitian truncation is self-adjoint, so this construction is exact even with interactions retained inside that finite matrix. Ref. [93] proves the clock-neutral/deparametrized equivalence explicitly for the free GFT; the interacting cutoff removal is not supplied by that paper and is not imported here as a theorem.

The same exact mechanism is available canonically: the Lorentzian LQG master-constraint programme constructs a positive symmetric master constraint with a Friedrichs self-adjoint extension, and its direct-integral zero spectral fiber defines the physical Hilbert space [94, 95]. This is an exact nonperturbative canonical witness for how the geometric rigging map can exist. It is not a proof that the raw Lorentzian EPRL foam sum equals that canonical projector.

Clock-test-space convergence theorem. The earlier three-hypothesis spectral statement is unnecessarily strong. Let H_Λ be self-adjoint on the regulated GFT Hilbert space and define the deparametrized constraint

$$C_\Lambda = p_\chi + H_\Lambda \quad \text{on} \quad L^2(\mathbb{R}_\chi) \otimes \mathcal{H}_{\text{GFT}}.$$

For product test vectors $\Phi = f \otimes \phi$ and $\Psi = g \otimes \psi$ with $f, g \in \mathcal{S}(\mathbb{R})$, group averaging is exactly

$$\eta_\Lambda(f \otimes \phi)[g \otimes \psi] = 2\pi \langle \phi, h_{fg}(H_\Lambda)\psi \rangle, \quad h_{fg}(\lambda) = \overline{\widehat{f}(-\lambda)} \widehat{g}(-\lambda).$$

Indeed, p_χ and H_Λ act on separate tensor factors, and the τ integral gives $2\pi\delta(p + \lambda)$ in clock-momentum space. Since f and g are Schwartz, $h_{fg} \in C_0(\mathbb{R})$. Therefore

$$H_\Lambda \xrightarrow{\text{s.r.}} H \implies \eta_\Lambda(\Phi)[\Psi] \longrightarrow \eta(\Phi)[\Psi].$$

The limiting form is positive,

$$\eta(f \otimes \phi)[f \otimes \phi] = 2\pi \int_{\mathbb{R}} |\widehat{f}(-\lambda)|^2 d\mu_{\phi\phi}(\lambda) \geq 0,$$

and it annihilates C on the test space.

The same Schwartz smearing supplies the causal boundary value. The spectral density of C_Λ on these vectors is the convolution

$$\rho_\Lambda(E) = \int \overline{\widehat{f}(E - \lambda)} \widehat{g}(E - \lambda) d\mu_\Lambda(\lambda).$$

It is uniformly bounded, Lipschitz in E , and integrable because the spectral measure is finite and the clock kernel is Schwartz. Strong-resolvent convergence at fixed $\varepsilon > 0$ followed by the Plemelj limit therefore gives

$$G_F = (C - i0)^{-1} = \text{PV } C^{-1} + i\pi\delta(C), \quad \text{Im } G_F(\Phi, \Phi) = \pi\rho_{\Phi\Phi}(0) = \frac{1}{2}\eta(\Phi)[\Phi] \geq 0.$$

Thus the former density-domination and pointwise- $\rho(0)$ hypotheses, and the separate limiting-absorption assumption, are consequences on the Schwartz clock test space. Only strong-resolvent convergence remains for this particular all-state rigging-map route. It is a sufficient mathematical construction, not a requirement that the physical cutoff be removed.

The theorem applies to deparametrized evolution. The equation $p_\chi + H = 0$ does not project onto $\ker H$, and χ must be a physical relational matter clock. If χ is the usual massless-scalar GFT clock, the low-energy spectrum contains two TT gravitational helicities and the clock scalar. It contains no additional capacity-gravity scalar. A condensate Bogoliubov mode identified with the clock must satisfy $c_s^2 \rightarrow 1$ as $k \rightarrow 0$. A master constraint $\mathbf{M} \geq 0$ defines a separate zero-fiber problem and cannot replace H in the clock equation. The masslessness test for the selected branch is

$$(H_{\text{TT}}) \quad H_n \xrightarrow{\text{s.r.}} H_{\text{Ein}}, \quad \inf \sigma(H_{\text{Ein}}|_{\text{TT}}) = 0, \\ H_{\text{Ein}}|_{\text{TT}} = c|\mathbf{k}| + O(k^3 L_{\text{block}}^2), \quad \text{with two positive-residue TT helicities.}$$

The selected metric-Regge theorem above establishes (H_{TT}) , and the perfect-action identity pulls it back to an exact fixed- $j = 3$ realization. It does not establish the native-measure condition (H_{phase}) . The distinct canonical completion retains its own open test,

$$(H'_{\text{can}}) \quad \mathbf{M}_\Lambda^{(3)} \xrightarrow{\text{s.r.}} \mathbf{M}^{(3)} \text{ with a non-topological zero fiber carrying TT Einstein support.}$$

Equivalence between the covariant and canonical completions is not assumed. Fixed-complex q -deformed Lorentzian EPRL models supply finite nonzero- Λ regulators [96]; they do not establish the native phase condition or regulate the empty $\Lambda = 0$ branch universally.

The status is therefore

$$\begin{aligned} &\text{Gaussian determinant bridge: exact,} \\ &\text{quadratic-source Legendre/KL mismatch: exact,} \\ &\text{marked support preservation: exact on the established branch,} \\ &\text{maximal-fusion blocking: exact,} \\ &\text{renewal triangularity: exact,} \\ &\text{selected-branch Einstein theorem } (H_{\text{TT}}): \text{ closed,} \\ &\text{Einstein-compatible fixed-} j = 3 \text{ completion: exists,} \\ &\text{capacity premises uniquely fix } \mathcal{G}_v: \text{ excluded,} \\ &\text{derived native phase } (H_{\text{phase}}): \text{ undefined until } \mathcal{G}_v \text{ is fixed,} \\ &\text{capacity-decorated rigging-map lift: exact,} \\ &\text{finite-regulator group averaging: exact for self-adjoint truncations,} \\ &\text{clock-test positivity and causal boundary value: exact,} \\ &\text{canonical } (H'_{\text{can}}): \text{ open and not required by the selected covariant host.} \end{aligned}$$

The vacuum rigging-map problem contains no unresolved *capacity* step. The selected metric-Regge branch satisfies the Einstein TT infrared theorem (H_{TT}) , and it has a compatible fixed- $j = 3$ primitive realization. The host itself remains underdetermined: the current premises leave $\mathcal{G}_v$ and its geometric measure as selected data. A new geometric principle must fix both before (H_{phase}) becomes a well-posed derived theorem about the nonperturbative measure. A canonical-covariant correspondence would be a further result. The raw EPRL sum does not supply that projector.

Candidate selection principle. The face-wise reading of the relational lock (Appendix H.9a, paragraph 4a) is the one candidate for the additional geometric principle named above that is already present in the postulates. It does not choose $\mathcal{G}_v$ among gauge-covariant tensors; it restricts the configuration space on which any such tensor acts to equilateral causal triangulations with fixed edge $a = 2.530L_*$, on which the Regge weight is $(\ln 2/2\pi) \sum_h N_h \Theta_h$ with no free coefficient. Under that reading μ_{native} is the CDT measure at couplings fixed by the theory, and (H_{phase}) becomes the well-posed numerical question of Appendix J.10: whether that ensemble, with the capacity weighting switched on, is extended with a gapless TT sector at its predicted point. The reading is recorded as a premise, not a theorem.

H.11 Coherence under renewal and reconstruction of the retained marked Hilbert sector

The question. The renewed register contains no information about its input, whereas quantum interference is carried by off-diagonal operators. Coherence can survive only if the interfering degrees of freedom remain in the retained system and the discarded complement carries no path record. The calculation below first states this division exactly. It then asks whether the retained nine-state mark must be assigned a complex Hilbert space or whether that structure follows from the displayed fusion and Lorentzian action. It adds no fourth foundational postulate.

Exact renewed-output theorem. For

$$\mathcal{E}_*(X) = \rho_* \text{Tr } X,$$

the Heisenberg dual is fixed by

$$\text{Tr}[\mathcal{E}_*(X)O] = \text{Tr}[X\mathcal{E}_*^\dagger(O)].$$

Substitution gives

$$\boxed{\mathcal{E}_*^\dagger(O) = \text{Tr}(\rho_* O)I.}$$

Every later observable on the renewed output therefore pulls back to a multiple of the identity, and every traceless input operator is annihilated. Equivalently, for any reference R ,

$$(\text{id}_R \otimes \mathcal{E}_*)(\rho_{RA}) = \rho_R \otimes \rho_*.$$

The replacement channel is entanglement-breaking on the renewed register. A global dilation can still preserve the old state in its complementary output, as the history register in H.8 does. The theorem excludes coherence from the replaced output only; it does not exclude global coherence.

Complementary-record coherence theorem. Let X denote the defect position and D its retained internal marked fiber. For two freely transported branches, write the relevant part of a one-step isometry as

$$V(|x\rangle_X |q\rangle_D |0\rangle_E) = |F(x)\rangle_X U_D |q\rangle_D |e_x\rangle_E,$$

and similarly for y . The environment E denotes only what is discarded when the defect state is read. Directly tracing E gives

$$|x\rangle\langle y| \longmapsto \underbrace{\langle e_y | e_x \rangle}_{\gamma_{xy}} |F(x)\rangle\langle F(y)| \otimes U_D |q\rangle\langle q| U_D^\dagger.$$

Thus

$$\boxed{\mathcal{C}_{xy}(1) = \gamma_{xy} \mathcal{C}_{xy}(0), \quad \gamma_{xy} = \langle e_y | e_x \rangle.}$$

For independent record factors over n updates,

$$\mathcal{C}_{xy}(n) = \left(\prod_{k=1}^n \gamma_{xy}^{(k)} \right) \mathcal{C}_{xy}(0).$$

The equation separates the two possible microscopic readings. If the vertex leaves a cell-addressed record, $|e_x\rangle = |x\rangle_E$ and $|e_y\rangle = |y\rangle_E$, then $\gamma_{xy} = 0$ for $x \neq y$: position coherence dies in one update. If the persistent internal fiber remains inside $X \otimes D$ and the discarded free-renewal output is branch-independent, $|e_x\rangle = |e_y\rangle$, then $\gamma_{xy} = 1$ and renewal produces no intrinsic position decoherence. Intermediate overlaps produce ordinary partial decoherence.

Calling a record “worldline-attached” is not by itself sufficient. A spatially localized record that moves along two different worldlines is still a which-path record if it is discarded. In this paper the phrase means the stronger structure: the marked response fiber is retained as part of the coherently transported defect, while the discarded renewal register contains no branch address. Genuine interactions with detectors or an environment may, and ordinarily do, make the discarded states distinguishable.

What $Q \perp V_3$ does and does not prove. Primitive fusion gives the orthogonal decomposition

$$V_{3/2} \otimes V_{3/2} = V_3 \oplus Q, \quad Q = V_0 \oplus V_1 \oplus V_2,$$

and hence

$$P_Q P_3 = 0, \quad \text{Tr } P_Q = 9, \quad \text{Tr } P_3 = 7.$$

If a discarded record actually resolves these two sectors, the same overlap theorem sets every Q – V_3 off-diagonal to zero. This gives exact marked/unmarked sector dephasing under a sector-resolving export. It is compatible with charge-sector superselection, but it is not yet a derivation of the full electric-charge superselection rule. That stronger statement requires the mark to be identified with the generator of a local $U(1)$ gauge symmetry and requires the physical observable algebra to preserve its Gauss-law sectors. Orthogonality of two representation subspaces alone does not force the environment to measure them.

Forced vertex constraint, not a new premise. The complete microscopic Hilbert space at a marked vertex must therefore be read as

$$\mathcal{H}_v = \mathcal{H}_{\text{ren}} \otimes \mathcal{H}_X \otimes (\mathbb{C}|0\rangle \oplus Q),$$

where $\mathcal{H}_{\text{ren}}$ is replaced by ρ_* , while $\mathcal{H}_X \otimes Q$ is transported coherently. In the free sector the dilation of W_* must satisfy

$$\boxed{\langle e_y | e_x \rangle = 1 \quad \text{for unmeasured position branches } x, y}$$

after the canonical translation identifying their background renewal states. The exact equality defines the minimal ideal free vertex; electron-interference experiments impose a finite lower bound on the product of overlaps over their actual coherence time [28]. A cell-fixed alternative is not another interpretation of the same action. It is an experimentally excluded countervertex.

Sections 3.3 and 22 already require the operational branch to reproduce ordinary quantum interference. The overlap theorem converts that existing requirement into an explicit microscopic test of W_* , so the condition adds no ontological premise. Electron interference would falsify an action that violates it. The finite calculation verifies the dual-channel identities, the Q – V_3 projector orthogonality, and the zero, unit, and intermediate record-overlap cases to machine precision.

Fast/slow architecture. The result separates the roles that renewal had previously blurred. The fast closure register supplies the objective replacement process and its exported history correlations. The persistent marked fiber carries internal phase and position coherence. The fast output cannot feed old off-diagonal data back into present-cell observables because $\mathcal{E}_*^\dagger(O)$ is scalar; the slow coherent sector can nevertheless evolve unitarily because it is not the argument of $\mathcal{E}_*$. Renewal and interference are therefore compatible provided the complete vertex respects the tensor split above.

Complementary outputs of primitive fusion. The two sides of the primitive split have different infrared roles,

$$V_{3/2} \otimes V_{3/2} = \underbrace{V_3}_{\text{matched geometric channel}} \oplus \underbrace{Q}_{\text{retained mismatch record}}.$$

Appendix H.10 proves (H_{TT}) on the maximal V_3 ray: two positive-residue, massless helicity-2 modes with $E = c|\mathbf{k}|$, together with an exact fixed- $j = 3$ pullback whose nonmaximal gravitational complement is gapped. The reconstruction below concerns the orthogonal Q output. It does not replace the graviton theorem; it explains why the retained record acquires quantum state geometry. On the factorized vacuum branch the finite capacity/record decoration adds no graviton pole.

The retained field is the fusion complement. The local Hilbert reconstruction begins without assuming a complex state space. Let

$$Q = V_0 \oplus V_1 \oplus V_2, \quad \dim Q = 9,$$

be the complement of maximal primitive fusion. H.9 gives an equivariant isometry $L : Q \rightarrow \mathcal{H}_{\text{mark}}$ with

$$L^\dagger L = P_Q, \quad LL^\dagger = I_9.$$

The marked coordinate is therefore the same retained degree of freedom in a different basis,

$$\boxed{d = L\chi, \quad \chi = P_Q\Phi = L^\dagger d.}$$

It is not an additional nine-state field with an independently assignable free propagator.

Quadratic inheritance from the displayed action. Restrict the action of Q.9 to fluctuations in Q about the selected V_3 background. Its quadratic Hessian is

$$\mathcal{H}_Q = \mathcal{K}_{\text{kin}} + M_c^2 I_Q + \kappa(6P_0 + 5P_1 + 3P_2),$$

where the condensate may shift the common term M_c^2 but cannot remove the displayed relative splitting. Since $\delta\chi = L^\dagger \delta d$, the chain rule gives

$$\delta^2 \Gamma_f = \delta d^\dagger L \mathcal{H}_Q L^\dagger \delta d, \quad \boxed{\mathcal{H}_D = L \mathcal{H}_Q L^\dagger.}$$

An extra one-mark quadratic term would be a new defect-only invariant and would define a nonminimal action. The hard-core constraint does not alter this result on the fixed-one-mark sector.

If the relative frame varies, $L = L(g(x))$, the same change of variables also turns the primitive kinetic term into

$$Z(D_\mu d)^\dagger D^\mu d, \quad D_\mu = \partial_\mu + A_\mu, \quad \boxed{A_\mu = L \partial_\mu L^\dagger},$$

with $A_\mu^\dagger = -A_\mu$. Thus one field identification supplies both the moving-complement connection and the inherited mismatch spectrum. After Lorentzian continuation the three coupled-spin sectors have

$$\boxed{\omega_J^2(k) = k^2 + M_c^2/Z + \frac{\kappa}{Z}(6, 5, 3)_J, \quad J = 0, 1, 2,}$$

so their phases cannot all advance at the same rate when $\kappa > 0$.

The real dynamical algebra. Before introducing complex amplitudes, write the nine retained normal coordinates and their canonical momenta as

$$v = (q, p) \in W \equiv \mathbb{R}^9 \oplus \mathbb{R}^9, \quad \Omega_0 = \sum_{a=1}^9 dq_a \wedge dp_a.$$

The moving complement generates $A \in \mathfrak{so}(9)$. The non-common Lorentzian phase supplies a real symmetric traceless generator S ; its orbit under $\mathfrak{so}(9)$ spans all of $\text{Sym}_0(9)$. Their phase-space action is

$$X(A, S) = \begin{pmatrix} A & -S \\ S & A \end{pmatrix}, \quad A^T = -A, \text{quad} S^T = S, \text{quad} \text{tr } S = 0.$$

There are $36 + 44 = 80$ independent generators. Their commutators preserve Ω_0 and close as

$$\text{Lie}(\mathfrak{so}(9), iS) = \mathfrak{su}(9).$$

The common oscillator phase adds the center and gives $\mathfrak{u}(9)$; it does not affect projective states.

Unique positive complex structure. Define the real operator

$$\mathcal{J} = \begin{pmatrix} 0 & -I_9 \\ I_9 & 0 \end{pmatrix}.$$

It satisfies $\mathcal{J}^2 = -I_{18}$ and commutes with every $X(A, S)$. Its uniqueness is elementary. Any real K commuting with all diagonal $\mathfrak{so}(9)$ actions must have scalar blocks,

$$K = \begin{pmatrix} aI & bI \\ cI & dI \end{pmatrix}.$$

Commutation with every symmetric generator forces $d = a$ and $c = -b$, so

$$\text{Comm}(\mathfrak{g}) = \text{span}_{\mathbb{R}}\{I, \mathcal{J}\}.$$

The condition $K^2 = -I$ leaves $K = \pm \mathcal{J}$. Only the displayed sign makes

$$g(u, v) = \Omega_0(u, \mathcal{J}v) = u^T v$$

positive. Hence

$$\boxed{\mathcal{J} \text{ is the unique positive complex structure preserved by the one-mark dynamics.}}$$

Writing $z_a = q_a + ip_a$ is now notation for this derived real operation, not an independent postulate. The Hermitian product

$$h(u, v) = g(u, v) + i\Omega_0(u, v)$$

becomes $h(z, w) = z^\dagger w$, and therefore

$$\boxed{\mathcal{H}_{\text{mark}} \simeq \mathbb{C}^9.}$$

Why the physical states are rays. Normalize $z^\dagger z = 1$. The hard-core marked algebra already contains the matrix units

$$d_a^\dagger d_b = |a\rangle\langle b|,$$

and $\mathfrak{su}(9) \oplus \mathbb{R}I$ spans every Hermitian 9×9 response operator. For such an operator T , the response $f_T(z) = z^\dagger T z$ is unchanged by $z \mapsto e^{i\theta} z$. Conversely, if normalized z and w give the same value of f_T for every traceless Hermitian T , then

$$\mathrm{Tr}\left[T(zz^\dagger - ww^\dagger)\right] = 0 \quad \text{for all } T.$$

Thus $zz^\dagger - ww^\dagger = cI$. Equal unit traces give $c = 0$, and equality of the rank-one projectors gives $w = e^{i\theta} z$. The complete local response algebra therefore separates normalized states exactly modulo common phase:

$$\boxed{\mathcal{S}_{\text{mark}} = S^{17}/U(1) = \mathbb{CP}^8.}$$

Projectivization is an operational consequence of the available responses, not an asserted ray rule.

Fubini–Study geometry and the factor of two. The quotient of the Hermitian metric gives

$$ds_{\text{FS}}^2 = dz^\dagger dz - |z^\dagger dz|^2.$$

With $z_a = \sqrt{P_a} e^{i\phi_a}$ and $\sum_a P_a = 1$,

$$\boxed{ds_{\text{FS}}^2 = \frac{1}{4} \sum_a \frac{dP_a^2}{P_a} + \sum_a P_a \left(d\phi_a - \sum_b P_b d\phi_b \right)^2.}$$

The probability block is one quarter of the Fisher metric. The symplectic normalization follows directly from the real canonical variables,

$$dq_a \wedge dp_a = \frac{1}{2} dP_a \wedge d\phi_a.$$

Consequently, with $\Phi_a = \hbar\phi_a$ and the metric scaled by $\hbar$,

$$\boxed{\Omega_K = \frac{1}{2} \sum_a dP_a \wedge d\Phi_a, \quad ds^2 = \sum_a \left[\frac{\hbar}{4P_a} dP_a^2 + \frac{P_a}{\hbar} d\Phi_a^2 \right]_{\text{mhorizontal}}.}$$

The factor $1/2$ in Ω_K is fixed by $q_a = \sqrt{P_a} \cos \phi_a$ and $p_a = \sqrt{P_a} \sin \phi_a$.

Independent Fisher check at the fresh mark. Rotational invariance and the stationarity equation give

$$\langle C_a C_c \rangle = \frac{1}{2\eta_*} \delta_{ac}.$$

For independent present and history strands, the normalized marked response is

$$r_{ab} = 2\eta_* C_a^P C_b^H, \quad \langle r_{ab} r_{cd} \rangle = \delta_{ac} \delta_{bd}.$$

Perturbing the square-root probability amplitude by $A_\epsilon = A_0(1 + \epsilon^{ab} r_{ab}) + O(\epsilon^2)$ gives statistical score $s_{ab} = 2r_{ab}$. Hence

$$I_{ab,cd}^{\text{Fisher}} = 4\delta_{ac}\delta_{bd}, \quad g_{ab,cd}^{\text{mark}} = \delta_{ac}\delta_{bd},$$

and therefore

$$\boxed{g_{\text{mark}} = \frac{1}{4} I_{\text{Fisher}} = g_{\text{FS}}}$$

at the fresh marked tangent point. Extending this equality over all of $\mathbb{CP}^8$ requires the corresponding finite response family away from the fresh state; the local normalization is exact.

Many-mark composition. The same physical bridge used in P.6 survives projection onto two neighboring retained complements. For primitive pairs (1, 2) and (3, 4),

$$\begin{aligned} H_{QQ} &= (P_Q^{12} \otimes P_Q^{34})(H_2 \otimes H_3)(P_Q^{12} \otimes P_Q^{34}) \\ &= A_R \otimes A_L, \end{aligned}$$

where $A_R = P_Q(I \otimes H)P_Q|_Q$ and $A_L = P_Q(H \otimes I)P_Q|_Q$. Exact rational spin-3/2 evaluation gives

$$\text{rank } A_R = \text{rank } A_L = 8, \quad \text{Tr } A_R = \text{Tr } A_L = 0, \quad \text{Tr } A_R^2 = \text{Tr } A_L^2 = 38.$$

Thus the bridge is nonzero, nonlocal, and cannot be written as a sum of one-mark Hamiltonians. With the two local algebras, the tensor-product lemma of P.6 yields

$$\boxed{\text{Lie}(\mathfrak{su}(9)_1, \mathfrak{su}(9)_2, iA_R \otimes A_L) = \mathfrak{su}(81).}$$

Induction along a connected marked network gives

$$\boxed{\mathcal{H}_N = (\mathbb{C}^9)^{\otimes N}, \quad \mathfrak{g}_N = \mathfrak{su}(9^N),}$$

provided simultaneous retained marks exist and every spanning-tree bridge survives the complete gauge/intertwiner projection of the geometric host.

Projective phase is not electric charge. The ray construction supplies the kinematic principal bundle $S^{17} \rightarrow \mathbb{CP}^8$ and its local representative connection

$$\mathcal{A}_\mu = -iz^\dagger \partial_\mu z, \quad \mathcal{A}_\mu \mapsto \mathcal{A}_\mu + \partial_\mu \theta.$$

This does not identify the bundle with electromagnetism. In fact, the free decorated block preserves the response-mark number n_d , but the incidence vertex does not:

$$[n_d, V_Q] = \sum_{ab} \left(d_{ab}^\dagger \xi_a^P \xi_b^H - \text{h.c.} \right) \neq 0.$$

The temporary H.9 response mark is therefore not the conserved electric charge. Any electromagnetic generator must belong to the persistent charged defect and must still produce Gauss law, a gapless Maxwell field, and the observed coupling.

Relation to Hamilton–Killing reconstruction and remaining scope. Caticha’s reconstruction supplies an independent general theorem connecting Fisher geometry, a compatible Kähler structure, complex amplitudes, and linear unitary evolution [26, 27]. Here the finite one-mark complex structure, ray space, Kähler geometry, and full unitary algebra have instead been obtained from the displayed fusion and quadratic action; Caticha’s result is a consistency check and a guide for the continuum extension. What remains is to show that the final constrained GFT host preserves the marked bridges, construct indefinite record capacity or a controlled continuum/Fock limit, derive relativistic fields and their vacuum, and derive electromagnetic charge and Maxwell dynamics. None of those requirements weakens the local theorem or the exact renewal and overlap results.

Closure statement.

renewed-output coherence blindness: exact,
record-overlap multiplier: exact,
$Q \perp V_3$: exact,
cell-addressed marked export: excluded by interference,
branch-independent retained marked fiber: required vertex condition,
$\mathbb{C}^9/\mathbb{CP}^8$ one-mark reconstruction: exact in the selected quadratic completion,
$g_{\text{mark}} = I_{\text{Fisher}}/4$ at the fresh tangent point: exact,
$\mathfrak{su}(81)$ two-mark closure: exact before final host projection,
$\mathfrak{su}(9^N)$ network extension: conditional on surviving projected bridges,
continuum/Fock theory, electromagnetic charge, and relativistic QED: open.

Appendix I: Mass and Gauge Extensions

Appendix I develops possible particle and gauge extensions of the entanglement logic. The closed weak-field derivation does not depend on them.

I.1 Charged-lepton spectrum from the shell algebra

The electron is the ground-state one-bit charged defect — the $N = 0$ configuration whose seven-channel dressing sets the substrate length (Appendix D.4). The muon and tau are heavier shell excitations of the same defect. Each adds one radial entanglement shell, realized microscopically as the exclusion of one face label from the seven-state alphabet of Appendix B. The closure machinery loses its nondegenerate weighting after the third branch, so the construction contains three charged leptons. Their baseline masses follow from the shell entropy and singlet overlaps; the decorated marked-transfer vertex supplies the finite response that earlier versions left in the residuals. Because the ratios carry no dimensionful input, they test the same marked action that enters G_* .

With that picture fixed, the spectrum is written as a shell expansion,

$$\log \frac{m_N}{m_e} = B_0 N + A_0 N^2, \quad N = 0, 1, 2,$$

with the electron the $N = 0$ ground state and the muon and tau the $N = 1, 2$ radial entanglement-shell excitations of the same core structure.

Three-generation termination from closure-spectrum collapse. Combinatorial injectivity alone allows up to $N = 3$ shells, since the reduced alphabet must retain at least four distinct labels to support an injective face assignment ($7 - N \geq 4$). The sharper structural bound comes from the admissibility-closure machinery. For the reduced ensemble at shell N , the closure invariant K^2 inherits a discrete spectrum whose dispersion gives the admissibility kernel $e^{-\eta K^2}$ real weight. For $N = 0, 1, 2$ the K^2 spectrum has multiple distinct values, and the closure equation $\langle K^2 \rangle_\eta = 3/(2\eta)$ has a non-degenerate solution.

At $N = 3$ the spectrum collapses. With only four labels left, every injective assignment to the four faces uses all of them, and K^2 depends on the labels only through the permutation-invariant sums S and Σ^2 , so every microstate of the reduced four-label ensemble carries the same value of K^2 . With a single spectral value K_0^2 the closure equation $\langle K^2 \rangle_\eta = 3/(2\eta)$ still fixes $\eta = 3/(2K_0^2)$ formally; what fails at $N = 3$ is selection, not solvability. The kernel $e^{-\eta K^2}$ acts trivially when

every microstate carries the same K^2 , so it weights nothing and leaves no admissibility dispersion to define a branch. Shell $N = 3$ therefore does not define a non-degenerate admissibility-closed branch in the same sense as the earlier shells: the cutoff rests on the collapse of the K^2 dispersion, not on the closure equation admitting arbitrary η .

The number of charged-lepton generations is consequently the number of shells with a non-degenerate closure branch,

$$N \in \{0, 1, 2\} \quad \implies \quad \text{three generations,}$$

a structural bound stronger than the combinatorial count. The closure-spectrum collapse fixes the generation count at three; injectivity alone would allow four.

Linear coefficient from the first-shell reduced-alphabet entropy. The linear coefficient is the entropy cost of adding one shell excitation. With one face label excluded, the reduced ensemble has

$$\Omega_1 = 2 P(6, 4) = 720 = 6!,$$

i.e., the full symmetric group S_6 with orientation degeneracy. Direct admissibility evaluation on the reduced ensemble gives $g_1 = \ln \Omega_1 = \ln 720$ to within 0.1%, since the admissibility correction is small on the reduced ensemble where the alphabet symmetry is nearly intact. Hence

$$B_0 = \ln \Omega_1 = \ln 720 = 6.579 \dots,$$

compared with the empirical fit $B_0 = 6.586$ (agreement at 0.1%).

Why the same 720 at every shell. The per-shell factor is the *same* $\Omega_1 = 720$ for each added shell, not a decreasing sequence $720 \cdot 240 \dots$ in which each shell excludes a further label. At the tau, cumulative exclusion would give $m_\tau/m_e \simeq 1151$ (−67%), whereas the repeated factor gives 3454.6 (−0.65%), so the data require the same reduction at each shell. The structural reason is the rank of the inter-shell coupling. The shells couple only through the permutation-invariant singlet projector P_{sing} , whose spectrum is $\{1, 0, 0, 0, 0, 0, 0\}$: rank one. A rank-one projector pins exactly one shared direction, and it is the same direction however many shells are already stacked, so each added shell surrenders only that one shared singlet direction and retains the other six — the same direction at every shell, not a fresh label removed cumulatively. The injective count realizes this exactly: $P(6, 4)/P(7, 4) = 360/840 = 3/7$, so $1680 \rightarrow 720$ once and again at each subsequent shell. This also separates the two reduced-alphabet notions that otherwise look inconsistent. The *generation cutoff* above counts $7 - N$ surviving labels because it asks whether a non-degenerate closure *branch* still exists; the *mass factor* here is the rank-one singlet pinning one shared direction per shell. They are different operations, which is why the cutoff scales with $7 - N$ while the ladder carries a fixed 720.

Quadratic coefficient from singlet-projection shell algebra. The quadratic coefficient follows from three premises already in the framework: (i) mass arises from scalar capacity response (Postulate II); (ii) the scalar EFT response is quadratic in the total source; (iii) the coarse scalar branch projects onto the permutation-invariant singlet of the seven-state face alphabet.

Let

$$\mathcal{H}_7 = \text{span}\{|m\rangle : m = -3, \dots, 3\}, \quad |u\rangle = \frac{1}{\sqrt{7}} \sum_{m=-3}^3 |m\rangle, \quad P_{\text{sing}} = |u\rangle\langle u|.$$

A shell excitation that marks the face state $|a_r\rangle$ contributes to a coherent shell source

$$|J_N\rangle = \sum_{r=1}^N |a_r\rangle.$$

The coarse scalar field δS is permutation-symmetric on face labels and therefore couples only through P_{sing} . The scalar-channel response is quadratic in the total source, so the relevant object is the source norm

$$\langle J_N | P_{\text{sing}} | J_N \rangle = \sum_{r,s=1}^N \langle a_r | P_{\text{sing}} | a_s \rangle.$$

Because $\langle a | P_{\text{sing}} | b \rangle = 1/7$ for any a, b , this reduces to $N^2/7$ independent of which specific face labels are excluded. Including the binary orientation degeneracy of the boundary ensemble, the orientation-summed scalar transfer weight per ordered shell incidence is

$$\mathcal{M}_{rs} = 2 \langle a_r | P_{\text{sing}} | a_s \rangle = \frac{2}{7}.$$

The ordered-pair count decomposes as $N^2 = N + 2\binom{N}{2}$: N self-incidences along the diagonal and $2\binom{N}{2}$ directed cross-incidences. Two distinct steps enter here, and only the first is the quadratic EFT response. The quadratic singlet response fixes that there are N^2 ordered incidences, each carrying the per-incidence overlap $\mathcal{M}_{rs} = 2/7$; it does not by itself make the mass multiplicative. The passage from these N^2 additive incidences to a multiplicative factor is the log-overlap mass map: the mass is the exponential of a dressing free energy whose pairwise term is $\sum_{r,s} \ln \mathcal{M}_{rs} = N^2 \ln(2/7)$, the same log-overlap form that carries the gravitational dressing (Appendix H.3) and stated explicitly below. With that map, the N -shell dressing is

$$\prod_{r,s=1}^N \mathcal{M}_{rs} = \left(\frac{2}{7}\right)^{N^2},$$

so

$$A_0 = \ln \frac{2}{7} = -1.253\dots,$$

compared with the empirical fit $A_0 = -1.255$ (agreement at 0.15%).

The ordered-bilinear structure is not a free assumption. It is the unique scalar-channel response to a multi-source state under the three premises just stated: the source norm $\langle J_N | P_{\text{sing}} | J_N \rangle$ runs over both indices independently because the integrated-out scalar contribution is quadratic in J_N .

Mass ladder and numerical accuracy. Combining the two coefficients,

$$\boxed{\frac{m_N^{(0)}}{m_e} = 720^N \left(\frac{2}{7}\right)^{N^2}, \quad N = 0, 1, 2.}$$

Equivalently,

$$\log \frac{m_N}{m_{N-1}} = \ln 720 + (2N - 1) \ln \frac{2}{7},$$

so each added shell contributes $\ln 720$ plus an odd number of new scalar-overlap incidences (1, 3, 5, ..., summing to N^2). The second difference is fixed at

$$\Delta^2 \log m_N = 2 \ln \frac{2}{7} = -2.506\dots,$$

compared with the empirical -2.509 (0.15%). The marked vertex of H.9 then supplies

$$Z_\mu = 1 + \zeta_*, \quad Z_{\tau,2} = 1 + \frac{2}{7}\zeta_*.$$

Thus

$$\begin{aligned} \frac{m_\mu}{m_e} &= 720 \frac{2}{7} Z_\mu = 206.768280237 \quad (-0.535\sigma), \\ \frac{m_\tau}{m_e} &= 720^2 \left(\frac{2}{7}\right)^4 Z_\mu Z_{\tau,2} = 3477.343310 \quad (+0.481\sigma). \end{aligned}$$

The dressing map, explicitly. The three factors follow from one construction. Write the N -th charged lepton as a coherent source raised on the electron ground dressing by exciting N shells, $\mathcal{S}_N = \sum_{k=1}^N s_k$, where each s_k excludes one face label from the seven-state alphabet. Its dressing free energy is a log-overlap functional of the kind used for the gravitational dressing in Appendix H.3, and it separates into a per-shell term and a pairwise term. Each shell carries the reduced-alphabet entropy $\ln \Omega_1 = \ln 6! = \ln 720$, the permutation count of the six surviving labels, which builds the linear factor 720^N . Each ordered pair of shells contributes a singlet log-overlap: a single label projected on the closure singlet $u = \frac{1}{\sqrt{7}}(1, \dots, 1)$ of Section 8 has weight $|\langle u|m \rangle|^2 = 1/7$, and orientation summation doubles it to $2/7$. The N shells form N^2 ordered pairs, so the pairwise term is $N^2 \ln(2/7)$, and exponentiating the free energy returns

$$\frac{m_N^{(0)}}{m_e} = \Omega_1^N \left(\frac{2}{7}\right)^{N^2} = 720^N \left(\frac{2}{7}\right)^{N^2}.$$

The coefficients are fixed: $720 = 6!$, the N^2 pair count is combinatorial, and $2/7$ is the orientation-summed singlet projection behind $\Omega_{\text{tet}} = 2P(7, 4) = 1680$. The marked factors are finite traces of the same decorated vertex used in the electron scale.

The support exponent, tested against the spectrum. The ladder also tests the exponent in the support-to-length identification of Appendix D.4, which reads the physical ratio as the first power of the entropy support, $\lambda/L_* \propto e^H$. Letting that power float, $\lambda/L_* \propto e^{\alpha H}$, turns each lepton ratio into a measurement of α , since $m_N/m_e = [720^N(2/7)^{N^2}]^\alpha$:

$$\alpha_N = \frac{\ln(m_N/m_e)_{\text{PDG}}}{\ln[720^N(2/7)^{N^2}]}, \quad \alpha_\mu = 1.0010, \quad \alpha_\tau = 1.0008.$$

The leading shell values give $\alpha_\mu = 1.0010$ and $\alpha_\tau = 1.0008$, while square-root and quadratic support maps miss by orders of magnitude. The decorated action does not float this exponent. It keeps the first-power support map and computes the finite response factors separately.

Status. The three-generation termination, baseline shell coefficients, and marked response are fixed inside the displayed finite action. The collapse at $N = 3$ is the permutation-invariance of K^2 ; 720 , $2/7$, and N^2 are reduced-alphabet, singlet-projection, and ordered-pair counts; and Z_μ , $Z_{\tau,2}$ are the enumerated marked traces. The remaining particle-sector work concerns other defect classes, not an unfixed charged-lepton coefficient.

The electron remains the elementary weak-field anchor; the heavier charged leptons are correlated outputs of the same marked shell algebra, not closure-defining inputs. Composite hadrons remain part of the dressed bound-state entropy program. The neutrino and quark sectors are not treated here. Extending the closure-spectrum-collapse argument to those defect classes remains a separate task.

I.2 Gauge structure and the conditional projective color sector

The framework's primary target remains the gravitational and dark sector. The Abelian and weak sectors below are substrate-natural hosting arguments whose physical Standard Model identification remains external. Color now has a stronger status. One conditional microscopic identification — a colored defect carries one persistent open tetrahedral-routing leg, transported reversibly until singlet closure — fixes the projective qutrit transport algebra and the adjoint form of the local pure-gauge action. A minimal kinetic completion then fixes its primitive adjoint transfer. This does not derive the global $SU(3)$ endpoint lift, chiral quarks, hypercharge assignments, quark masses, confinement, or the finite matching to a continuum QCD scheme.

Baseline-redundancy template. The same logic that organizes the gravity sector extends to any conserved-charge sector. For a conserved charge Q , introduce an entropy-like potential $S_Q(x)$ and require that physical observables depend only on differences of that potential, not on its absolute baseline. Localizing that redundancy requires a compensating connection. In the Abelian case,

$$D_\mu S_Q = \partial_\mu S_Q - q A_\mu, \quad S_Q \rightarrow S_Q + \alpha(x), \quad A_\mu \rightarrow A_\mu + \frac{1}{q} \partial_\mu \alpha,$$

giving Maxwell-type dynamics for A_μ . The non-Abelian generalization, for multiplet-valued entropic potentials transforming under a compact Lie group G , gives Yang–Mills covariant derivatives and field strengths in the standard form.

The template tells us what form a gauge interaction takes once a conserved-charge sector with baseline redundancy is present. It does not select which specific groups are physically realized.

Substrate-natural components. Two ingredients of Standard Model gauge structure are naturally accommodated by the substrate.

$U(1)_Y$. The baseline-redundancy template naturally accommodates an Abelian conserved-charge sector of the hypercharge type. The associated entropy potential is conserved-charge in character, and localizing its baseline redundancy produces the corresponding Maxwell-type dynamics. The argument is independent of the gravity-sector derivation, applying the same template to a separate conserved-charge potential.

$SU(2)_L$. The substrate naturally carries an $SU(2)$ -representation structure through its half-integer face data. On the marked-fusion incidence branch, the three-dimensional vector response and the complement theorem select the primitive spin $j_0 = \frac{3}{2}$ (Section 5 and Appendix B); without that identification, the same spin is the surviving discrete branch. The static K^2 ensemble breaks this $SU(2)$ to its $U(1)$ Cartan through the magnetic-quantum-number projection, but the full $SU(2)$ remains present in the quantum face data discarded by that projection. Identifying this representation-theoretic $SU(2)$ with the internal weak group $SU(2)_L$, including chirality and doublet assignments, remains external to the present derivation.

Three tetrahedral routes and the closed-cell obstruction. Four faces admit exactly three opposite-face pairings,

$$A = (12 | 34), \quad B = (13 | 24), \quad C = (14 | 23).$$

Tetrahedral permutations act transitively on this set, so no route is preferred. On a closed effective cell define

$$R_A = m_1 m_2 + m_3 m_4, \quad R_B = m_1 m_3 + m_2 m_4, \quad R_C = m_1 m_4 + m_2 m_3.$$

Their sum is fixed by the closure invariant,

$$R_A + R_B + R_C = \frac{1}{2} \left[\left(\sum_i m_i \right)^2 - \sum_i m_i^2 \right] = 72 - \frac{3}{2} K^2.$$

The symmetric route is therefore the scalar closure mode already read by gravity. The exact 1680-state sum at η_* gives the connected covariance

$$C_R = \begin{pmatrix} 27.45571280 & -7.84449077 & -7.84449077 \\ -7.84449077 & 27.45571280 & -7.84449077 \\ -7.84449077 & -7.84449077 & 27.45571280 \end{pmatrix}.$$

With

$$P_1 = \frac{1}{3} \mathbf{1} \mathbf{1}^T, \quad P_2 = I - P_1,$$

this becomes

$$C_R = 11.76673126 P_1 + 35.30020358 P_2.$$

The singlet eigenvalue is fixed analytically,

$$\lambda_1 = \frac{1}{3} \text{Var}(R_A + R_B + R_C) = \frac{3}{4} \text{Var}(K^2).$$

A closed cell thus supplies $\mathbf{3}_{\text{route}} = \mathbf{1}_{\text{closure}} \oplus \mathbf{2}_{\text{imbalance}}$. It cannot furnish an irreducible color triplet as a closed scalar observable.

Primitive route geometry. The route carrier belongs to the primitive spin layer, before neighboring slots fuse into the effective $j_{\text{eff}} = 3$ link. Couple the four primitive $j_0 = 3/2$ slots according to A , B , or C , taking each pair through its maximal $J = 3$ channel and closing the two intermediate spins to a singlet. Standard $SU(2)$ recoupling gives

$$\langle A|B \rangle = \langle A|C \rangle = \langle B|C \rangle = -\frac{1}{20}.$$

The route Gram matrix is therefore

$$G = \begin{pmatrix} 1 & -1/20 & -1/20 \\ -1/20 & 1 & -1/20 \\ -1/20 & -1/20 & 1 \end{pmatrix} = \frac{9}{10} P_1 + \frac{21}{20} P_2.$$

Its eigenvalue ratio is

$$\frac{\lambda_2}{\lambda_1} = \frac{7}{6},$$

the same exact ratio that supplies the isotropically averaged one-label-exclusion source benchmark $\ln(7/6)$ in Appendix C.5. This equality follows before any color datum is introduced. It is a cross-sector recoupling identity, not a determination of the strong coupling.

The layer ordering is fixed by the construction. Two primitive shared-face slots first form $V_{3/2} \otimes V_{3/2}$, maximum-capacity fusion selects V_3 , and the four resulting effective links carry the K^2 closure ensemble. The color route therefore resides in an open primitive record that survives between fusion events. Appendix H.9 gives the complementary result that the nonmaximal primitive fusion channels $V_0 \oplus V_1 \oplus V_2$ are exactly the nine-state representation already required by the present/history marked fiber.

Persistent open routing and the gauge algebra. The conditional color identification is precise: a colored fermionic defect carries one unclosed tetrahedral-routing leg, and transport preserves its correlations with an arbitrary reference until the leg closes into a singlet. The three unresolved route alternatives span the qutrit

$$\mathcal{H}_{\text{route}} \simeq \mathbb{C}^3.$$

A reversible quantum channel between equal finite-dimensional systems is unitary conjugation,

$$\mathcal{E}_U(X) = UXU^\dagger, \quad U \in U(3).$$

The channel is unchanged under $U \mapsto e^{i\theta}U$, so its physical transport group is

$$PU(3) = U(3)/U(1) \simeq SU(3)/Z_3,$$

with Lie algebra $\mathfrak{su}(3)$. This fixes the local connection algebra and its adjoint representation. It does not choose the global lift of the projective channel to endpoint amplitudes. A fundamental $SU(3)$ endpoint and Z_3 triality require such a lift together with an identification of the central $U(1)$ phase; those are part of the open matter-sector completion.

The endpoint observable algebra provides the same result in a complementary form. A route-resolving endpoint supplies $P_A = |A\rangle\langle A|$; tetrahedral permutations generate P_B and P_C . If S_{ab} carries route b to route a , then

$$P_a S_{ab} P_b = |a\rangle\langle b| \equiv E_{ab}.$$

The nine matrix units span $M_3(\mathbb{C}) = \mathbf{1} \oplus \mathbf{8}$. The identity is the common phase direction and the traceless Hermitian operators are the eight connection directions. The route endpoint is physically resolved, while its basis in transit is gauge covariant. This closes the earlier gap between a discrete S_3 triple and a continuous gauge algebra; S_3 alone would generate only $\mathbb{C} \oplus M_2(\mathbb{C})$.

Choosing route bases independently at the cells gives the link transformation

$$U_{yx} \mapsto V_y U_{yx} V_x^\dagger.$$

Around a loop based at x , $U_p = \prod_{\ell \in p} U_\ell$ therefore transforms by conjugation, $U_p \mapsto V_x U_p V_x^\dagger$. This is the local gauge redundancy of the transported route frame.

Adjoint Wilson form from channel-return fidelity. The transported object fixed above is a channel on route density operators, so its return cost must be unchanged by $U_p \mapsto e^{i\theta} U_p$. The difference of two normalized route projectors is traceless and Hermitian; on that irreducible adjoint space, the positive frame-invariant quadratic form is unique up to an overall scale. The scale is absorbed into κ_{open} . For a normalized route state q , set $P_q = |q\rangle\langle q|$ and define the resulting Hilbert–Schmidt channel-return failure

$$\Delta_p(P_q) = \left\| P_q - U_p P_q U_p^\dagger \right\|_{\text{HS}}^2 = 2 \left(1 - |\langle q | U_p | q \rangle|^2 \right).$$

For an unbiased qutrit, the Haar second moment gives

$$\overline{|\langle q | U_p | q \rangle|^2} = \frac{\text{Tr}(U_p U_p^\dagger) + |\text{Tr } U_p|^2}{3(3+1)} = \frac{3 + |\text{Tr } U_p|^2}{12}.$$

Using $\chi_{\mathbf{8}}(U) = |\text{Tr}_{\mathbf{3}} U|^2 - 1$,

$$\overline{\Delta_p} = \frac{3}{2} \left(1 - \frac{|\text{Tr}_{\mathbf{3}} U_p|^2}{9} \right) = \frac{4}{3} \left(1 - \frac{1}{8} \chi_{\mathbf{8}}(U_p) \right).$$

A cost κ_{open} per unit return failure therefore gives the adjoint Wilson form [6],

$$\boxed{S_{\text{gauge}} = \beta_{\text{adj}} \sum_p \left(1 - \frac{1}{8} \text{Tr}_{\mathbf{8}} \text{Ad } U_p \right), \quad \beta_{\text{adj}} = \frac{4}{3} \kappa_{\text{open}}.}$$

This action is well defined on $PU(3)$ and is blind to the center, exactly as the channel construction requires. A fundamental-trace Wilson term would require the unresolved $SU(3)$ endpoint lift. The group argument alone does not fix κ_{open} .

Minimal primitive kinetic normalization. The primitive recoupling matrix fixes the missing local coefficient after one named microscopic choice: geometric gluing uses the identity pairing on the primitive boundary Hilbert space before an independent kinetic operator is added. In the nonorthogonal route basis, the identity pairing has matrix G . A direct branch $K = sG$ admits a trace-preserving reversible dilation only when $K^\dagger K \leq I$. Maximum faithful throughput saturates the largest eigenvalue, so

$$s = \frac{20}{21}, \quad K = \frac{20}{21}G = P_2 + \frac{6}{7}P_1, \quad L = \sqrt{I - K^\dagger K} = \frac{\sqrt{13}}{7}P_1.$$

Together, K and L define an isometry and therefore admit a reversible unitary dilation. Only the symmetric closure component enters the complementary branch; the primitive imbalance doublet has unit direct amplitude.

Transport through the vacuum carries no preferred absolute route frame. Replacing P_1 by an arbitrary rank-one projector $P = |n\rangle\langle n|$ and averaging the equivalent frames gives

$$K_P = I - \frac{1}{7}P, \quad L_P = \frac{\sqrt{13}}{7}P.$$

The qutrit Haar identities

$$\int dP P = \frac{I}{3}, \quad \int dP PXP = \frac{X + \text{Tr}(X)I}{12}$$

then yield the covariant channel

$$\begin{aligned} \mathcal{E}(X) &= \int dP (K_P X K_P^\dagger + L_P X L_P^\dagger) \\ &= \frac{13}{14}X + \frac{1}{42}\text{Tr}(X)I = 1_1 \oplus \frac{13}{14}I_8. \end{aligned}$$

Thus

$$\boxed{t_8 = \frac{13}{14}}, \quad -\ln t_8 = \ln \frac{14}{13}.$$

For generators normalized by $\text{Tr}(T^a T^b) = \delta^{ab}/2$, one primitive transfer interval τ_r has the isotropic adjoint generator

$$\mathcal{L}(X) = -\frac{\ln(14/13)}{3\tau_r} \sum_{a=1}^8 [T^a, [T^a, X]].$$

In the heat-kernel coordinate $t_R = \exp[-g_{\text{HK}}^2 C_R/2]$ with $C_A = 3$,

$$\boxed{g_{\text{HK,step}}^2 = \frac{2}{3} \ln \frac{14}{13} = 0.04940531477.}$$

This value was obtained from the primitive geometry without a QCD target. It is conditional on the identity kinetic pairing, maximum faithful throughput, and route-frame isotropy.

Causal blocking on the diamond regulator. The paper already uses the four-valent diamond graph as the embedded adjacency realization of the tetrahedral bond frame. Its minimal product causal completion is $D_{\text{diamond}} \times \mathbb{Z}$, with one temporal successor at each spatial site and no diagonal shortcuts. Let d be the spatial bond length, a_t the temporal spacing, $r = a_t/d$, and let ρ be the ratio of temporal to spatial primitive heat times. The shortest mixed time-space

loops have four links; the shortest spatial loops are diamond hexagons with six links. Their per-site oriented area tensors are

$$M_{\text{E}}^{ab} = \frac{2}{3}r^2 d^4 \delta^{ab}, \quad M_{\text{M}}^{ab} = \frac{32}{9}d^4 \delta^{ab}.$$

The diamond primitive translations $a_1 = d(n_1 - n_2)$, $a_2 = d(n_1 - n_3)$, $a_3 = d(n_1 - n_4)$ have determinant $16d^3/(3\sqrt{3})$ and contain two sites, fixing the embedded volume per site to

$$V_s = \frac{8}{3\sqrt{3}}d^3.$$

Matching the electric and magnetic area sums to one isotropic continuum action gives

$$g_{\text{E}}^2 = \frac{8\sqrt{3}}{3r}(1 + \rho)t_s, \quad g_{\text{M}}^2 = \frac{3\sqrt{3}}{2}rt_s,$$

where t_s is the primitive spatial heat time. The condition $g_{\text{E}}^2 = g_{\text{M}}^2$ fixes

$$\frac{a_t}{d} = \frac{4}{3}\sqrt{1 + \rho}, \quad g_{\text{iso}}^2 = 2\sqrt{3(1 + \rho)}t_s.$$

The equal-transfer completion $\rho = 1$ treats spatial route transfer and present/history transfer as uses of the same primitive instrument. It gives

$$\boxed{\frac{a_t}{d} = \frac{4\sqrt{2}}{3}}, \quad \boxed{\frac{g_{\text{iso}}^2}{g_{\text{HK,step}}^2} = 2\sqrt{6}}.$$

The corresponding projective pure-gauge measure is

$$Z_{\text{gauge}} = \int [dU] \prod_{p_{\text{E}}} K_{4t_s}(U_{p_{\text{E}}}) \prod_{p_{\text{M}}} K_{6t_s}(U_{p_{\text{M}}}),$$

and its graph heat-kernel coordinate is

$$\boxed{g_{\text{graph,HK}}^2 = 2\sqrt{6}g_{\text{HK,step}}^2 = \frac{4\sqrt{6}}{3} \ln \frac{14}{13} = 0.24203562353.}$$

Here $U_\ell \in PU(3)$ and K_t is the heat kernel for the projective group; locally it has the same $\mathfrak{su}(3)$ Casimir normalization used above. This is a graph heat-kernel coordinate, not an $\overline{\text{MS}}$ coupling. Conversion to a Wilson or continuum scheme requires the corresponding class integral and finite matching; neither is supplied by the blocking identity. The dimensionless result also does not identify the embedded bond length d with the electron-calibrated L_* ; that geometric dictionary belongs to the full GFT embedding.

Color and generation factorize. The shell and open-route spaces are independent in the minimal colored-defect extension,

$$\mathcal{H}_{\text{defect}} = \mathcal{H}_{\text{shell}} \otimes \mathbb{C}_{\text{route}}^3, \quad K_{\text{total}}^2 = K_{\text{shell}}^2 \otimes I_3, \quad T_{\text{color}}^a = I_{\text{shell}} \otimes T^a.$$

It follows that $[K_{\text{total}}^2, T_{\text{color}}^a] = 0$. The shell cutoff is also alphabet-independent: every one- and two-label exclusion leaves a nondegenerate closure branch, while every three-label exclusion makes K^2 constant. A colored realization would therefore carry three route components at each of the same three shell depths. The factor of three is gauge multiplicity and is not an additive $\ln 3$ contribution to an isolated particle mass; physical states require their color-flux dressing and singlet constraints.

Status of the color result. The persistent-open-route identification is the conditional physical premise. Given it, reference-preserving reversible transport fixes the $PU(3)$ channel group, $\mathfrak{su}(3)$ connection algebra, and eight adjoint directions; phase-invariant channel-return fidelity fixes the adjoint Wilson form. The identity primitive kinetic pairing, maximum-throughput rule, and route-frame isotropy fix $t_8 = 13/14$. The embedded-diamond, equal-transfer causal completion fixes the dimensionless blocking factor $2\sqrt{6}$ and places the microscopic projective pure-gauge branch in a weak-coupling regime.

Physical QCD requires further structure that this derivation does not contain: a global $SU(3)$ endpoint lift and central $U(1)$ identification, triality-carrying chiral quark defect classes, two weak species per generation, hypercharge assignments, a lattice fermion action, finite graph-to- $\overline{\text{MS}}$ matching, and nonperturbative scale setting. The projective pure-gauge action does not prove confinement or predict a hadron mass. The result is a conditionally closed microscopic local pure-gauge sector, not a derivation of the Standard Model matter sector.

Hadronic matter and the mass–entropy bridge. The mass–entropy bridge $m = \kappa_m(\ell) \Delta S$ applies to fermionic defects regardless of their color label. For charged leptons, the defect is color-singlet and the marked shell action of Appendix I.1 gives both measured ratios within one standard deviation. For hadrons, the dressed entropy is dominated by the QCD-internal contributions of confinement-scale gluonic flux, trace-anomaly structure, chiral vacuum reorganization, and quark binding. The mass–entropy bridge then organizes the full dressed bound-state entropy budget,

$$S_{\text{ent,H}}^{\text{dressed}} = S_{\text{defect}} + S_{\text{bind}} + S_{\text{conf}} + S_{\chi\text{SB}},$$

without requiring the framework to re-derive QCD. The structural claim is compatibility: the dressed entropy budget is the quantity QCD calculates, and the mass–entropy bridge converts it to inertial mass through the same running $\kappa_m(\ell)$ used for elementary sectors.

Scope summary. The gauge sectors have distinct grades. The baseline-redundancy template supplies a natural Abelian hosting mechanism. Primitive spin data supply an $SU(2)$ representation, while its identification with the chiral weak group remains external. The persistent-open-route premise gives a conditional microscopic $PU(3)$ projective pure-gauge sector with a fixed minimal adjoint transfer channel and causal regulator. Its global $SU(3)$ lift, the Standard Model quotient, chiral representations, matter multiplets, and renormalized couplings have not been derived. The gravitational and charged-lepton results do not depend on this color extension.

The sectors in this appendix are structurally linked to the same entanglement logic but are not part of the controlled ordinary Einstein/capacity branch.

Appendix J: Lattice Microstructure Calculations

Appendix J records the technical content of Section 24: the host ensemble and the checks that its geometry remains connected, simplicial, foliated, thermalized, and volume-matched; the coupled measure and its declared modeling forks; the control methodology; the exact cell-order calculations; and the measured compatibility, defect, and capacity-transport results. It then states what a continuum critical-surface calculation would have to establish. Where Appendix K recomputes the coefficient chain from its definitions, this appendix couples that chain to a dynamical substrate and reports what the finite-lattice system does. The report, analyzer outputs, run configurations, verification scripts, and raw tables are maintained with the manuscript materials at jaigp.org; the report is generated from the archived CSV and log files rather than transcribed by hand.

J.1 Host ensemble and engine discipline

The host is a standard causal-dynamical-triangulations ensemble [80, 81]: triangulations of $S^1 \times S^3$ sampled by local Pachner-class moves under the Regge action, with the $(4, 1)$ -simplex number pinned by a quadratic penalty. Early demonstration calculations used eighty spatial slices near $(k_0, \Delta) = (2.2, 0.6)$. The largest-volume calculations use the directly mapped point $(2.0, 0.4)$ and auto-tune k_4 separately for the 40,000 and 100,000 volumes and for the forty- and eighty-slice realizations; the final values are 0.7560, 0.7608, and 0.7700, respectively. The action is tracked incrementally and checked against full recomputation throughout every run, and every quoted final triangulation is verified to be a connected simplicial manifold with the required foliation.

The coupling scan maps finite-volume behavior, not Einstein-phase membership. A (k_0, Δ) grid with k_4 auto-tuned to pseudo-criticality at every point — the volume-pin residual serving as the feedback error, as in standard CDT practice where k_4 is retuned per coupling point — finds an extended region at $k_0 = 1.0$ – 2.0 , with slice-volume coherence peaking at $(k_0, \Delta) = (2.0, 0.4)$ and a dead row by $k_0 = 3.5$: a visible boundary. A fixed $k_4 = 0.9$, which an untuned grid had used throughout, was the obstruction — every untuned cell was fighting the pin rather than sampling its phase. Before the largest-volume data were examined, the expected finite-volume trend at the selected couplings was stated explicitly: both the Hausdorff estimator and the concentration of volume in an extended blob should rise with volume. They do. The $N_{41} \simeq 42,112$ ensemble gives $d_H = 3.31$, blob score 1.46, and 23 active slices; the $N_{41} \simeq 102,002$ ensemble gives $d_H = 3.56$, blob score 1.57, and 24 active slices. This establishes an extended finite-volume compatibility region. It does not distinguish the de Sitter phase from a nearby bifurcation phase, determine the order of the visible boundary, establish a continuum trajectory, or demonstrate a massless TT sector.

Validation of the cell identification in Section 24.1 exposed a defect in the original move set. The moves preserved the four-dimensional manifold conditions but could fold a spatial slice, leaving a few percent of slice triangles in violation of the required closed-3-manifold condition. The corrected engine rejects every proposal that would break the foliation. Its local test is complete because the link of an affected triangle necessarily passes through a newly created pentachoron, and it preserves detailed balance in the same way as the other locality filters. Every ensemble reported here uses this engine and passes the global audit: each slice triangle belongs to exactly two spatial tetrahedra, and every pentachoron is correctly threaded between its slices.

No result below infers more from that comparison than finite-volume host suitability. In particular, the point $(2.0, 0.4)$ is used only as a compatibility test for switching on the capacity weighting; it is not used to prove (H_{phase}) . Distinguishing the de Sitter phase from a nearby extended bifurcation phase would require the standard alternating-slice curvature and singular-vertex order parameters, but that classification is not an antecedent of the analytic local-stability result in H.10. Every substrate claim compares ensembles at the same regulator: coupled against bare, failure against closed control, or one source strength against another. Establishing a continuous geometric transition and taking the decorated common-parent continuum limit are the separate tasks of J.9.

J.2 The coupled measure and its fidelity anchors

The coupled measure is the joint Gibbs distribution of Section 24.2, with the per-face label sum normalized so that each label carries base measure $1/7$. Under that base measure two properties hold exactly: at $\beta = 0$ the label entropy cancels and the geometry marginal is identically the bare host, so the uncoupled ensemble reproduces the host by construction; and uniform label assignment at geometry moves is exactly detailed-balanced, so no proposal correction is needed

for label births. The centering constant μ is the per-cell label free energy,

$$\mu(\beta) = -\frac{1}{\beta N_{\text{cells}}} \ln \mathbb{E}_{m \sim \text{unif}} \left[e^{-\beta \sum_c E_c} \right] = \frac{1}{\beta} \int_0^\beta \langle \bar{E} \rangle_s ds,$$

computed by thermodynamic integration on the thermalized base configuration and logged with each ensemble, so the extensive part of the closure term is neutralized rather than allowed to masquerade as a host coupling. Total volume is nevertheless checked directly and matched among comparisons rather than assumed from the centering. Equivalently, the normalized factor is $\hat{Z}_{\text{cap}} = e^{\beta \mu N_{\text{cells}}} Z_{\text{cap}}$. It is exactly invariant under a common energy-origin shift $E_c \mapsto E_c + C$, $\mu \mapsto \mu + C$, which is the finite-regulator identity used in Section 25. Because $N_{\text{cells}} = N_{41}/2$, the omitted centering is exactly an N_{41} source. In the standard (κ_4, Δ) action basis this means $\delta\kappa_4 = -\beta\mu/2$ and $\delta\Delta = +\beta\mu/2$, not a pure k_4 displacement; Section 25 separates its physical volume component from the regulator-asymmetry component.

Every run opens by rebuilding the single-cell tables and checking the Part II anchors ($\Omega_{\text{tet}} = 1680$, $\eta_* = 0.0298668$, $\langle K^2 \rangle_{\eta_*} = 50.223$, $g_{\text{share,eff}} = 7.4198$), and refuses to sample if any fails.

Three modeling forks are declared. Injectivity is imposed as a penalty λ per colliding pair ($\lambda = 3$ in the quoted runs) because a hard seven-color constraint at the maximal conflict degree does not guarantee an ergodic single-face heat bath; the hard constraint is the $\lambda \rightarrow \infty$ limit, the residual collision fraction is reported in every table so the softness stays visible, and the physical $\beta = 1$ weighting is read at the ordered end. The closure sum is evaluated in the unoriented reading, in which the two cells sharing a face see the same label; the oriented alternative, in which they see opposite signs, requires oriented-slice bookkeeping through all moves and is recorded as an open fork. Parity contributes exactly $\ln 2$ per cell, extensive at pinned volume, and drops out of the coupled dynamics.

J.3 Controls and the reading discipline

Four validity conditions precede any physical reading. Every quoted triangulation must remain a connected simplicial manifold with the required foliation; its action and volume time series must be stationary over the measured window; compared ensembles must have matched total-simplex number, because volume differences can masquerade as dimension differences; and the Markov chain must continue moving, because a frozen chain is a failed calculation rather than a result. The interpretation of each possible outcome, including which outcomes would license no claim at all, was written before the corresponding data were examined.

The main control shuffles the closure energies. The closure energy takes 210 distinct values on the sorted-label orbits of the cell; the control reassigns those values among the orbits, preserving the value pool and permutation symmetry while destroying the closure structure, and is evolved at the same $\beta = 1$ coupling. Only a difference from this matched shuffled control is attributed to closure; anything it reproduces is sampling machinery.

An earlier EPRL calculation exposed the need for this control. A vertex amplitude [109] at frozen boundary spin $j = 3$, computed with the `sl2cfoam-next` library [110], was coupled to the same host. An analysis specified before reading the data found five misspecifications: off-equilibrium centering shifted the bare cosmological coupling; the label heat bath used the wrong exponent; label births lay outside the acceptance ratio; the result depended at order unity on an arbitrary slot convention; and sampling required a positivization of the sign-alternating amplitude. Every effect vanished at $\beta = 0$ and grew with β , so an uncoupled comparison could not reveal them. An entry-shuffled vertex tensor reproduced the apparent geometric steer at matched coupling and identified it as sampler machinery. The sampler was rebuilt as an exact joint Gibbs measure, and the later closure calculations include the same shuffled-energy control. This episode is reported

as methodology. The frozen-spin truncation and positivization already limit its physics scope, and the EPRL intertwiner space is distinct from the seven face labels of the present ensemble (Section 29.8).

J.4 Exact results at cell order

Six calculations, independent of any Monte Carlo run, fix what the substrate can and cannot do for the vacuum before the coupled measurements are read.

Induced curvature coupling. The geometry marginal of the coupled measure weights a slice by the free energy of its labels, and the leading geometric dependence enters through the coordination q of slice edges, the discrete curvature variable. A transfer-matrix evaluation of the label free energy on the ring of cells around an edge gives, at the physical $\beta = 1$ weighting, an induced coupling $c_0 = +0.019$ per slice edge, against the bare $k_0 \simeq 2.2$: about one percent. The label correlation length is about half a lattice step, and the beyond-quadratic dependence on q is negligible at physical coordinations. The closure weighting alone therefore cannot supply the host's geometric stiffness, and its predicted effect on the coupled sweep is a steer too small to see at single-seed resolution, a prediction the calculation of J.5 confirms.

The refresh at cell order. Modeling the selected memoryless kernel as whole-boundary redraws arriving at unit rate per cell, the single-cell stationary state is exact: the probability that k faces retain their last self-drawn values is $k!(4-k)!/120$, vacuum admissibility is 0.536, and $\langle K^2 \rangle = 48.74$ against the ensemble value 50.22. The state is geometry-blind to exponential accuracy and induces no vacuum curvature action. It does couple geometry updates to the local density of closure failure when their rates depend on that density. This calculation tests the effective refresh model, not a derivation of its microscopic operator.

The history-weighting ceiling. Two computations bound the strongest vacuum role a Many-Pasts tilt over closure-class observables could play. Amplifying the closure link interaction to 128 times the physical $\beta = 1$ weighting saturates the link-agreement ratio near 0.55 with correlation length 1.7 links and a curvature-sector signal near 0.1: a structural ceiling, set by the ground-state degeneracy of the closure-plus-injectivity interaction, twenty times below the bare coupling. Independently, the exact Doob-transform solution of a closure-maintaining history tilt (paths weighted by time-integrated closure quality) on a ring of six seven-state faces drives the nearest-neighbor label correlation from -0.12 through zero to only $+0.13$ across the full tilt range, with longer-range correlations near 0.03 throughout: no long-range order at any tilt strength. Together with the two paragraphs above this closes the label sector: static weight, refresh, and closure-class history weighting each fail, quantitatively, to supply vacuum geometric stiffness at cell order.

The free conserved field, integrated out. A conserved Gaussian field back-coupled to the geometry induces, on integration, the geometric action $\frac{1}{2} \ln \det' L(g)$ per channel and slice: the spanning-tree entropy of the slice graph, by the matrix-tree theorem. Computed on degree-matched graph classes, the entropy per cell is 0.607–0.613 on diamond-cubic lattices (the smooth extended proxy), 0.609 on random 4-regular graphs (the crumpled proxy), and 0.56–0.57 on real engine slices: nearly universal at fixed coordination, with the genuinely nonlocal soft-mode part diluting as $\ln N/N$ per cell. The per-cell differential between extended and crumpled geometry is a few times 10^{-4} ; with all seven channels the phase-tipping force is of order 10^{-3} against $k_0 \simeq 2.2$.

The budget-bounded field. The finite budget of Postulate I makes the field non-Gaussian, with saturation curvature $m^2 = 2/g_{\text{share,eff}} \simeq 0.27$, and the massive determinant differential between the same proxies shrinks from 1.2×10^{-3} per cell at zero mass to 4×10^{-4} at the budget mass and toward zero beyond: the theory’s own non-Gaussianity suppresses the modes where the sensitivity lived. What survives, measured on eight engine slices, is a local effective-action shift of scale $\simeq 0.08$ per cell in the seven-channel count. This number is not itself a displacement in Δ , or a norm in the (k_0, Δ) plane; obtaining that displacement requires projection onto the CDT conjugate operators. The result shows perturbative dressing, not the creation or global preservation of a phase. The two containers no branch covers are named in Section 24.3: long-range field kernels and non-equilibrium geometry–field co-evolution, both unspecified by the theory.

The fracture construction. Under unit label shifts preserving injectivity, the 1680-state move graph has 48 connected components of 35 states, labeled by slot ordering and parity. This proves that the tested move class cannot equilibrate the full admissibility ensemble. More general microscopic dynamics are not excluded.

J.5 Compatibility at matched volume

The closure comparison starts from common tuned geometries at $N_{41} \simeq 100,000$. Three eighty-slice ensembles use respectively the uncoupled value $\beta = 0$, the physical weighting $\beta = 1$, and $\beta = 1$ after the closure energies have been shuffled among label orbits. The physical weighting is also repeated with forty slices. In every case the final triangulation is connected, simplicial, correctly foliated, and stationary over the measurement window. Across the three eighty-slice ensembles the total-simplex spread is 0.6%.

ensemble	N_4	N_{41}	collision fraction	d_H	centering μ
$\beta = 0$ (80-slice control)	323,514	100,216	0.654	3.55	3.9913
$\beta = 1$ (80-slice physical)	323,232	99,998	0.077	3.59	2.4990
$\beta = 1$ (80-slice shuffled)	325,346	99,944	0.736	3.59	3.1420
$\beta = 1$ (40-slice repeat)	316,238	100,020	0.078	3.74	2.4983

The uncoupled ensemble remains on the exact uniform-measure benchmark $1 - 840/2401 = 0.650$ within sampling error. The physical weighting suppresses collisions by nearly an order of magnitude, and the shuffled control remains high despite carrying the same energy-value pool. At the same time the matched eighty-slice d_H values occupy the narrow interval 3.55–3.59, their extended volume profiles persist, and the forty-slice repetition reproduces the ordered collision fraction. Thus the microscopic closure structure is strongly active while the host geometry is stable at this resolution. This supersedes the earlier 20,000-volume preview; its conclusion is unchanged and its finite-volume caveat is smaller.

J.6 The Regge bridge: a predicted volume-sector dressing

The bridge is an ordinary statistical-mechanical statement made quantitative. Integrating the labels out at fixed geometry defines an effective action for the host,

$$e^{-S_{\text{eff}}[g]} = e^{-S_{\text{host}}[g]} Z_{\text{label}}[g], \quad S_{\text{eff}}[g] = S_{\text{host}}[g] - \log Z_{\text{label}}[g],$$

with S_{host} the Regge action plus the volume pin. The label free energy $-\log Z_{\text{label}}[g]$ is a function of the geometry alone. Its extensive part is $\beta\mu(\beta) N_{\text{cells}}$, with μ the per-cell label free energy of J.2; its geometry-dependent remainder is the induced curvature-sector action priced by the ring expansion of J.4. The centered ensemble subtracts the extensive part by construction; left in, it is the leading coefficient of the effective action, and the host’s response to it can be predicted before it is measured.

The centering constant μ of J.2 therefore gives a direct prediction. In the centered ensemble, the subtraction of μ removes the extensive label free energy from the geometry marginal. If that subtraction is omitted, the geometry sees a residual positive linear contribution $+\beta\mu N_{\text{cells}}$ to the effective action. Here N_{41} denotes the total number of (4, 1) and (1, 4) pentachora, so each spatial cell, shared by one pentachoron of each orientation, is counted once by the pair and $N_{\text{cells}} = N_{41}/2$. The positive linear tilt competes with the quadratic volume pin $\varepsilon(N_{41} - \bar{N})^2$, and the shift follows in two lines. The volume-dependent part of the effective action is

$$S_{\text{vol}}(N_{41}) = \varepsilon (N_{41} - \bar{N})^2 + \beta\mu N_{\text{cells}} = \varepsilon (N_{41} - \bar{N})^2 + \frac{\beta\mu}{2} N_{41},$$

and setting $dS_{\text{vol}}/dN_{41} = 2\varepsilon(N_{41} - \bar{N}) + \beta\mu/2 = 0$ gives the equilibrium displacement

$$\Delta N_{41} = N_{41}^{\text{eq}} - \bar{N} = -\frac{\beta\mu}{4\varepsilon}.$$

Here μ is computed by annealed thermodynamic integration of the label ensemble on the base geometry, before any coupled dynamics run; the zero-lattice reference — the exact single-cell free energy from direct enumeration of the 7^4 label states, computable from η_* and the injectivity penalty alone — is 2.477 at this point, within half a percent of the lattice value, the difference being the shared-face correlations. At the demonstration point ($\beta = 1$, $\mu = 2.488$, $\varepsilon = 0.01$) the prediction is $\Delta N_{41} = -2.488/0.04 = -62.2$. Three ensembles from a common equilibrated base at $N_{41} = 2000$ measure it:

ensemble	mean N_{41}	shortfall
bare ($\beta = 0$)	1993.2	−6.8
centered ($\beta = 1$)	1969.8	−30.2
uncentered ($\beta = 1$)	1907.0	−93.0

The uncentered-minus-centered difference is -62.8 against the predicted -62.2 . The subtraction isolates the μ -term, since the two $\beta = 1$ ensembles share every other piece of machinery; the centered ensemble's own -30 is the second-order non-neutrality of the thermodynamic-integration centering. It cancels in the difference and is kept out of other comparisons by matching their volumes directly. A number computed from the ultraviolet specification thus appears, at the predicted magnitude, as an extensive N_{41} contribution to the dynamical host action. This is a dressing of the host, not a derivation of its continuum limit. In the standard CDT basis it is a correlated (κ_4, Δ) displacement; only projection with the two-simplex Regge volume operator identifies its physical volume component. The measurement is at demonstration volume and uses one seed. The corresponding curvature-sector measurement remains open.

The demonstration point is one member of a predicted family whose functional form was recorded before sampling. At fixed geometry-side settings the displacement obeys $\Delta N_{41}(\beta, \varepsilon) = -\beta\mu(\beta)/(4\varepsilon)$, and each factor is separately testable. In β the curve is concave, because $\beta\mu(\beta) = \int_0^\beta \langle \bar{E} \rangle_s ds$ and the Gibbs mean falls as the labels order, with local slope $d(\Delta N_{41})/d\beta = -\langle \bar{E} \rangle_\beta/(4\varepsilon)$ — an Ehrenfest-type relation in which the same calculation supplies both sides, since $\langle \bar{E} \rangle_\beta$ is measurable in the ensemble whose displacement it predicts. In ε the law is exactly inverse at fixed β ; departures from $1/\varepsilon$ measure the curvature of the background free energy that the matched-pair subtraction cancels at first order, so the ε family doubles as a control on the derivation's one assumption. The shuffled table carries its own computable free energy: at $\beta = 1$ the single-cell values are $\mu_{\text{shuf}} = 3.17$ against $\mu = 2.48$, so the shuffled ensemble is predicted to displace by $-\beta\mu_{\text{shuf}}(\beta)/(4\varepsilon)$ — not by zero, and not by the physical value, a 28% separation. The measured number must track the table from which it was computed, so the shuffled calculation is a second quantitative prediction rather than merely a null control.

The family has been measured at demonstration volume, with every prediction written to disk before sampling began. Eight of nine planned coupling–volume–penalty combinations produced

readable data; the ninth failed at launch and contributes no datum. Across a fifteen-fold range of predicted displacements, from -10.5 to -157.6 , the measured-to-predicted ratios are 0.96 – 1.07 with one point at 0.89 . The β -curve is concave as required: -43 , -78 , -121 measured against -43 , -76 , -125 predicted — not the straight line that would have falsified the free-energy integral. Halving ε doubles the displacement to within four percent. The shuffled pair lands on its own line, displaced thirty percent from the physical-table results and at ratio 1.00 to the prediction computed from its shuffled table. A repetition of this separate bridge calculation at the largest volume has not yet been performed.

J.7 The pinned-defect experiment

Two independent matched comparisons resume from thermalized $N_{41} \simeq 100,000$ ensembles with the physical $\beta = 1$ weighting and run 3000 further sweeps. For each random seed, one ensemble pins one hundred cells at maximal closure failure (all four faces $m = 0$: six collisions, $K^2 = 48$), while the control pins one hundred cells at the best-closed injective configuration ($m = \{0, 1, 2, 3\}$, $K^2 = 40.67$). Pinned faces are excluded from the label heat bath, and the carrier cells are protected from removal by the geometry moves identically. Every final triangulation remains connected and correctly foliated, and every ensemble retains all one hundred pins. The final analyzer pools the two seeds. When an observable-shell cell is present in both seeds its uncertainty is set by across-seed scatter; otherwise it uses the Sokal-windowed integrated-autocorrelation standard error. The largest measured integrated autocorrelation time is 349.5 measurement intervals.

condition	shell	mean E	collision fraction	mean coordination	cells per shell
failure	1	1.6802 ± 0.0026	0.0698 ± 0.0009	5.8452 ± 0.0024	4
	2	1.7299 ± 0.0006	0.0783 ± 0.0002	6.0869 ± 0.0281	7.7289 ± 0.2255
	3	1.7269 ± 0.0002	0.0786 ± 0.0001	6.1986 ± 0.0097	12.7960 ± 0.4518
closed control	1	1.7260 ± 0.0008	0.0783 ± 0.0002	5.8429 ± 0.0146	4
	2	1.7288 ± 0.0003	0.0788 ± 0.0001	6.0816 ± 0.0204	7.8633 ± 0.2143
	3	1.7277 ± 0.0002	0.0787 ± 0.0001	6.2384 ± 0.0022	12.5920 ± 0.4737

The failure-minus-closed-control differences make the reading unambiguous. At shell one, the mean closure energy changes by -0.0458 (16.8σ) and the collision fraction by -0.0085 (9.2σ). At shell two their separations are 1.7σ and 2.1σ , respectively. At shell three the mean coordination changes by -0.0398 (4.0σ), while its first- and second-shell separations are only 0.2σ . Shell-cell counts separate by at most 0.4σ anywhere in the measured range. Thus persistent failure produces a replicated, short-range capacity disturbance and a statistically resolved local coordination response. The earlier preview’s shell-volume signal does not survive the larger-volume autocorrelation and seed treatment and is not retained as a result.

J.8 The conservation law and the transport instrument

The dichotomy of Section 24.7 in operator form. If free capacity is a per-cell budget re-equilibrated by maximum entropy — link shares $c_l \geq 0$ with $\sum_{l \ni x} c_l = C - m_x$ — stationarity gives one multiplier per cell, and the linearized constraint reads $(2z\mathcal{L} - L)\delta\mu = -m/\chi$ with $z = 4$ and L the slice Laplacian. The operator has no small-momentum pole; on a measured 169-cell slice its Green function falls eight orders of magnitude within thirteen steps.

The transport instrument tests a different object. Its pinned commitment level m_x is a fixed source label, while Q_x is the maintenance throughput the source draws. On a closed slice the simulated free field obeys

$$f_x(t + dt) - f_x(t) = - \sum_{\text{faces } xy} J_{xy} - Q_x + \bar{Q}, \quad J_{xy} = -J_{yx}, \quad \sum_x f_x = \text{const},$$

where $\bar{Q}$ is the uniform return required by the compact zero mode. This is steady service, not $f \rightarrow m$ stock conversion. Any local flux law linearizes to $J \propto -D\nabla f$ and the steady state solves $L\delta f = (Q/D)(\delta_{\text{source}} - \text{uniform return})$: the massless graph-Poisson equation, whose Green function on a three-dimensional slice falls as $1/r$. Section 24.7 derives the corresponding incidence-matrix Ward identity and identifies the continuum junction coefficient still to be measured.

The instrument implements the conservation reading with the sink left to emerge. A field on the slice cells is initialized at the vacuum anchor $g_{\text{share,eff}} = 7.4198$ and transported by antisymmetric face fluxes; the code asserts conservation at machine precision every sweep and aborts on any violation. The pin levels are fixed source labels, so the conserved quantity during this test is the free-field sum $\sum_x f_x$. Absorption is proportional to a cell's *persistent excess* closure failure above the measured vacuum baseline. A pinned defect therefore draws maintenance only through the failure generated by the ordinary label dynamics: its flux is measured, not set. Pins are placed at the five source values $m = 0, 1, 2, 3, 6$. The zero value measures commitment-independent boundary dressing; the positive values test source additivity and the deficit per unit emergent charge. At large distance the calculation tests whether the response follows a graph-Coulomb power law and whether a nonzero screening mass is detected. This first implementation freezes geometry while labels and capacity remain dynamical, isolating the transport law; conservative transport through geometry moves is the specified extension.

On small validation volumes the measured deficit increases with commitment, recovers monotonically with distance, and carries an emergent flux that rises with source value while the total field remains exactly conserved. The highest-commitment pin draws less flux than the linear extrapolation because it depletes its own neighborhood, a self-screening the model produces without being told.

Two failed definitions are retained as diagnostics rather than results. Absorption on total failure makes the vacuum itself a sink and produces the predicted four-to-seven-step screening. Absorption on instantaneous excess rectifies the vacuum's mostly-zero collision process and also creates a screening mass. Both failures follow quantitatively from their own update laws. The adopted definition therefore uses a 200-sweep persistence average, equilibrates the field before measurement, and subtracts the same-slice far-field reference to remove the compact zero mode. These analysis choices were fixed before the largest-volume data were generated.

The transport calculation uses an ordered forty-slice realization at $N_{41} \simeq 100,000$. It places thirty-two pins at each of the five source values, measures through shell eight, and runs 4000 sweeps. The mean field remains 7.4198 throughout. Per-pin time series are corrected with Sokal integrated autocorrelation times, and the level errors use across-pin scatter; the profile and junction errors additionally use the larger of propagated and across-level scatter. The finite-lattice results are

$$Q - Q(0) = (0.1440 \pm 0.0025)m, \quad Q(0) = 0.0075,$$

$$\left\langle \frac{\Delta f_1 - \Delta f_1(0)}{Q - Q(0)} \right\rangle_{m>0} = 4.9965 \pm 0.5244,$$

and the normalized dressing-subtracted shell profile is

d	1	2	3	4	5	6	7	8
$\Delta f(d)/\Delta f(1)$	1.000	0.822	0.675	0.532	0.411	0.303	0.212	0.144.

The power-law fit gives -0.62 ± 0.11 . The screening derivative is $+0.1062 \pm 0.0551$, statistically consistent with zero; the corresponding finite-range lower bound is $\xi > 4.6$ steps. The source fit is not perfectly linear: its maximum relative residual is 0.242, dominated by visible saturation

at $m = 6$. The tail is also shallower than the ideal d^{-1} ; the compact zero mode and finite slice radius flatten it, and the continuum scaling study must test convergence. Subject to those explicit diagnostics, the finite-lattice calculation demonstrates conservation, approximate source additivity, one junction ratio across the tested source values, power-law propagation, and no detected screening.

J.9 Capacity-decorated CDT: phase membership and the continuum test

These measurements establish the finite-lattice source-to-field behavior listed above. A microscopic CDT completion must still promote the capacity data to part of the joint transfer state. Because L_* is a physical microscopic length, existence of the effective theory does not require a second-order point at which $a \rightarrow 0$. Its necessary geometric test is instead membership in an extended semiclassical phase whose transfer matrix contains a massless transverse-traceless sector and whose large-volume observables approach four-dimensional metric behavior. A continuous critical surface is a stronger universality and cutoff-artifact-removal test, not the existence condition for the finite- L_* theory. For each triangulation T , the minimally sufficient configuration bundle is

$$\mathfrak{X}_{\text{cap}}(T) = \{b_c, f_c, c_{A,c}, \theta_A, H_{A,c;\ell}\}.$$

Here b_c is the existing closure state, f_c is transported free renewal bandwidth, and $c_{A,c}$ is the mass-equivalent committed allocation carrying the physical source label A . The variables θ_A and $H_{A,c;\ell}$ are respectively the source/worldline renewal phase and the internal history output at renewal depth ℓ . The index ℓ labels circuit records, not a fifth spacetime coordinate. The source-supported observables are

$$M_{c,A}^{\text{lat}} = \sum_c c_{A,c},$$

$$\sigma_A = \frac{M_{c,A}^{\text{lat}}}{M_{c,A}^{\text{max}}},$$

$\mathcal{C}$ = single-stream integrability of the combined committed flow.

There is no local per-channel “saturated” state and no microscopic coherence bit. After vacuum normalization the coarse availability observable is obtained from f_c , while σ_A is computed against the independently fixed per-source ceiling. The transfer kernel must route formation and release through the existing marked defect vertex, so $c_{A,c}$ is not an externally prescribed matter source. For a stable mark those stock-changing rates vanish; its ordinary closure dynamics instead determine the maintenance draw from f_c . Terminal exhaustion is omitted from this bulk transfer matrix because it belongs to the separate $q_{\text{geo}} = 0$ boundary theory.

The ultraviolet configuration space is the disjoint union over causal triangulations,

$$\mathfrak{X}_{\text{UV}} = \bigsqcup_{T \in \mathfrak{T}_{\text{CDT}}} \mathfrak{X}_{\text{cap}}(T),$$

with joint partition function

$$Z(\mathbf{g}) = \sum_T \frac{1}{C_T} e^{-S_{\text{CDT}}[T; \kappa_0, \Delta, \kappa_4]} Z_{\text{cap}}[T; \mathbf{g}_{\text{cap}}].$$

The microscopic theory fixes $\mathbf{g}_{\text{cap}}$; the physical capacity slice is therefore

$$\mathcal{P}_{\text{cap}} = \{(\kappa_0, \Delta, \kappa_4; \mathbf{g}_{\text{cap}}^*)\}.$$

Integrating out the capacity variables gives $S_{\text{eff}} = S_{\text{CDT}} - \ln Z_{\text{cap}}$, making their displacement of the host critical manifold a measurable output.

The measured capacity backreaction is perturbative in the host action, but the quoted 0.08 per cell is not yet a coordinate displacement in the CDT coupling plane. To obtain one, the geometry-dependent term must be projected onto the operators conjugate to the CDT couplings:

$$-\log Z_{\text{cap}}[T] \simeq aN_0 + bN_{41} + cN_{32} + \cdots,$$

and then matched to

$$S_{\text{CDT}} = -(\kappa_0 + 6\Delta)N_0 + \kappa_4(N_{41} + N_{32}) + \Delta(2N_{41} + N_{32}).$$

Only that projection defines $(\delta\kappa_0, \delta\Delta, \delta\kappa_4)$ or a measured boundary shift. Thus $0.08 \neq \delta\Delta$, and it cannot be compared directly with a plotted distance to a phase boundary. H.10 needs no such comparison: it proves local stability of an already existing Einstein phase under a finite, pole-free, gauge-compatible correction smaller than the host TT stiffness. Phase membership itself must still come from the geometric host.

Increasing the number of simplices at a generic coupling point is not by itself evidence of either phase membership or a continuum limit. The existing CDT runs establish only that strong capacity ordering is compatible with an extended finite-volume geometry at the sampled regulator; they are not used to prove (H_{phase}) . The finite- L_* route must instead show a stable extended phase, four-dimensional large-volume metric behavior, and decisively a massless TT transfer sector. If one additionally seeks cutoff removal, κ_4 must approach its critical value so the four-volume can diverge and a remaining relevant direction t must approach a continuous geometric transition at which

$$\frac{\xi_{\text{geom}}}{a} \sim |t|^{-\nu} \longrightarrow \infty.$$

Four-dimensional CDT contains candidate continuous transition lines that can support this construction [87, 88]. In the enlarged decorated theory define

$$\mathfrak{C}_* = \{\mathbf{g} : \kappa_4 = \kappa_4^c, \quad \xi_{\text{geom}}/a = \infty\}.$$

That stronger cutoff-removal route succeeds only if

$$\boxed{\mathcal{P}_{\text{cap}} \cap \mathfrak{C}_* \neq \emptyset.}$$

If a capacity coupling is irrelevant, it shifts the location of the critical surface without adding a tuning. If it is relevant, its already-fixed microscopic value must lie on the surface. If the geometry and capacity sectors form a new joint fixed point, its exponents become predictions of the decorated theory. Failure of the fixed capacity slice to intersect $\mathfrak{C}_*$ rejects the cutoff-removal embedding, but does not by itself reject a finite- L_* theory already demonstrated to lie inside the required semiclassical phase.

The volume-operator normalization itself is already fixed kinematically. With Euclidean asymmetry α ,

$$\mathcal{V}_4 = a^4 \left[\frac{\sqrt{8\alpha - 3}}{96} N_{41} + \frac{\sqrt{12\alpha - 7}}{96} N_{32} \right].$$

After the mixed non-volume directions have been projected out, the two independently read source coefficients must satisfy

$$\boxed{\frac{t_{41}}{a^4 v_{41}} = \frac{t_{32}}{a^4 v_{32}} = \rho_{\text{vol,R}}.}$$

Equivalently, the target-blind double ratio

$$\boxed{\mathcal{R}_V(a) := \frac{t_{41} v_{32}}{t_{32} v_{41}} \longrightarrow 1}$$

must converge before either coefficient is converted to Λ_R . This equality uses two measurements and has no adjustable relative normalization. At fixed $\xi = N_{32}/N_{41}$ it reduces to $Z_V^{\text{geom}} = (v_{41} + \xi v_{32})/(1 + \xi)$. The scaling trajectory and its operator projection remain to be measured.

The required finite-size program is:

1. auto-tune κ_4 at each (κ_0, Δ) , repeat the phase scan across increasing N_4 , measure $\alpha(a)$ and $\xi(a)$, set a from a target-blind geometric observable, and verify the two-source equality $t_{41}/(a^4 v_{41}) = t_{32}/(a^4 v_{32})$ after critical-eigenoperator projection;
2. identify a continuous boundary using susceptibility peaks, Binder-cumulant crossings, histogram shape, transfer-matrix gaps, and pseudo-critical drift;
3. diagonalize the connected covariance matrix of the action-conjugate geometric and capacity operators,

$$C_{ij} = \langle \mathcal{O}_i \mathcal{O}_j \rangle_c, \quad \mathcal{O}_i = -\frac{\partial \ln w}{\partial g_i},$$

to identify the relevant mixed eigenoperator and the renormalized q observable;

4. verify the augmented-event incidence identity configuration by configuration and show that any screening mass vanishes with increasing volume;
5. measure D_q , χ_q , and α_{maint} separately, including their lattice-spacing factors, before forming $\chi_q \alpha_{\text{maint}}/D_q$;
6. apply the electron-derived L_* and G_* only after the dimensionless continuum extrapolation and test the junction condition $\chi_q \alpha_{\text{maint}}/D_q = 8\pi G_*/c^2$.

The phase and continuum tests do different work. Phase membership supplies the long-wavelength geometric branch at physical L_* ; a critical surface would additionally remove the geometric cut-off and test universality. The incidence Ward identity separately protects the hydrodynamic zero mode of the proposed bandwidth carrier. None supplies the others, and conservation alone does not establish the metric bridge, source purity, causal response, or junction normalization. At fixed lattice spacing, the larger host has the expected stronger extended-region indicators, and the transport calculation demonstrates conservation, approximate source additivity, a source-independent lattice junction ratio, power-law propagation, and no detected screening over the measured range. The massless TT transfer sector, asymptotic four-dimensional phase behavior, visible boundary's order, optional decorated critical trajectory, target-blind operator and lattice-spacing matching, and absolute junction to $8\pi G_*/c^2$ remain open. No continuum claim is assigned to the completed finite-volume results.

J.10 Predicted bare couplings and the native point

Under the face-wise lock of Appendix H.9a, paragraph 4a, the host is the equilateral CDT class with spatial edge $a = 2.530L_*$ and $G = G_* = L_*^2$ ($\hbar = c = 1$). The bare couplings are then outputs. Writing the Euclideanized Regge action in the standard form

$$S_E = -(\kappa_0 + 6\Delta)N_0 + \kappa_4(N_{41} + N_{32}) + \Delta(2N_{41} + N_{32})$$

and using the exact four-dimensional action of [82] with $k = a^2/(8\pi G_*)$ and the counting identity $N_2^{\text{TL}} = 2N_0 + N_{41} + 2N_{32}$ that follows from their Dehn–Sommerville relations, the equilateral point $\tilde{\alpha} = 1$ gives

$$\boxed{\kappa_0 = \frac{\sqrt{3}a^2}{8G_*} = 2 \ln 2 = 1.3863, \quad \Delta = 0,}$$

independently of the bare cosmological term. Along $\tilde{\alpha}$ with $\lambda = 0$ (Appendix O.5), κ_0 rises slowly, 1.39, 1.47, 1.93, 2.33 at $\tilde{\alpha} = 1, 2, 5, 8$, and Δ rises from 0 through 0.11 and 0.26 to 0.36. The

implementation reproduces the published coefficients of the Lorentzian and Euclidean actions at $\alpha = \pm 1$ to three decimals, and its inversion of the standard de Sitter point $(\kappa_0, \Delta) = (2.0, 0.4)$ returns $\tilde{\alpha} = 0.71$, $a^2/G = 26$, and a bare cosmological coefficient $\lambda \simeq 86$, in the direction and magnitude of the published $\tilde{\alpha}(\Delta)$ curve at $\kappa_0 = 2.2$ [81]. The inversion exposes a structural fact: at the standard de Sitter point the positive Δ is almost entirely induced by the bare cosmological term, $\Delta_\lambda = \lambda[\sqrt{8\tilde{\alpha} - 3} - \sqrt{12\tilde{\alpha} - 7}]/96$, while the geometric asymmetry contributes about 0.01. The lattice spacing is a second cross-check: $a = 2.53L_*$ is 2.53 Planck lengths on the matched branch, against the roughly two Planck lengths that semiclassical de Sitter fits assign to the CDT lattice spacing [84].

The time asymmetry. A $(4, 1)$ simplex places its apex above the base circumcenter at Lorentzian proper-time height $h = a\sqrt{\alpha + 3/8}$, so the number of renewal ticks per slice step is $n = h/L_*$. Two kinematic floors precede any dynamics: $n > R/L_* = 1.55$ for time-like edges to be time-like, and $n > 2.48$ for the Euclideanized $(3, 2)$ simplices to be nondegenerate ($\tilde{\alpha} > 7/12$). One tick per slice is impossible; the slice separation is at least three ticks. The manuscript fixes the event four-volume in two places that disagree once the face area is fixed. The convention of Section 25, one native cell of four-volume L_*^4 , gives $a^4 v_{41}(\tilde{\alpha}) = L_*^4$ at $\tilde{\alpha} = 1.06$ and $a^4 v_{32}(\tilde{\alpha}) = L_*^4$ at $\tilde{\alpha} = 1.04$, hence

$$\boxed{\tilde{\alpha} \simeq 1.05, \quad n = 3.0 \text{ ticks per slice,}}$$

with the two simplex volumes agreeing to two percent and $n = 3$ coinciding with the depth-three tetrahedral update of Appendix H.8. The alternative reading, one regular cell of three-volume $a^3/(6\sqrt{2}) = 1.91L_*^3$ swept through one tick, gives $v_* = 1.91L_*^4$ and $\tilde{\alpha} = 2.3\text{--}2.9$. Either way $\tilde{\alpha}$ is of order one. The native curve crosses the published bifurcation–de Sitter line only at $\tilde{\alpha} \simeq 7.1$, seven ticks per slice and an event volume of $3.1L_*^4$, which no reading of the text supplies.

The capacity asymmetry term. The uncentered capacity free energy $\beta\mu N_{\text{cells}} = \beta\mu N_{41}/2$ is an N_{41} -only term and therefore a pure Δ displacement, $\delta\Delta = +\beta\mu/2 = +1.24$ (Appendix J.2). It cannot supply the de Sitter asymmetry, for two independent reasons. The closure energy $K^2 = 48 - (S^2 - \Sigma^2)/3$ carries a conventional origin, and $E_c \mapsto E_c + C$ leaves $\hat{Z}_{\text{cap}}$ exactly invariant, so no origin-independent value of the extensive coefficient exists; full per-cell centering is the unique consistent prescription and $\Lambda_i = 0$ stands. And an N_{41} -only term has no N_0 part, so $\delta\kappa_0 = -6\delta\Delta$: the uncentered value sends κ_0 to -6.1 , and even the $\delta\Delta \simeq +0.35$ that the bifurcation line would require sends it to -0.7 . The same arithmetic is why the standard de Sitter point needs $a^2/G \simeq 26$ rather than 6.4.

The native point and its test. The theory’s native coupling point is therefore

$$\boxed{(\kappa_0, \Delta)_{\text{native}} = (2 \ln 2, \simeq 0.01)}$$

under the L_*^4 convention, or $(1.5\text{--}1.6, 0.13\text{--}0.17)$ under the swept-cell convention. On the published bare-measure phase diagram [83] the first lies within the scatter of the second-order $B\text{--}C_b$ line at this κ_0 and the second inside the bifurcation phase; neither is the de Sitter phase. Three readings are open. The face-wise lock may be wrong, returning metricity to the selected status of Appendix H.9a, paragraph 4. The published diagram may not be the right map: it is drawn for the bare $1/C_T$ measure, whereas the theory’s measure carries $e^{-\eta_* K^2}$ with injectivity on every face, a strong non-extensive constraint (Appendix J.5 records that it moves the slice collision fraction from 0.65 to 0.08) whose quadratic curvature dressing $c_0 \simeq 0.019$ does not bound its effect on phase boundaries. Or the native point is critical, sitting on CDT’s second-order line without tuning. The second reading keeps every step derived, and it is decided by a calculation with the existing engine rather than by extrapolation: run the weighted

ensemble at the predicted couplings and read the phase diagnostics of Appendix J.1. The registered protocol is the tuned grid of Appendix J.1 at $N_{41} = 10^4$, eighty slices, κ_4 auto-tuned, at $\kappa_0 \in \{1.0, 1.386, 1.8\} \times \Delta \in \{0, 0.1\}$ and at (1.55, 0.15), read against the same engine's grid at $\Delta \geq 0.2$. Extended: adjacent-slice correlation, blob score, and d_H comparable to the engine's reference point (2.0, 0.4) at the same volume (0.48, 2.78, 2.56), with maximum slice fraction below 0.05. Time-collapsed: maximum slice fraction of order one with a single active slice. A pass is followed by the volume-pair confirmation of Appendix J.1; a fail is recorded against the face-wise lock. Either outcome is a finite-volume phase diagnostic at the grade of Appendix J.1, not a continuum or massless-TT statement.

Result at $N_{41} = 10^4$. The registered grid was run with the engine of Appendix J.1 (eighty slices, four hundred sweeps, κ_4 auto-tuned over three bursts, one seed per point); every triangulation passed the manifold and foliation audits. No point collapsed.

κ_0	Δ	κ_4	d_H	blob	active	slicecorr	maxfrac
1.000	0.00	0.680	2.79	3.99	15	0.29	0.050
1.386	0.00	0.752	2.70	5.05	9	0.17	0.063
1.800	0.00	0.808	2.58	6.61	5	0.03	0.083
1.000	0.10	0.720	2.67	3.15	13	0.17	0.039
1.386	0.10	0.780	2.69	4.24	9	0.30	0.053
1.800	0.10	0.788	2.65	2.57	18	0.00	0.032
1.550	0.15	0.756	2.65	2.99	21	-0.01	0.037
2.000	0.40	0.752	2.56	2.78	12	0.48	0.035

The last row is the engine's reference point from the July grid at the same volume. The native point (2 ln 2, 0) is extended: nine active slices, a maximum slice fraction of 0.063 against the order-one value of a time-collapsed geometry, and $d_H = 2.70$, as high as any point in the engine's ten-thousand-simplex grid. Its volume profile is more concentrated than the reference (blob 5.0 against 2.8, fewer active slices, adjacent-slice correlation 0.17 against 0.48). Along $\Delta = 0$ the profile concentrates monotonically with κ_0 (active slices $15 \rightarrow 9 \rightarrow 5$, blob $4.0 \rightarrow 5.0 \rightarrow 6.6$), so the native point lies midway along the concentration direction. The reading-B point (1.55, 0.15) is indistinguishable from the July $\Delta = 0.2$ row.

Volume ladder and collapse control.

point	N_{41}	d_H	blob	active	slicecorr	maxfrac
native (2 ln 2, 0), seed 0	5,000	2.42	2.94	14	0.20	0.037
native, seed 0	10,000	2.70	5.05	9	0.17	0.063
native, seed 0	20,000	2.71	5.02	13	-0.06	0.063
native, seed 0	40,000	2.48	4.72	13	0.04	0.059
native, seed 1	20,000	1.69	8.53	5	-0.09	0.107
native, seed 1	40,000	2.88	9.42	6	-0.04	0.118
control (1.386, -0.3)	10,000	2.56	12.3	7	-0.09	0.154
control (2.5, -0.3)	10,000	2.43	10.7	10	-0.09	0.134
reference (2.0, 0.4)	10,000	2.56	2.78	12	0.48	0.035
reference (2.0, 0.4)	40,000	3.00	1.77	18	0.20	0.022

The control fixes what collapse looks like in this engine at this volume: at $\Delta = -0.3$ the maximum slice fraction is 0.13–0.15 and the blob score 11–12, not the order-one values of the published time-collapsed phase, so the published collapse signature does not transfer to this engine's conventions any more than the published de Sitter boundary does. Against that calibration the first seed at the native point is a stable extended geometry, unchanged from ten to twenty thousand simplices in d_H , blob, and maximum slice fraction, with the blob rising from five to ten thousand and then flat; the registered criterion is met on the first rung and flat on the second, and the maximum slice fraction did not fall. The second seed at twenty thousand

simplices lands elsewhere: $d_H = 1.69$, blob 8.5, five active slices, maximum slice fraction 0.107, much closer to the $\Delta = -0.3$ control. Two seeds at the same couplings in different basins is the signature of a point on or near the boundary between the extended and concentrated phases of this engine, not of a point inside either. The native point is therefore graded boundary-adjacent in the engine’s own phase map, consistent with the published diagram’s placement of $\Delta = 0$ near the second-order line. The forty-thousand-simplex rung of the first seed does not resolve it in the extended direction: the point stays uncollapsed (maximum slice fraction 0.059, thirteen active slices, pin residual 0.012), but the blob score is flat to falling ($5.02 \rightarrow 4.72$) and d_H falls from 2.71 to 2.48, the opposite of the reference point’s rise with volume in Appendix J.1. The registered volume-pair criterion therefore fails at this rung. The confound that the grid protocol uses four hundred sweeps whereas the rising d_H of Appendix J.1 came from four-thousand-sweep runs was tested directly: the reference point (2.0, 0.4) at forty thousand simplices under the same four-hundred-sweep protocol gives $d_H = 3.00$, up from 2.56 at ten thousand, with the profile spreading (blob 1.77, eighteen active slices, maximum slice fraction 0.022). The protocol therefore does reproduce the volume trend of the de Sitter-candidate region at the reference, and the native point’s failure to show it is not a thermalization artifact. A second seed at the native point at forty thousand simplices lands in the concentrated basin ($d_H = 2.88$, blob 9.4, six active slices, maximum slice fraction 0.118), as it did at twenty thousand. The registered volume-pair test therefore fails at the predicted couplings while passing at the reference under identical conditions. Of the three readings above, the second (inside the extended de Sitter-candidate phase) is excluded at this grade; the first (the face-wise lock is wrong, and metricity returns to the selected status of Appendix H.9a, with the coupling formula retained as what the couplings would be under that lock) and the third (a critical, seed-unstable boundary point) remain, and separating them would require a production-protocol study with several seeds, which is registered but not performed.

The shape sector does not change this. One escape was tested and closed. The theory hands geometry face areas, so the native variable set is that of area-Regge calculus, in which each disphenoid cell’s two shape parameters fluctuate and the shape-mismatch modes are massive [101]; CDT, which freezes edge lengths, is the rigid limit of that ensemble, and one might hope the fluctuating shapes carry what the rigid test lacked. A linearized area-Regge computation on a periodic Kuhn-subdivided hypercubic lattice (generic flat background; flat deficits, the Schläfli identity, and Hessian symmetry verified to 10^{-6}) reproduces the known structure for the full theory — ten massless modes per vertex, four of them lattice diffeomorphisms and six with $\lambda \propto k^2$, and forty massive modes with gap $|\lambda| \simeq 0.5\text{--}0.8$ in lattice units. With the spatial areas frozen at a_{ch} , as the fixed $j = 3$ capacity faces require, the continuous sector retains for spatial momenta only two k^2 modes of transverse-shift type and for momenta along the slicing direction none: the purely spatial transverse-traceless polarizations are frozen with the spatial metric. The shape modes therefore cannot carry gravitons; block-scale spatial geometry is carried by the slice connectivity, which is the CDT mechanism, and the combinatorial test above is the relevant one. The same modes do not fix the time asymmetry either: their one-loop effective potential for $\tilde{\alpha}$ is dominated by the measure and by the arbitrary background deformation (opposite slopes under flat-in-area and scale-free measures, slope differing threefold between backgrounds), so $\tilde{\alpha}$ remains a modulus at this order. The one positive output is the gap: with $ma \simeq 0.75$ and $a = 2.53L_*$, the mismatch modes decouple beyond $L_{\text{block}} = a/(ma) \simeq 3.4L_*$, inside the provisional bracket.

Appendix K: Numerical and Consistency Checks

Appendix K recomputes the numerical consequences of the stated derivations and collects the substrate-scale ledger. These checks support reproducibility but do not establish the ontology independently. Section 24 and Appendix J perform a separate task by coupling the same coef-

ficient chain to a dynamical substrate. The complete self-contained reproduction code appears in terminal Appendix R.

The numbers recomputed here, and where each enters the main text, are: the admissibility entropy $g_{\text{share,eff}}$ (Section 13, Appendices B and D.4); the substrate length L_* (Sections 2 and 13, Appendix D.4); the induced scale G_* (Sections 10 and 13, Appendices D.4 and K); the galactic scale a_0 (Section 15, Appendix C.7); the diamond-lattice Green constant $G_{\text{tet}}(0)$ (Appendix C.5); the edge-kernel chain $J_{\text{bare}} \rightarrow \gamma$ and the return sum Σ_{ret} (Appendices C.3–C.4); the charged-lepton ladder with its support-exponent cross-check (Section 13, Appendix I.1); and the primitive route, adjoint-channel, and causal-blocking identities of the conditional color branch (Appendix I.2).

K.1 Cross-sector numerical checks

The cross-check program includes:

- the one-bit fermionic defect check $\Delta S_f = \ln 2$;
- the rooted-shell convergence check $\sigma_{\text{ind}}^{(2)} \simeq \sigma_{\text{ind}}^{(3)}$;
- the UV closed-branch moments $\langle K^2 \rangle_{\eta_*}$, $\text{Var}_{\eta_*}(K^2)$, and a_{UV} ;
- the Gaussian closure-amplitude identity, nine-state response Gram matrix, unitary pair dilation, 21-block edge Hessian, and 56,800-state routing enumeration;
- the closed routing covariance, primitive route Gram spectrum, marked-fusion representation match, $t_8 = 13/14$ channel, and $2\sqrt{6}$ diamond blocking factor;
- cross-sector consistency among the electron anchor, the substrate length L_* , the induced scale G_* , the Green-matched Newton closure, and the galactic scale a_0 .

These checks independently reproduce the coefficient chain in every sector where closure is claimed. The derivations remain the source of the coefficients.

K.2 Reproducibility ledger for the substrate scale

The substrate-length calculation is short enough to record as a numerical ledger. Enumerating the 1680 oriented injective tetrahedral states with labels $m = -3, \dots, 3$, weighting them by $e^{-\eta K^2}$, and solving

$$\langle K^2 \rangle_{\eta} = \frac{3}{2\eta}$$

gives

$$\eta_* = 0.02986684439352237, \quad g_{\text{share,eff}} = 7.419800023570903.$$

The one-pass seven-sector history support and its transverse export are then

$$e^{7g_{\text{share,eff}}} = 3.602860521062804 \times 10^{22},$$

$$\frac{2}{3}e^{7g_{\text{share,eff}}} = 2.401907014041869 \times 10^{22}.$$

Using $\lambda_e = \hbar/(m_e c) = 3.861592671986303 \times 10^{-13}$ m and $r = e^{-7g_{\text{share,eff}}}$ first gives the baseline

$$L_*^{(0)} = -\frac{3}{2}\lambda_e \ln(1-r) = 1.607719470158885 \times 10^{-35} \text{ m},$$

while the marked vertex gives

$$\zeta_* = 0.005123584484947, \quad Z_e = 1.005308283809514.$$

Therefore

$$L_* = Z_e L_*^{(0)} = 1.616253701392569 \times 10^{-35} \text{ m},$$

and

$$G_* = \frac{c^3 L_*^2}{\hbar} = 6.674289077220912 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}.$$

The source-side Green constant is obtained independently from the diamond-lattice integral in Appendix C.5. Uniform-grid quadrature with endpoint extrapolation gives

$$G_{\text{tet}}(0) = 0.448220394388 \dots,$$

confirming the exact Joyce value

$$G_{\text{tet}}(0) = \frac{3\Gamma(1/3)^6}{2^{14/3}\pi^4}.$$

This is the value used in the source theorem. These numbers are not additional inputs; they are the numerical evaluation of the finite spectrum, the seven-sector history support, and the graph Green function already defined in the derivation.

K.3 Conditional color and record-retention audits

The color audit first recomputes the weighted route covariance directly from the same 1680 states used for admissibility. It returns

$$C_R = 11.7667312558 P_1 + 35.3002035759 P_2,$$

and verifies

$$\lambda_1 = \frac{3}{4} \text{Var}(K^2)$$

to numerical precision. The primitive recoupling matrix has spectrum

$$\text{spec } G = \left\{ \frac{9}{10}, \frac{21}{20}, \frac{21}{20} \right\}, \quad \frac{\lambda_2}{\lambda_1} = \frac{7}{6}.$$

After maximum-throughput normalization, the direct and complementary branches satisfy

$$K = P_2 + \frac{6}{7}P_1, \quad L = \frac{\sqrt{13}}{7}P_1, \quad K^\dagger K + L^\dagger L = I.$$

The phase-invariant loop cost obeys the representation identity

$$\overline{\Delta_p} = \frac{3}{2} \left(1 - \frac{|\text{Tr}_3 U_p|^2}{9} \right) = \frac{4}{3} \left(1 - \frac{1}{8} \text{Tr}_8 \text{Ad } U_p \right),$$

and is unchanged by multiplying any lift of U_p by a common phase. This checks that the derived action is well defined on the projective channel group without assuming the still-open fundamental endpoint lift. The Haar identities of Appendix I.2 then give a unital trace-preserving channel with singlet eigenvalue one and eight equal traceless eigenvalues $13/14$. The heat-kernel coordinate and equal-transfer diamond blocking evaluate to

$$g_{\text{HK,step}}^2 = \frac{2}{3} \ln \frac{14}{13} = 0.04940531477,$$

$$2\sqrt{6} = 4.89897948557, \quad g_{\text{graph,HK}}^2 = \frac{4\sqrt{6}}{3} \ln \frac{14}{13} = 0.24203562353.$$

These computations audit the conditional projective pure-gauge construction. They are not a comparison with a continuum QCD coupling.

The generation cutoff has also been checked without choosing a preferred excluded alphabet. Exhaustive enumeration over all reduced alphabets gives nonzero closure variance for every one-label exclusion (all seven cases, variance range 9.95556–16.17778) and every two-label exclusion (all 21 cases, range 1.24444–26.02667). Every three-label exclusion (all 35 cases) gives zero variance. The cutoff therefore follows from the remaining alphabet size and the permutation symmetry of K^2 , not from a specially selected sequence of labels.

The record-retention formula has a separate normalization check. For any orthogonal set of fine records merged into $\bar{R}$, summing $p(h, R_i)$ over both h and i gives $p(\bar{R})$, and the conditional distribution in Section 22 therefore sums to one. Record erasure changes the accessible partition of the same joint measure; it adds no transition weight and cannot alter the unconditioned Born marginals.

Appendix L: Fork accounting for the scale chain

This appendix records the construction choices behind the induced value of G_* and quantifies the surrounding discrete-choice fork space. The comparison shows how much freedom was available in the scale chain.

Provenance. By the author’s recollection, a target entropy near 7.42 was identified early in the framework’s development by inverting the measured Newton constant, with the ensemble constructed to deliver it jointly with other structural constraints; the documentary record does not reach that genesis layer. The realized ensemble was fixed and motivated in print by information-independence arguments before the induced- G_* chain existed. Its original electron scale and the baseline charged-lepton ladder landed at discrepancies of roughly one percent and one-half percent. Those residuals were public parts of the construction before the decorated marked vertex was developed.

Why the residuals suggested one missing object. The amplitudes can be compared on the same scale. The original Newton value was low by about 1.05%, while the length entering it was low by half that amount because $G_* \propto L_*^2$. The three required multiplicative uplifts were

baseline quantity	required uplift	structural reading
$L_*^{(0)}$ inferred from G	0.5309%	electron feedback
$m_\mu^{(0)}/m_e$	0.5124%	universal marked response
$m_\tau^{(0)}/m_e$	0.6562%	universal response plus second shell

Dividing the tau uplift by the muon uplift leaves a further 0.1431% second-shell contribution. The pattern is not three identical residuals, and it was not used to impose a universal rescaling. It is more specific: two observables require nearly the same charged-transfer amplitude, while the third requires that common amplitude followed by one smaller shell passage. This was the motivation for seeking one marked response with different graph traces.

Why the repair had to be additive. By that stage the tetrahedral construction was already doing several independent jobs. Its label space and admissibility weight fixed the closure spectrum and sharing entropy; its edge projection fed the ordinary weak-field coefficient; its transverse export entered the scale, horizon normalization, and galactic branch; and its shell algebra produced the baseline charged-lepton ladder. Removing or retuning any of those ingredients to close one residual would propagate through the rest of the microscopic and macroscopic chain. A viable repair therefore had to preserve the unmarked tetrahedral ensemble and its ordinary transfer operator.

This requirement narrowed the search to a defect-bound additive sector: it must be invisible in the unmarked vacuum, reuse the closure response already present in the renewed cell, activate only on occupied charged-channel pairs, and route through the existing electron and shell graph. These conditions explain the form of the calculation in Appendix H. They do not by themselves prove its nine-state fiber, 21 pair records, Majorana determinant, or routing polynomials; those are the independent action-level results and audits. Showing this part of the development is appropriate because the residuals were already known. It makes clear which features were motivated by the discrepancy, which were protected by the existing theory, and which had to be derived rather than chosen.

The vertex in Appendix H was consequently found with the discrepancies known. It contains no continuously adjusted coefficient: $g_{\text{share,eff}}$, η_* , $2/7$, the nine-state trace, 21 pair records, and the three routing polynomials are fixed by the displayed action. That fact makes the result an action-level postdiction rather than a fit, but it does not make it a blind prediction. Its added evidential content is that one field space and one routing graph account for G , m_μ/m_e , and m_τ/m_e simultaneously, while the nearby shared-mode and off-diagonal alternatives fail. The cluster capacity bound, committed abundance, redshift direction of a_0 , and low-acceleration rotation-curve regime remain post-fixation tests of the original ensemble; none entered either the early entropy calibration or the marked residual calculation.

The audit. The exact substrate length is proportional to $-Z_e \ln(1 - e^{-7g_{\text{share,eff}}})$, with leading behavior $L_* \propto Z_e e^{-7g_{\text{share,eff}}}$, so the calibration is meaningful only relative to the space of constructions that could have been written instead. The original grammar contains six label counts, five multiplicity rules, five closure coefficients, four support exponents, and five export prefactors, for 3000 candidate constructions. Those counts predate the marked vertex and must be recomputed with its $Z_e(g, \eta)$ dependence before they are used quantitatively.

The marked-sector audit separately enumerates 576 nearby formulas obtained by varying the response dimension, pair count, direction factor, determinant power, statistics, and overlap. Only the decorated-vertex formula lies within one muon standard deviation, and that exclusivity must be read carefully. The one-sigma muon window on ζ_* is about four parts per million relative, far finer than the spacing of any discrete formula menu, so at most one candidate could ever land inside it, and the failure of the other 575 carries little information on its own. What the menu records is qualitative brittleness: a shared edge register, directed complex determinant, stable bosonic determinant, or six-state symmetric response gives a visibly different result, and no probability measure is assigned to the menu. The quantitative audit is the one-integer scan of Section 13.5, which holds ζ_* fixed and asks the measured Newton constant to select the routing integer: $n = 7$ is the unique survivor over $|n| \leq 200$, its neighbors fail at $2.3\text{--}2.4\sigma$, and the tau's second-shell rational is supported but not uniquely resolved. The dark-energy sector carries the opposite provenance: its accumulation law, equation-of-state shape, and branch windows were fixed before the distance-data comparison of Appendix O.12 was run, so that comparison is a test rather than a calibration.

The galactic scale depends only linearly on $g_{\text{share,eff}}$, so it tests the coupling form more strongly than the exact ensemble choice. The corrected G_* and charged-lepton ratios test the shared marked vertex through three different graph factors, with the postdiction status stated above.

Appendix M: Why the saturated CMB carrier must be committed capacity, not a lagged response

The microwave background requires a gravitating component that carries the growing mode of the baryon perturbations while rejecting their acoustic oscillation. This appendix records the

quantitative exclusion of every relaxational realization of that component and the impossibility argument that selects the constraint reading of Section 20.

Growth-limited kernels. For development dynamics $dW/dt = (W_{\max} - W)/\tau$ with $\tau = \beta t_{\text{ff}}(\rho_{\text{local}})$, the measured cluster radial decline of the source weight fixes $\beta \simeq 15.6$, while saturation of the cosmological bath by recombination requires $\beta \lesssim 0.56$: the window is empty by a factor of order thirty. The radial ratio between R_{500} and R_{200} , a β -independent prediction of the free-fall clock, agrees with the data at the several-percent level, so the exclusion is of the absolute clock, not of the radial structure.

Lagged response. A first-order lag transmits a fraction $T = [1 + (\omega\tau)^2]^{-1/2}$ of an oscillation at frequency ω . The transmitted component acts as additional effective baryon loading $(1/\epsilon - 1)T$ in the acoustic driving; Einstein–Boltzmann computation shows a 0.3% temperature-spectrum tolerance bounds $T \leq 2 \times 10^{-3}$, requiring $\beta \gtrsim 37$ against the development requirement $\beta \lesssim 0.56$: empty by a factor of order sixty-six. More generally, by recombination a third-peak mode has completed only a few oscillations, so no causal filter of any order achieves the required rejection.

Sound-speed classification. For $L = f(X)$, perturbations carry $c_s^2 = f_X/(f_X + 2Xf_{XX})$. Generic unconstrained continuations away from the saturation surface have nonzero propagation speed, while plateau approaches $f = f_{\max} - A/X^n$ give $c_s^2 = -1/(2n+1)$ and are gradient-unstable. Both natural cap readings — stationarity of the canonical momentum and of the energy density — impose $f_X + 2Xf_{XX} = 0$, the pole of c_s^2 , and carry no healthy propagating perturbation. The only locus with $c_s^2 = 0$ and a healthy density response is the multiplier constraint surface $X = 1/2$ used in Section 20. Appendix O shows that carrier loss does not continue this field onto any of the unconstrained scalar branches: the source-label constraint is removed, the metric-visible remainder returns to the ordinary source-associated capacity bookkeeping, and only the homogeneous marked-history projection changes the cosmological term.

Relaxation plus a hard cap cannot carry the secular component. Within dynamics consisting of relaxation toward a demand together with a hard cap, with the cell weight as the only state variable, no coupling of the weight to the demand carries a secular component of the modulation while rejecting its oscillation: a pinned weight retains no memory of the modulation, and any instantaneous coupling of bounded periodic inputs is itself bounded and periodic. A secular component requires an additional conserved integrating variable. The conserved spatial density of committed cells is that variable, and it is native to the saturated phase rather than added to it.

Appendix N: Action Reconstruction, Parent-Theory Audit, and Transverse-Sector Target

This appendix asks whether the ordinary capacity functional is a second scalar action or the gravitational constraint action written in a different variable. The static longitudinal calculation answers that question. The final subsections give the specified leading transverse EFT and isolate its remaining microscopic and nonlinear-covariance requirements.

N.1 One parent metric, one matter coupling

The minimal covariant parent of the ordinary branch is

$$I_0[g, \psi] = I_{\text{EH}}[g] + I_{\text{GHY}}[g] + I_{\text{matter}}[g, \psi],$$

$$I_{\text{EH}} = \frac{c^3}{16\pi G} \int_{\mathcal{M}} d^4x \sqrt{-g} (R - 2\Lambda).$$

Here and throughout the covariant formulas, $x^0 = ct$. Thus $d^4x = dx^0 d^3x$ and the Einstein–Hilbert coefficient is $c^3/(16\pi G)$. After the split $dx^0 = c dt$, the ADM action written with t in seconds carries $c^4/(16\pi G)$. This convention makes the final Newton action an ordinary $\int dt L$ action. All matter, including radiation, couples to the one physical metric. Its stress tensor is

$$T_{\mu\nu} = -\frac{2c}{\sqrt{-g}} \frac{\delta I_{\text{matter}}}{\delta g^{\mu\nu}},$$

and the equations are

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}.$$

The Bianchi identity and the matter equations give $\nabla_\mu T^{\mu\nu} = 0$. No explicit coupling $S_{\text{ent}} T^\mu{}_\mu$ is present. Such a term would vanish as a source for classical radiation, would have a nontrivial metric variation through $\delta T^\mu{}_\mu / \delta g^{\alpha\beta}$, and would duplicate the matter response already carried by the metric equation.

The capacity variable enters after gauge choice and constraint reduction:

$$\delta S = \delta S[g, \psi]_{\text{reduced}}.$$

This is analogous to using the Newtonian potential as a reduced gravitational variable. A reduced variable can have a useful functional without being a new covariant matter field.

N.2 Direct reduction of the Einstein action

ADM form and boundary terms. With the GHY term included, the bulk gravitational action is

$$I_{\text{ADM}} = \frac{c^4}{16\pi G} \int dt d^3x N \sqrt{h} \left({}^{(3)}R + K_{ij} K^{ij} - K^2 - 2\Lambda \right) + I_\infty.$$

This line uses $t = x^0/c$ and is therefore consistent with the covariant c^3 normalization above. For a static scalar perturbation in Newtonian gauge,

$$N = \sqrt{1 + \frac{2\Phi}{c^2}} = 1 + \frac{\Phi}{c^2} + O(c^{-4}), \quad N^i = 0, \quad h_{ij} = \left(1 - \frac{2\Psi}{c^2}\right) \delta_{ij}, \quad K_{ij} = 0.$$

The asymptotic term I_∞ subtracts the flat reference contribution and cancels the total divergence generated when the spatial curvature is integrated by parts.

Define $\phi = \Phi/c^2$, $\psi_N = \Psi/c^2$, and write $h_{ij} = e^{2\zeta} \delta_{ij}$, with

$$\zeta = \frac{1}{2} \ln(1 - 2\psi_N) = -\psi_N - \psi_N^2 + O(\psi_N^3).$$

In three dimensions,

$${}^{(3)}R = e^{-2\zeta} [-4\nabla^2 \zeta - 2(\nabla \zeta)^2].$$

Before integration by parts, the expansion through second order is

$$N\sqrt{h} {}^{(3)}R = 4\nabla^2 \psi_N + 4\phi \nabla^2 \psi_N + 4\psi_N \nabla^2 \psi_N + 6(\nabla \psi_N)^2 + O(3).$$

The first term is a boundary term. Integrating the next two Laplacians by parts and combining all surface pieces with I_∞ gives

$$N\sqrt{h} {}^{(3)}R = \frac{2}{c^4} [(\nabla \Psi)^2 - 2\nabla \Phi \cdot \nabla \Psi] + \nabla_i B^i + O(c^{-6}).$$

The boundary integral of B^i is removed by the reference-subtracted variational prescription. For nonrelativistic matter with ρ the rest-mass density, expansion of $-mc^2 \int d\tau$ gives

$$I_{\text{matter}}^{(1)} = - \int dt d^3x \rho \Phi.$$

Consequently,

$$I_{0,\text{scal}}^{(2)}[\Phi, \Psi] = \int dt d^3x \left\{ \frac{(\nabla \Psi)^2 - 2\nabla \Phi \cdot \nabla \Psi}{8\pi G} - \rho \Phi \right\}.$$

Constraint equations. Variation with respect to the lapse potential gives

$$\delta_\Phi I_{0,\text{scal}}^{(2)} = 0 \implies \nabla^2 \Psi = 4\pi G \rho.$$

Variation with respect to the scalar part of the spatial metric gives

$$\delta_\Psi I_{0,\text{scal}}^{(2)} = 0 \implies \nabla^2 (\Phi - \Psi) = 0.$$

The elimination order is essential: Φ and Ψ are varied independently, both constraint equations are obtained, and only then is their solution substituted back. Imposing $\Phi = \Psi$ in the two-potential action before variation would erase one constraint and is not an equivalent reduction. For asymptotically flat isolated sources, the harmonic difference vanishes:

$$\Phi = \Psi.$$

Substituting this solution back into the two-potential action yields

$$I_{\text{Newton}}[\Phi] = \int dt d^3x \left[-\frac{(\nabla \Phi)^2}{8\pi G} - \rho \Phi \right].$$

The sign of the reduced kinetic term is not a ghost signal. Φ is a nondynamical constraint potential, not a propagating scalar oscillator. In the full covariant parent, the lapse and shift impose constraints and the propagating gravitational content is the two tensor polarizations.

N.3 Capacity field redefinition and coefficient identity

On the renormalized static branch, the capacity functional is

$$I_{\text{cap}}^{\text{static}} = \int dt d^3x \left[-\frac{\gamma}{2} (\nabla \delta S)^2 + \kappa \rho \delta S \right].$$

The most general linear local field redefinition compatible with a constant vacuum background is $\delta S = A\Phi$. Matching the signs of the source and the weak-field clock shift fixes $A < 0$. The bounded relation $q = 1 - \delta S/S_\infty = N^2$ at first order fixes

$$A = -\frac{2S_\infty}{c^2}.$$

Substitution gives

$$I_{\text{cap}}^{\text{static}} = \int dt d^3x \left[-\frac{2\gamma S_\infty^2}{c^4} (\nabla \Phi)^2 - \frac{2\kappa S_\infty}{c^2} \rho \Phi \right].$$

The equality

$$G = \frac{c^2 \kappa}{8\pi \gamma S_\infty}$$

then makes the kinetic coefficient and source coefficient agree simultaneously:

$$\boxed{I_{\text{cap}}^{\text{static}} = \frac{2\kappa S_{\infty}}{c^2} I_{\text{Newton}}.}$$

The off-shell identity proves that the two expressions are the same classical functional, up to a field-independent normalization, throughout the reduced static field space. Agreement of their Poisson equations alone would not establish this equivalence. The statement uses matched Dirichlet or asymptotically flat boundary conditions. With a finite boundary, equality also requires mapping the Newton boundary functional through the same linear field redefinition.

Normalization freedom. The observable static equation depends only on

$$\frac{\kappa}{\gamma S_{\infty}}.$$

A change of entropy units

$$\delta S \rightarrow a \delta S, \quad S_{\infty} \rightarrow a S_{\infty}, \quad \gamma \rightarrow \frac{\gamma}{a^2}, \quad \kappa \rightarrow \frac{\kappa}{a}$$

leaves both the action and the observable bridge invariant. Multiplying the entire reduced action by a further constant also leaves its classical equation invariant. The UV calculation can fix these conventions for fluctuation normalization and correlation functions, but the classical Newton constant is fixed only once through the invariant combination. A convenient unit-normalized representative has $Z_S = 1$, equivalently

$$\kappa = \frac{c^2}{2S_{\infty}}, \quad \gamma = \frac{c^4}{16\pi G S_{\infty}^2},$$

but the manuscript's canonical UV convention need not be changed to use the equivalence.

Why canonical scalar stress cannot do this job. If $\delta S = O(\rho)$ about a constant background, then

$$T_{\mu\nu}^{(\delta S)} \sim \partial_{\mu} \delta S \partial_{\nu} \delta S = O(\rho^2).$$

The bridged potential is $O(\rho)$. It therefore cannot be attributed to the canonical stress of a second scalar. In the reconstruction above, the linear response comes from the metric constraint and the apparent scalar functional is simply its reduced representation.

N.4 Parent-action options and degree-of-freedom audit

Route	Variables / action	Modes	Audit result
Metric-only / composite capacity	$I_0[g, \psi]$; δS defined after weak-field constraint reduction and $q_{\text{geo}} = (\nabla R)^2$ in spherical symmetry	two tensors	Passes the weak-field action identity, couples radiation correctly, gives baseline no slip and GR PPN, and passes the spherical exterior test. This is the preferred ordinary branch. A generic local covariant definition of capacity outside the controlled reductions remains open.

Route	Variables / action	Modes	Audit result
Algebraic ADM multiplier	$I_0 + \int \sqrt{h} \lambda (N^2 - q)$	no new mode only because the construction is empty	δq gives $\lambda = 0$ when q has no other bulk term. The relation only renames a foliation-dependent lapse and does not yield the capacity source equation. Excluded as a parent completion.
Clock-constrained auxiliary capacity	$I_0 + \int \sqrt{-g} \Lambda (qX + 1), X = g^{\mu\nu} \partial_\mu \tau \partial_\nu \tau$	trivial if q is purely algebraic; extra scalar or zero-pressure mode once dynamics is supplied	The q equation again drives Λ to zero unless additional q dependence is introduced. Kinetic or higher-derivative terms generically introduce a preferred foliation, mimetic zero-pressure behavior, or a scalar stability problem. Retain only as a control model with a full Hamiltonian analysis.
Scalar-tensor / disformal	$I[g, q] + I_{\text{matter}}[\tilde{g}, \psi],$ $\tilde{g}_{\mu\nu} = C(q)g_{\mu\nu} + D(q)u_\mu u_\nu$	two tensors plus at least one scalar	Universal matter coupling can be arranged, but fifth forces, gravitational slip, PPN shifts, and radiative stability become new obligations. It is not needed for the ordinary branch and is not selected by the present evidence.

The metric-only route is minimal in a precise sense: it adds no propagating field, uses the full stress tensor, and reproduces both controlled reductions. This does not prove that every substrate sector is metric-only. It proves that the ordinary longitudinal capacity mode must not be counted again as an independent force.

N.5 Exact spherical reduction and the Misner–Sharp first integral

Let

$$ds^2 = h_{ab}(x) dx^a dx^b + R^2(x) d\Omega^2.$$

We use $x^0 = ct$, so $d^2x = dx^0 dr$. If the orbit-space integral is instead written as $dt dr$, its prefactor is $c^4/(4G)$. The curvature identity is

$${}^{(4)}R = {}^{(2)}R + \frac{2}{R^2} [1 - (\nabla R)^2 - 2R\Box R].$$

After the angular integral,

$$I_{\text{EH}} = \frac{c^3}{4G} \int d^2x \sqrt{-h} [R^{2(2)}R + 2\{1 - (\nabla R)^2\} - 4R\Box R].$$

Integrating the last term by parts together with the reduced GHY term gives

$$I_{\text{sph}} = \frac{c^3}{4G} \int d^2x \sqrt{-h} [R^{2(2)}R + 2(\nabla R)^2 + 2] + I_{\text{matter}}^{(2)} + I_{\partial}^{(2)}.$$

The reduced boundary term is not left implicit. For a product boundary $\partial\mathcal{M}_4 = \partial\mathcal{M}_2 \times S^2$,

$$K^{(4)} = K^{(1)} + \frac{2}{R} n^a \nabla_a R.$$

The second term cancels the boundary contribution generated when $-4R\Box R$ is integrated by parts. With the standard orientation sign ε , the surviving term is

$$I_{\partial}^{(2)} = \varepsilon \frac{c^3}{2G} \int_{\partial\mathcal{M}_2} dy \sqrt{|\gamma_{(1)}|} R^2 K^{(1)} + I_{\text{joint}} + I_{\text{ref}}.$$

The vacuum Euler–Lagrange equations are

$$R^{(2)}R - 2\Box R = 0,$$

$$2R(h_{ab}\Box R - \nabla_a\nabla_b R) + h_{ab}[(\nabla R)^2 - 1] = 0.$$

Define

$$q_{\text{geo}} \equiv (\nabla R)^2, \quad M_{\text{MS}} \equiv \frac{c^2 R}{2G}(1 - q_{\text{geo}}).$$

Contracting and differentiating the metric equation, with the R equation used to remove the curvature term, gives

$$\nabla_a M_{\text{MS}} = 0$$

in vacuum. Hence

$$q_{\text{geo}} = 1 - \frac{2GM}{c^2 R}.$$

In the static areal gauge $R = r$, asymptotic flatness fixes the remaining normalization and returns

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2,$$

$$q_{\text{geo}} = N^2.$$

Thus q is a composite geometric scalar in spherical symmetry, and its vacuum profile is a first integral. No auxiliary scalar is required. The result is nonperturbative but symmetry restricted.

N.6 Covariant capacity observable beyond the controlled reductions

The action reconstruction fixes two limits but does not yet provide one local scalar valid in every geometry. In a stationary spacetime with a timelike Killing field ξ^μ normalized so that $\xi^\mu \xi_\mu \rightarrow -c^2$ at the asymptotic reference boundary, the lapse has the invariant representative

$$q_K = -\frac{\xi^\mu \xi_\mu}{c^2} = 1 + \frac{2\Phi}{c^2} + O(c^{-4}).$$

In spherical symmetry the invariant representative is instead

$$q_{\text{MS}} = (\nabla R)^2 = 1 - \frac{2GM_{\text{MS}}}{c^2 R}.$$

They coincide in the static Schwarzschild exterior. Outside stationary or spherical sectors neither a preferred Killing field nor an areal-radius dilaton exists, so neither formula is a general definition.

This is not evidence that a new fundamental field is required. A Newtonian potential is not a local curvature scalar: it depends on relational boundary data and gauge choice, whereas local curvature invariants measure tidal derivatives. The natural general object is therefore allowed to be quasilocal and state dependent,

$$q_{\text{cap}}(x) = \mathcal{Q}[g_{\mu\nu}, \rho_q; \mathcal{D}_x, \mathcal{A}(\mathcal{D}_x)],$$

where $\mathcal{D}_x$ is a relationally specified causal diamond or world tube, $\mathcal{A}(\mathcal{D}_x)$ is its observable algebra, and ρ_q is the quantum state relative to the chosen vacuum. This notation is a target class, not a proposed closed formula.

An informational realization could use a vacuum-relative entropy or modular-energy functional on $\mathcal{A}(\mathcal{D}_x)$ followed by a bounded constitutive map. Because regional relative entropy is unbounded while $q_{\text{cap}} \in [0, 1]$, a global linear identification is impossible. Schematically one would need

$$q_{\text{cap}} = \mathfrak{f}\left(\frac{S_{\text{rel}}}{S_\infty}, \text{geometric data}\right), \quad \mathfrak{f}(0, \dots) = 1, \quad 0 \leq \mathfrak{f} \leq 1,$$

with its first variation reproducing the weak-field bridge and its spherical restriction reproducing q_{MS} .

Bianconi’s geometric-relative-entropy action is a published example showing that the local, volumetric part of this target is mathematically viable: a positive metric-response operator enters through a trace logarithm, its low-curvature expansion contains the Einstein–Hilbert action, and its Legendre transform gives the convex mismatch energy used in Section 25 [25]. It is not a solution for $\mathcal{Q}$. Her response operator is independently dynamical and contributes second-derivative terms to the field equations, whereas the present target must introduce no new Cauchy data in the ordinary branch. The comparison fixes a useful functional neighborhood near equilibrium; the bounded, composite, metric-only completion remains to be derived here.

A successful definition must satisfy all of the following:

1. it is diffeomorphism invariant once its region, state, and reference data are specified relationally;
2. it introduces no independent Cauchy data and hence no extra propagating gravitational degree of freedom;
3. it reduces to q_K in the stationary weak-field sector and to q_{MS} in spherical symmetry;
4. its response is causal or explicitly quasilocal, with no dependence on an arbitrary coordinate foliation;
5. it is compatible with the metric Ward identity and the full stress-tensor source;
6. it supplies a controlled bounded continuation near saturation rather than identifying bounded capacity globally with unbounded relative entropy.

Determining $\mathcal{Q}$ is the general constitutive problem left by the action reconstruction.

N.7 The $q_{\text{geo}} = 0$ variational boundary

The exterior action should be defined first on a stretched timelike surface

$$\mathcal{B}_\epsilon = \{q_{\text{geo}} = \epsilon\}, \quad \epsilon > 0.$$

For Dirichlet metric data, include

$$I_{\text{GHY}}[\mathcal{B}_\epsilon] = \varepsilon \frac{c^3}{8\pi G} \int_{\mathcal{B}_\epsilon} d^3y \sqrt{|\gamma|} K,$$

the reference term at infinity, and the appropriate joints. Only after the variation is defined should the null limit $\epsilon \rightarrow 0^+$ be taken, using the null-boundary form and a fixed normalization of the generators. This supplies a well-posed exterior Einstein variational principle.

Three logically different claims must not be conflated:

1. The reduced metric equations produce a marginal sphere where $q_{\text{geo}} = 0$. This is derived.
2. The substrate capacity fraction equals q_{geo} up to that surface. This is a constitutive identification, strongly matched in static spherical vacuum.
3. The physical spacetime ends there and does not realize $q_{\text{geo}} < 0$. This is the bounded-domain postulate and requires new boundary dynamics.

The standard gravitational boundary term fixes neither a channel Hamiltonian nor a reflectivity. The missing microscopic functional may be denoted

$$\Gamma_{\partial q}[\sigma_{AB}, \zeta_{\partial}; \rho_{\partial}],$$

where ζ_∂ are boundary channel variables and ρ_∂ is their state. It must determine:

- whether the GR ingoing condition $\mathcal{R}(\omega) = 0$ is recovered;
- whether a stretched layer $q_{\text{geo}} = \epsilon$ is dynamically selected;
- the relaxation spectrum and any echo kernel;
- the shape variation and boundary stress balance;
- consistency of the derived $a_{mch} = 4 \ln 2 L_*^2$ area normalization under any future microscopic boundary Hamiltonian (the kinematic count itself is fixed in F.5).

Until this functional is derived, the Schwarzschild exterior is a result, while interior excision and boundary phenomenology are hypotheses.

N.8 Carrier-resolved transverse EFT and metric contact

This subsection gives a self-contained action audit of the galactic branch. The leading EFT contains four distinct ingredients: a source-space nesting rule, a thermal occupation contact, a conservative auxiliary-field action, and a no-slip metric contact. The first three determine which stress enters the galactic response and how its static potential is computed. The metric contact determines lensing. Their GFT derivation and fully nonlinear covariant variation remain open.

Carrier projector and nesting. For an active carrier P , let $\mathcal{R}_P$ be the retained record algebra and χ_P its support. Define

$$\Pi_P X = \chi_P \mathbb{E}[X \mid \mathcal{R}_P].$$

Active carriers form an antichain. On that set, and for a descendant $A \subset P$,

$$\Pi_P^2 = \Pi_P, \quad \Pi_P \Pi_Q = 0 \quad (P \neq Q), \quad \Pi_P \Pi_A = \Pi_P.$$

The full Einstein action uses $T_{\mu\nu}$, while the transverse functional uses $\Pi_P T_{\mu\nu}$ and sums only over active carriers:

$$\Gamma_{\perp}^{\text{nest}} = \sum_{P \in \mathfrak{A}} w_P \Gamma_{\perp, w_P}[g_{\pm}; \Pi_P T_{\pm}, \mathcal{R}_P].$$

This separation counts every localized source in ordinary gravity and counts it once, at the retained carrier resolution, in the galactic excess.

Tetrahedral kinetic normalization. Let $q_i(x)$, $i = 1, \dots, 4$, denote fluctuations on the four equivalent tetrahedral face directions $\mathbf{n}_i$, normalized by

$$\mathbf{n}_i \cdot \mathbf{n}_j = \begin{cases} 1, & i = j, \\ -1/3, & i \neq j, \end{cases} \quad \sum_i \mathbf{n}_i = 0.$$

For the normalization argument use the Euclidean quadratic term

$$\Gamma_{E, \text{kin}} = \frac{Z_0}{2} \int d^4 x_E \sum_i \partial_\mu q_i \partial_\mu q_i.$$

The q_i are dimensionless occupation fluctuations, and Z_0 has units of action per length squared. Lorentzian response is obtained by analytic continuation to the signature of Appendix A. The four-dimensional permutation representation splits into a uniform singlet and a three-dimensional sum-zero sector. On the latter define

$$A^a = \frac{\sqrt{3}}{2} \sum_i n_i^a q_i, \quad \sum_i q_i = 0.$$

Using $\mathbf{n}_i \cdot \mathbf{n}_j = 4\delta_{ij}/3 - 1/3$ gives the exact norm identity

$$\mathbf{A}^2 = \sum_i q_i^2, \quad \Gamma_{E,\text{kin}} = \frac{Z_0}{2} \int d^4x_E \partial_\mu \mathbf{A} \cdot \partial_\mu \mathbf{A}.$$

The same result follows from Schur's lemma: the tetrahedral three-dimensional representation is irreducible, so a symmetry-invariant quadratic operator is proportional to the identity on it. A slowly varying baryonic field selects $\mathbf{e}_L = \nabla \Phi_{\text{bar}}/|\nabla \Phi_{\text{bar}}|$ and permits

$$\mathbf{A} = X\mathbf{e}_L + \mathbf{Y}, \quad \mathbf{e}_L \cdot \mathbf{Y} = 0.$$

For a locally fixed $\mathbf{e}_L$, the longitudinal mode and both transverse components inherit the same coefficient,

$$Z_L = Z_T = Z_0.$$

Writing $Y_1 = R \cos \vartheta$ and $Y_2 = R \sin \vartheta$ gives

$$(\partial Y_1)^2 + (\partial Y_2)^2 = (\partial R)^2 + R^2(\partial \vartheta)^2,$$

so the radial occupation mode that mixes with X also carries Z_0 . The two compact transverse phases have angular Haar volume $(2\pi)^2$ for one phase cell. The acceleration-scale construction conditionally loads one sharing entropy into that cell and couples it reversibly to the horizon state. This compact measure is kinematic; it does not identify the eigenfrequencies of a propagating transverse field. Spatial variation of $\mathbf{e}_L$ produces derivative connection terms suppressed by the curvature scale of the source direction. Higher-derivative and background-dependent operators can split the coefficients; the equality is a leading long-wavelength result.

One-capacity-invariant potential. After the relative sector of Appendix H is gapped, define the longitudinal and transverse occupations $N_L = \sigma^\dagger P_L \sigma$ and $N_T = \sigma^\dagger P_T \sigma$. The minimal infrared premise is that matter sees one total-capacity invariant,

$$\Delta C = w_L N_L + w_T N_T - N_*.$$

This premise is stronger than tetrahedral symmetry. The general local quartic potential contains

$$V_{\text{gen}} = \mu_L N_L + \mu_T N_T + \frac{\lambda_L}{2} N_L^2 + \lambda_{LT} N_L N_T + \frac{\lambda_T}{2} N_T^2 + \dots$$

Its quartic coupling matrix has rank at most one when

$$\lambda_{LT}^2 = \lambda_L \lambda_T.$$

It has rank one only if the matrix is nonzero. A stable one-invariant potential further requires the matrix to be positive semidefinite, so $\lambda_L, \lambda_T \geq 0$ and not both vanish. Under those conditions $\lambda_L = u_C w_L^2$, $\lambda_T = u_C w_T^2$, and $\lambda_{LT} = u_C w_L w_T$ with $u_C > 0$ and suitable signed weights. The linear terms must align with the same weights. A thermodynamic capacity-pressure variable makes the stable sign explicit through

$$V_{\text{eff}}(\Delta C) = \sup_{\lambda_C} \left[\lambda_C \Delta C - \frac{\chi_C}{2} \lambda_C^2 \right].$$

The stationary pressure $\lambda_C = \Delta C / \chi_C$ gives $V_{\text{eff}} = (\Delta C)^2 / (2\chi_C)$. A path-integral Hubbard–Stratonovich representation requires the contour and Lorentzian/Euclidean sign to be chosen consistently; the equation above is the thermodynamic Legendre form. More generally $V_{\text{eff}} = F(\Delta C)$ with $F'(0) = 0$ and $F''(0) > 0$.

Near a transverse background, $N_L = N_{L0} + \zeta_L X + \dots$ and $N_T = R^2 = Y_1^2 + Y_2^2$, so

$$\Delta C = \alpha_C + g_C X + \beta_C R^2.$$

For fields $\varphi_i = (X, R)$ on the stationary surface $\Delta C = 0$,

$$H_{ij} = \left. \frac{\partial^2 V_{\text{eff}}}{\partial \varphi_i \partial \varphi_j} \right|_{\Delta C=0} = F''(0) \partial_i \Delta C \partial_j \Delta C.$$

This outer product has rank one wherever $\nabla \Delta C \neq 0$; it has rank zero at a point where that gradient vanishes. Explicitly,

$$H = F''(0) \begin{pmatrix} g_C^2 & 2g_C \beta_C R_0 \\ 2g_C \beta_C R_0 & 4\beta_C^2 R_0^2 \end{pmatrix}, \quad C_\times^2 = A_L A_T.$$

At fixed δX , minimizing the quadratic form gives $\delta R_* = -C_\times \delta X / A_T$ and $C_\times / A_T = \sqrt{A_L / A_T}$. These are static thermodynamic identities. The Hessian has a tangent zero eigenvalue and an orthogonal eigenvalue $A_L + A_T$. It cannot supply two independent mode energies, and it is not used as the propagator that generates the RAR. A metric vertex orthogonal to the soft tangent also cannot obtain the observed response by exchanging that tangent mode. The rank-one result is therefore retained only as a one-capacity consistency check.

This result is protected at the effective level when all local corrections remain functions of the same invariant ΔC . To state the decoupling condition explicitly, define $D = w_T N_L - w_L N_T$ and

$$V(C, D) = F(C) + \frac{\Delta_D}{2} D^2 + \eta_D C D^2 + J_D D + \dots, \quad C \equiv \Delta C.$$

The stationary solution is $D = -J_D / (\Delta_D + 2\eta_D C) + \dots$. For large Δ_D , integrating out D changes $F(C)$ only through constants and powers of Δ_D^{-1} . This is an effective decoupling statement, not proof that the bare vertex is rank one. A standalone $m_X^2 X^2 / 2$ outside $F(\Delta C)$ would raise the Hessian rank and generate a Yukawa deformation; the X^2 curvature tied to the entries of the rank-one outer product does not.

The susceptibility is not fixed by the normalized 1680-state internal distribution. It depends on fluctuations of total condensate occupation,

$$\chi_C = \langle \hat{C}^2 \rangle - \langle \hat{C} \rangle^2, \quad \hat{C} = w_L \hat{N}_L + w_T \hat{N}_T,$$

in a specified GFT state. In an energy-normalized convention the corresponding susceptibility includes $\beta_H \equiv 1/(k_B T_H)$. For a single mode $\hat{C} = c_0 \hat{N}$, a thermal state gives $\chi_C = c_0^2 \bar{N}(1 + \bar{N})$, while a coherent condensate gives $\chi_C = c_0^2 \bar{N}$. The closure distribution fixes neither state nor $\bar{N}$. The susceptibility controls the massive direction and nonlinear corrections but cancels from the rank-one ratio.

Thermal contact. The compact phase-cell loading gives

$$a_0 = \frac{g_{\text{share, eff}}}{4\pi^2} c H_0$$

after reversible coupling to the horizon state. For carrier P , define

$$x_P = \sqrt{\frac{|\nabla \Phi_{b,P}|}{a_0}}, \quad H_{\text{cell},P} = k_B T_H x_P : N_P : .$$

The corresponding equilibrium free energy is

$$F_{\text{th},P} = k_B T_H \ln(1 - e^{-x_P}), \quad \frac{\partial F_{\text{th},P}}{\partial E_P} = \frac{1}{e^{x_P} - 1} = n_B(x_P).$$

The horizon loading, reversible coupling, and the identification $E_P / (k_B T_H) = x_P$ are matching premises. A microscopic state calculation must derive them.

Static auxiliary action. Project the density and solve its Newtonian auxiliary field,

$$\rho_P = \frac{u_\mu u_\nu}{c^2} \Pi_P T^{\mu\nu}, \quad \nabla^2 \Phi_{b,P} = 4\pi G \rho_P.$$

With $z_P = |\nabla \Phi_{b,P}|^2 / a_0^2$ and $x_P = z_P^{1/4}$, define

$$\mathcal{Q}_\perp(z) = \int_0^z \frac{ds}{e^{s^{1/4}} - 1}, \quad \mathcal{Q}(z) = z + \mathcal{Q}_\perp(z).$$

Then $\mathcal{Q}'(z) = 1 + n_B(z^{1/4})$. The carrier action is

$$I_{Q,P} = - \int dt d^3x \left\{ \frac{2 \nabla \Phi_P \cdot \nabla \Phi_{b,P} - a_0^2 \mathcal{Q}(z_P)}{8\pi G} + \rho_P \Phi_P \right\},$$

and the transverse contribution is $I_{\perp,P} = I_{Q,P} - I_{N,P}$, with $I_{N,P}$ obtained by setting $\mathcal{Q}(z) = z$. Its two Euler–Lagrange equations are

$$\nabla^2 \Phi_{b,P} = 4\pi G \rho_P, \quad \nabla^2 \Phi_P = \nabla \cdot [\mathcal{Q}'(z_P) \nabla \Phi_{b,P}].$$

Consequently, for $\phi_{\perp,P} = \Phi_P - \Phi_{b,P}$,

$$\boxed{\nabla^2 \phi_{\perp,P} = \nabla \cdot [n_B(x_P) \nabla \Phi_{b,P}].}$$

This is the minimal conservative nonspherical completion in the QUMOND class [19]. Spherical sources recover the exponential RAR. Nonspherical sources require the displayed Poisson solve and generally have geometry-dependent curl corrections.

Closed-time-path metric response. The open-system definition remains useful for deriving the leading contact from a microscopic state. Integrating out transverse substrate variables ξ^A in their physical state $\rho_\perp$ gives

$$e^{i\Gamma_\perp[g_+,g_-]/\hbar} = \text{Tr}_\perp \left(U_\perp[g_+] \rho_\perp U_\perp^\dagger[g_-] \right).$$

The total effective functional is

$$\Gamma_{\text{eff}} = I_0[g_+, \psi_+] - I_0[g_-, \psi_-] + \Gamma_\perp[g_+, g_-; \psi_+, \psi_-].$$

The physical metric equation is obtained by varying the difference field and then setting the two histories equal. Unitarity requires

$$\Gamma_\perp[g, g] = 0, \quad \Gamma_\perp[g_+, g_-]^* = -\Gamma_\perp[g_-, g_+].$$

Diffeomorphism invariance of the influence functional must yield the Ward identity that makes the induced response covariantly conserved.

The specified leading stationary response is

$$h_{\mu\nu}^{\perp,P} = -\frac{2\phi_{\perp,P}}{c^2} (\bar{g}_{\mu\nu} + 2u_\mu u_\nu), \quad \Delta \Phi_{\perp,P} = \Delta \Psi_{\perp,P}.$$

It is generated by the quasistatic contact

$$\boxed{\Gamma_\perp^{\text{QS}} = -\frac{1}{2} \sum_{P \in \mathfrak{A}} w_P \langle h_\Delta, \mathcal{E} h_{\perp,P} [\Pi_P T_c] \rangle.}$$

This functional vanishes when the two histories coincide and is real in the stationary mean. It gives $T_{\perp,P}^{00} = \rho_{\perp,P} c^2$ and $T_{\perp,P}^{ij} = 0$, hence zero scalar anisotropic stress. It also leaves the vacuum graviton propagator unchanged. The causal material response described below supplies damped transport poles without adding a vacuum metric polarization.

At linear order, the Ward identity follows from the Bianchi identity for $\mathcal{E}$. Nonlinear conservation requires covariant variation of the carrier records:

$$\Pi_P[\varphi^* g, \varphi^* \psi, \varphi^* \mathcal{R}] = \varphi^* \Pi_P[g, \psi, \mathcal{R}].$$

Treating Π_P as fixed during metric variation does not establish the nonlinear identity. The microscopic completion must supply the record variables and their variation.

Quadratic data and response convention. Here g_+ and g_- label the two CTP branches. Let $h_{\mu\nu}^{(1,2)}$ be their metric perturbations and define the average and difference fields $h_c = (h^{(1)} + h^{(2)})/2$ and $h_\Delta = h^{(1)} - h^{(2)}$. With $x^0 = ct$, the linear metric coupling is

$$\delta I_\perp = \frac{1}{2c} \int d^4x \sqrt{-g} h_{\mu\nu} \hat{T}_\perp^{\mu\nu}.$$

Define the densitized source

$$J_{\mu\nu} \equiv \frac{\sqrt{-g}}{2c} h_{\mu\nu}.$$

Let $J^c = (J^{(1)} + J^{(2)})/2$ and $J^\Delta = J^{(1)} - J^{(2)}$. Then the universal quadratic form in this source convention is

$$\Gamma_\perp^{(2)} = \int d^4x d^4y J_{\mu\nu}^\Delta(x) \hat{\Pi}_R^{\mu\nu\alpha\beta}(x, y) J_{\alpha\beta}^c(y) + \frac{i}{2} \int d^4x d^4y J_{\mu\nu}^\Delta(x) \hat{\mathcal{N}}^{\mu\nu\alpha\beta}(x, y) J_{\alpha\beta}^\Delta(y).$$

The retarded kernel is

$$\boxed{\hat{\Pi}_R^{\mu\nu\alpha\beta}(x, y) = +\frac{i}{\hbar} \theta(x^0 - y^0) \left\langle \left[\hat{T}_\perp^{\mu\nu}(x), \hat{T}_\perp^{\alpha\beta}(y) \right] \right\rangle_{\rho_\perp} + \hat{\Pi}_{\text{local}}^{\mu\nu\alpha\beta}(x, y).}$$

The sign follows from the stated $+\frac{1}{2} h_{\mu\nu} T^{\mu\nu}$ action coupling and the Kubo susceptibility convention. The local or *seagull* term comes from the explicit metric variation of $\hat{T}_\perp^{\mu\nu}$ and of the covariant measure. It is required, together with the commutator, for the diffeomorphism Ward identity. Define $\Delta \hat{T}_\perp^{\mu\nu} = \hat{T}_\perp^{\mu\nu} - \langle \hat{T}_\perp^{\mu\nu} \rangle_{\rho_\perp}$. The noise kernel is the correspondingly normalized symmetrized correlator,

$$\hat{\mathcal{N}}^{\mu\nu\alpha\beta}(x, y) = \frac{1}{2\hbar} \left\langle \left\{ \Delta \hat{T}_\perp^{\mu\nu}(x), \Delta \hat{T}_\perp^{\alpha\beta}(y) \right\} \right\rangle_{\rho_\perp}.$$

A microscopic calculation begins with the transverse Hessian and metric vertex,

$$\mathcal{K}_{AB}(x, y) = \frac{\delta^2 I_\perp}{\delta \xi^A(x) \delta \xi^B(y)}, \quad V_A^{\mu\nu}(x; y) = \frac{\delta^2 I}{\delta \xi^A(x) \delta g_{\mu\nu}(y)}.$$

In the physical state,

$$G_R^{AB}(x, y) = +\frac{i}{\hbar} \theta(x^0 - y^0) \text{Tr}(\rho_\perp [\xi^A(x), \xi^B(y)]),$$

and schematically

$$\hat{\Pi}_R^{\mu\nu\alpha\beta} = V_A^{\mu\nu} G_R^{AB} V_B^{\alpha\beta} + \hat{\Pi}_{\text{contact}}^{\mu\nu\alpha\beta}.$$

This correlator/Hessian calculation is the microscopic audit that must reproduce the specified contact, its projector, and its local Ward term.

Lensing and point-source limit. The no-slip contact gives one effective density,

$$\rho_{\text{lens},P} = \rho_P + \frac{1}{4\pi G} \nabla \cdot [n_B(x_P) \nabla \Phi_{b,P}].$$

For a point mass in the deep regime, $A = \sqrt{GMa_0}$ and

$$g = \frac{A}{r}, \quad \rho_{\perp} = \frac{A}{4\pi Gr^2}, \quad \Sigma_{\perp} = \Delta \Sigma_{\perp} = \frac{A}{4GR}, \quad \alpha_{\infty} = \frac{2\pi A}{c^2}.$$

This fixes the leading lensing normalization. Resolved disks and lenses require direct solutions of the nonspherical auxiliary equation.

Causal response and stability. The transverse transport equation is

$$\tau_0 \ddot{\phi}_{\perp,P} + \dot{\phi}_{\perp,P} - D \nabla^2 \phi_{\perp,P} = -4\pi G D \rho_{\perp,P}, \quad \frac{D}{\tau_0} = c^2, \quad \tau_0^{-1} = H_0.$$

The Fourier response relative to the static solution and its Laplace poles are

$$F(\omega, k) = \frac{Dk^2}{Dk^2 - \tau_0 \omega^2 - i\omega}, \quad F(0, k) = 1,$$

$$s_{\pm} = \frac{-1 \pm \sqrt{1 - 4\tau_0 Dk^2}}{2\tau_0}.$$

Both poles have negative real part and propagate with characteristic speed c . The galactic corrections begin at $O(v^2/c^2)$, so this causal completion adds no static spectral factor.

Local nesting test. For the Solar System, the Milky Way is the active carrier and the Sun is a resolved Einstein descendant. The transverse functional never evaluates a nonlinear response on $\mathbf{g}_{\odot} + \mathbf{g}_{\text{MW}}$. Thus

$$Q_2^{\odot,\perp} = 0,$$

while the smooth parent field contributes only a tide of order $g_{\text{MW}}/R_0 \simeq 9.2 \times 10^{-31} \text{ s}^{-2}$. The same rule makes wide binaries Newtonian apart from that tide. This is a source-space projection; a spatial cutoff alone would leave the coarse Solar cross-term and would double-count descendant halos.

Noise and dissipation. For N_{eff} independent occupation cells,

$$\text{Var}(\bar{n}) = \frac{n_B(1+n_B)}{N_{\text{eff}}}, \quad \frac{\sigma_g}{g} = \sqrt{\frac{n_B}{N_{\text{eff}}(1+n_B)}} \leq N_{\text{eff}}^{-1/2}.$$

The symmetrized covariance is positive. As an illustrative Markov bath model,

$$\dot{n} = -\gamma(n - n_B), \quad \chi_{nE}^R(\omega) = -\beta n_B(1+n_B) \frac{\gamma}{\gamma - i\omega},$$

$$\mathcal{N}_{nn}(\omega) = \frac{2\gamma n_B(1+n_B)}{\omega^2 + \gamma^2}.$$

These expressions illustrate the KMS fluctuation–dissipation pairing; they are not derived from the GFT state. The bath coupling, γ , and the correlation volume remain open.

Closure audit. The leading EFT now fixes the thermal contact, static nonspherical action, carrier nesting, no-slip metric response, point-source lensing law, stable causal transport, Solar quadrupole, wide-binary limit, and positive occupation noise. The conditional inputs are the horizon loading, $E/(k_B T_H) = x$, the active-carrier projector, the no-slip tensor vertex, and the canonical transport parameters. The remaining theory work is their microscopic GFT derivation, full nonlinear variation of the carrier records, and the bath equilibration calculation. Resolved nonspherical galaxy and lensing maps and the ultrafaint-dwarf phase boundary remain observational tests.

N.9 Action-level outcome

The action problem now divides into four parts:

ordinary longitudinal capacity = reduced Einstein constraint action,
spherical nonlinear capacity = composite Misner–Sharp invariant,
galactic transverse response = specified carrier-resolved leading EFT,
with a microscopic embedding still required,
capacity-exhaustion boundary = open microscopic boundary functional.

This division solves the original double-counting problem and closes the ordinary weak-field parent action. The transverse sector is also explicit at leading effective order: the active-carrier projector removes parent–descendant cross-terms, the thermal contact and auxiliary action give the nonspherical RAR, and the metric contact gives no-slip lensing without changing the vacuum graviton pole. A simplicial GFT completion must reproduce the cell Hamiltonian, projector, phase structure, tensor vertex, and local Ward term. The full nonlinear variation of carrier records and the bath equilibration calculation remain open. The boundary-channel functional at $q_{\text{geo}} = 0$ is separate work.

Appendix O: The Cosmological Term from Fixed Event Measure and Marked Source-Clock Release

The dark-energy sector remains conditional. Fixed primitive event measure gives the continuum reduction, and a coefficient-free dilation into the nine-state marked register gives the accumulation law. A caustic can create multiple clock branches, but identity-free committed capacity cannot record which stream is which. Release begins only when irreversible baryonic mixing removes the distinguishable Lagrangian carrier of a source clock. Faithful renewal then fixes complete rerouting. The event probability is an astrophysical measure of progenitor information. A three-dimensional passive-tracer surrogate supplies an executable kernel; a self-gravitating cosmological zoom remains the decisive measurement. O.14 states the remaining premises and systematics.

O.1 Fixed primitive event measure and the unimodular reduction

The microscopic history is a sequence of admissibility-closed network configurations, one per substrate tick (Appendix G). The sharp primitive face map resolves each update reversibly into a transmitted geometric sector and a marked complementary record,

$$V_{3/2} \otimes V_{3/2} \longrightarrow V_3 \oplus (V_1^P \otimes V_1^H), \quad 16 \rightarrow 7 + 9,$$

supplying a concrete primitive event. One event occupies one microscopic spacetime cell of measure $v_* \sim L_*^4$, so the continuum four-volume is the coarse reading of an event count, $\mathcal{V}_4 = v_* N$.

Continuum premise. *The primitive event count supplies a fixed, nondynamical volume form: continuum geometry resides in connectivity and relational channel data at fixed microscopic event measure, and the metric determinant is not an independent local degree of freedom.* Under this premise the continuum variation is unimodular, $g_{\mu\nu}\delta g^{\mu\nu} = 0$, and variation of the Einstein–Hilbert and matter actions gives the trace-free equation

$$R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu} = 8\pi G_* \left(T_{\mu\nu} - \frac{1}{4}Tg_{\mu\nu} \right).$$

If the matter sector is covariantly conserved, the contracted Bianchi identity gives $\nabla_\nu(R + 8\pi G_*T) = 0$, hence $R + 8\pi G_*T = 4\Lambda$ for a spacetime constant, and

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_*T_{\mu\nu}, \quad T_{\mu\nu}^{(\Lambda)} = -\rho_\Lambda g_{\mu\nu}, \quad \boxed{w_\Lambda = -1}$$

exactly, with $\rho_\Lambda = \Lambda/8\pi G_*$. The cosmological term is an integration constant conjugate to the primitive history volume, not a local vacuum-energy coupling [111, 112].

Vacuum-shift decoupling. A constant shift $\mathcal{L}_m \mapsto \mathcal{L}_m - \rho_{\text{vac}}$ leaves the traceless source invariant and contributes only an overall factor $e^{-i\rho_{\text{vac}}\mathcal{V}_4}$ to a fixed-event history sector, which cancels from normalized history weights. It shifts the integration constant, $\Lambda \mapsto \Lambda + 8\pi G_*\rho_{\text{vac}}$, but cannot independently curve spacetime. This decoupling is classical and holds at the level of normalized history weights; its stability under quantization requires the fixed-measure property to survive coarse-graining, which is not established here [113]. The equilibrium normalization of Section 25 supplies the complementary statement that the equilibrium bulk contribution is exactly zero; the present reduction explains why residual constant shifts are gravitationally sequestered rather than merely subtracted.

Falsifier of the reduction. If the coarse-grained microscopic Hessian of the condensate embedding contains an independently propagating determinant (conformal-volume) mode, the unimodular reduction does not follow and ordinary vacuum-energy sensitivity returns. This is a definite target for the fixed- N_4 simulation program: hold the event count fixed, vary connectivity and edge geometry, and test whether a scalar volume mode survives in the effective spectrum.

O.2 Volume-singlet theorem for the marked drain

The trace-free field equation by itself permits a local relation $\nabla_\nu\Lambda = 8\pi G_*J_\nu$ whenever a non-conserved coarse current is exact. That generic identity is more permissive than the fixed-event microscopic theory. The present branch contains only one direction conjugate to primitive four-volume.

Consider a coarse history slab containing N primitive events. Let $\mathbf{1} = (1, \dots, 1)$ be the event-space volume vector and let $\mathbf{r} = (r_1, \dots, r_N)$ be any marked release profile. Define

$$P_0 = \frac{\mathbf{1}\mathbf{1}^T}{N}, \quad P_\perp = I - P_0,$$

so that

$$\mathbf{r} = \bar{r}\mathbf{1} + \mathbf{r}_\perp, \quad \bar{r} = \frac{1}{N} \sum_e r_e, \quad \sum_e r_{\perp e} = 0.$$

The primitive four-volume is $\mathcal{V}_4 = v_*N$. The volume-conjugacy lemma of O.5 and Section 25 states that the only extensive, connectivity-independent direction of the coarse action maps to this total event count. Appendix O.13 fixes the same post-refresh drain coefficient η_* at every carrier-failure event. Therefore, for $r_e = \eta_*q_e$,

$$\boxed{\Pi_V \mathbf{r} = \eta_* \bar{q} \mathcal{V}_4, \quad v_* \sum_e r_{\perp e} = 0.}$$

The orthogonal marked profile has no overlap with the unique volume conjugate. Promoting it to a local Λ_e would introduce independent cell-by-cell volume couplings, precisely the determinant/volume degrees of freedom excluded by the fixed-measure premise of O.1.

On a cosmological source-clock slice Σ_t , the continuum projection is

$$P_0 f \equiv \langle f \rangle_\Sigma = \frac{1}{V_\Sigma} \int_{\Sigma_t} dV f.$$

With $q(x, t) = \Gamma_{\text{rel}}(x, t) \rho_u(x, t)$, the selected accumulation law is consequently

$$\boxed{\dot{\rho}_\Lambda = \eta_* \langle q \rangle_\Sigma, \quad \delta \rho_\Lambda(\mathbf{k}, t) = 0 \quad (\mathbf{k} \neq 0).}$$

Thus Λ is the global marked-history volume mode, not a local vacuum field. This theorem is conditional on the same fixed-event continuum and volume-conjugacy premises already required by O.1 and O.5; it adds no new cosmological degree of freedom.

O.3 Source-associated capacity after carrier loss and the accumulation law

While a baryonic progenitor label remains recoverable, the source-associated allocation may occupy the saturated constraint surface of Section 20. Carrier loss removes that source-label constraint. It does not create a new material species. The renewal theorem sends the replaceable closure register to the universal state ρ_* , and the marked complement is an internal history output rather than an ordinary spacetime fluid. Renewal also cannot create a second persistent ordinary gravitational pole (Appendix H.11 and N.8–N.9). Accordingly the orthogonal history output has no independent zeroth-derivative metric stress:

$$\boxed{T_{H,\perp}^{\mu\nu} \big|_{k^0} = 0, \quad \delta T_{H,\perp}^{\mu\nu} = O[(kL_{\text{block}})^2]}$$

within the local derivative expansion. Any such higher-gradient influence is a correction to the retained metric/capacity response, not a new conserved component.

The metric-visible remainder is instead associated with the retained coarse baryonic carrier Y . At scales larger than that carrier, covariance and the one-metric worldtube theorem give the leading monopole stress

$$\boxed{T_C^{\mu\nu} = \rho_C U^\mu U^\nu + \Delta T_C^{\mu\nu}, \quad \frac{|\Delta T_C|}{\rho_C} = O\left(\frac{\sigma_Y^2}{c^2}, k^2 R_Y^2\right).}$$

This is an effective source-locked capacity stress, not a particle ontology. The zero-pressure leading form follows from the absence of an independent local scalar pressure operator at this order; finite carrier size and velocity dispersion supply the displayed corrections.

It is useful to keep the source-resolved and reallocated ledgers internally. Let $q = \Gamma_{\text{rel}} \rho_u$, let n^μ be the normal to the cosmological source-clock slice, and let R denote reallocated metric-visible capacity, not an independent field. The unique transfer consistent with the volume projection is

$$\boxed{Q_u^\mu = -q u^\mu, \quad Q_\Lambda^\mu = \eta_* P_0[q] n^\mu, \quad Q_R^\mu = q u^\mu - \eta_* P_0[q] n^\mu,}$$

with $Q_u^\mu + Q_R^\mu + Q_\Lambda^\mu = 0$. For the total metric-visible source-associated capacity $C = u + R$,

$$\boxed{\nabla_\mu T_C^{\mu\nu} = -\eta_* \langle q \rangle_\Sigma n^\nu.}$$

On the homogeneous background,

$$\dot{\bar{\rho}}_C + 3H\bar{\rho}_C = -\eta_* \bar{q}, \quad \dot{\rho}_\Lambda = \eta_* \bar{q},$$

and with F_{rel} the cumulative released source fraction,

$$\bar{\rho}_C(a) = \rho_{c,0}^{\text{wb}} a^{-3} [1 - \eta_* F_{\text{rel}}(a)],$$

$$\rho_\Lambda(a) = \rho_{\Lambda,i} + \eta_* \rho_{c,0}^{\text{wb}} \int_{a_i}^a a'^{-3} dF_{\text{rel}}(a').$$

Setting $\rho_{\Lambda,i} = 0$ as in O.5 and writing $\omega_c^{\text{wb}} = \omega_b/\epsilon$ gives

$$\omega_{\Lambda,0} = \eta_* \omega_c^{\text{wb}} I_{\text{rel}}, \quad I_{\text{rel}} = \int_0^1 a^{-3} dF_{\text{rel}},$$

and the present normalization is solved self-consistently from

$$h^2 = \omega_r + \omega_b + \omega_c^{\text{wb}} [1 - \eta_* F_{\text{rel}}(1)] + \eta_* \omega_c^{\text{wb}} I_{\text{rel}}, \quad \Omega_{\Lambda,0} = \frac{\omega_{\Lambda,0}}{h^2}.$$

Sign and effective equation of state. Under the one-signed history orientation of Section 22, the homogeneous marked share accumulates monotonically. An observer fitting $\rho_\Lambda(a)$ as a separately conserved component infers

$$w_{\text{eff}}(a) = -1 - \frac{a^{-3} dF_{\text{rel}}/d \ln a}{3 \int^a a'^{-3} dF_{\text{rel}}}.$$

Thus active one-signed release gives an effective $w < -1$ and approaches -1 as release completes; a persistent low-redshift $w > -1$ branch would require the bookkeeping flow to reverse.

Perturbation closure. For every nonzero spatial mode the volume theorem gives $\delta Q_\Lambda^\mu = 0$. Hence

$$\delta Q_R^\mu(\mathbf{k}) = -\delta Q_u^\mu(\mathbf{k}), \quad \delta Q_C^\mu(\mathbf{k}) = 0, \quad \mathbf{k} \neq 0.$$

The spatial fluctuation of the release hazard therefore redistributes the internal source label but cancels from the leading total metric source. The peak-background response derived in O.12, $\delta\Gamma = \mathcal{R}_\Gamma(a)\Delta_m$, remains a prediction for where release occurs, but it is not an additional cosmological clustering parameter.

Define

$$\gamma_C(a) = \frac{\eta_* \bar{q}}{\bar{\rho}_C} = \frac{\eta_* \dot{F}_{\text{rel}}}{1 - \eta_* F_{\text{rel}}}.$$

The covariant source equation above is the primary perturbation statement. In the subhorizon Newtonian limit its scalar content can be written as

$$\begin{aligned} \dot{\delta}_C &= -\Theta_C + \gamma_C \delta_C + O(\dot{\Phi}, H\Psi), \\ \dot{\mathbf{v}}_C + (H - \gamma_C)\mathbf{v}_C &= -\frac{1}{a}\nabla\Psi + O\left(\frac{\sigma_Y^2}{c^2}, k^2 R_Y^2\right). \end{aligned}$$

There is no local $\delta\Lambda$, no independent sound-speed or anisotropic stress parameter, and no new k^2 -enhanced force. The principal short-scale operator is therefore the ordinary one-metric gravitational system, with the release history entering only through the homogeneous background drain and the small normalization rate γ_C .

O.4 What fixes η

The Hilbert-space dimensions do not: the isometry fixes information routing, not energy allocation, so $\eta = 9/16$ is not implied. The clean definition requires the infrared energy operator of the marked register. Let U_{rel} be the marked source-clock release map and Π_{mark} the marked projector; then

$$\eta = \frac{\text{Tr}\left[U_{\text{rel}}\rho_{\text{comm}}U_{\text{rel}}^\dagger \Pi_{\text{mark}}H_{\text{out}}\Pi_{\text{mark}}\right]}{\text{Tr}[\rho_{\text{comm}}H_{\text{comm}}]}.$$

Appendix O.13 states the conditional operator theorem that fixes $\eta = \eta_*$ when the energetic ledger debits the post-refresh closure state. H.11 makes independence from the renewed K^2 register exact under that ordering; it does not derive the ordering itself. The marked fugacity $\zeta_* = 0.00512$ remains part of the charged recurrence calculation and is not a cosmological event probability.

O.5 The initial constant

The accumulation law explains the growth of Λ . Its initial value $\rho_{\Lambda,i}$ is the usual unimodular integration ambiguity, which unimodular gravity alone cannot fix. Fixed event measure supplies a normalized microscopic history measure and a definite equilibrium state. Under the two conditions stated below, these inputs fix the initial value.

The argument has three steps. First, the theory is microcanonical in the primitive event count. Because N is fixed, a sector factor $e^{-\alpha N}$ is constant on every history being compared and cancels from normalized weights. The admissibility kernel fixes the remaining per-event normalization. Second, the equilibrium centering of Section 25 removes the complete extensive bulk coefficient at every regulator. The centered grand potential has zero homogeneous density, and this result is invariant under shifts of the microscopic energy origin. The uncentered control run displaces the equilibrium volume by the predicted amount. Third, $\mathcal{V}_4 = v_* N$ makes an extensive term CN exactly the vacuum-volume action $(C/v_*) \int d^4x \sqrt{-g}$. The removed coefficient is therefore the antecedent of the initial cosmological term, and

$$\boxed{C = 0 \implies \Lambda_i = 0.}$$

The grading is explicit. $C = 0$ is closed at finite regulator; the vanishing of the equilibrium capacity density is closed on every continuum subsequence with a defined volume operator; the total $\Lambda_i = 0$ is conditional on two named items — the volume-conjugacy lemma (the only extensive, connectivity-independent contribution to the coarse effective action maps uniquely to the volume operator) and the common geometry–capacity continuum limit of Section 25, which is the same gate the rest of the theory requires. Neither is a dark-energy assumption.

The comparison with causal dynamical triangulations clarifies the fixed- N hypothesis. In CDT the coupling conjugate to cell number is the bare cosmological constant, and it is not zero: it must be tuned to a critical value. A normalized measure therefore cannot imply a vanishing volume coupling for discrete gravity in general. The resolution is that CDT is grand-canonical in cell number — histories of different N_4 compete, and the volume coupling weights that competition — while the present theory is microcanonical in the event count: N is the fixed primitive measure, so the coupling CDT must tune is a constant on every sector this theory compares and never becomes observable. Generic discrete gravity therefore carries a cosmological-constant tuning problem. In the present construction, Postulate I makes the event measure fixed. Conditional on a continuum completion, this hypothesis removes that tuning freedom.

O.6 Commitment epoch: a correction to the recruitment reading

Section 20 contains two readings of the committed initial conditions. The recruitment narrative (allocation $\rho_c = \rho_b/\epsilon$ pointwise at a recombination-era commitment epoch) gives the committed field the Silk-damped baryon transfer function. A first-crossing computation on that field returns $\sigma_0 \simeq 0.34$ for the unsmoothed committed field today under standard growth, with first-caustic probability $P(\lambda_1 > 1/\sigma_0) < 5 \times 10^{-8}$: no caustics, no dark energy, and no cosmic web. This registered negative result falsifies the late, pointwise-tracking reading. The Einstein–Boltzmann check of Section 20 already used a standard cold component, which is consistent only with commitment completing before the acoustic evolution of the fitted modes: primordial adiabaticity, then cold propagation. The CMB sector and the dark-energy sector therefore vote jointly for early (pre-acoustic) commitment, and the recombination-era saturation estimate must be re-derived at the earlier epoch. This is recorded as a required revision of the recruitment argument.

O.7 Identity-free source labels cannot record stream assignment

Per substrate tick $\tau_* = L_*/c$, every unpinned closure register is replaced by an independent draw,

$$p_*(b) = Z_*^{-1} e^{-\eta_* K^2(b)}, \quad K_*(b, b') = p_*(b'),$$

and the old register is exported to history. A committed allocation $c_{A,c}$ carries a Lagrangian source label A and a continuous mass weight, but no additional particle or stream identity. Its local environment can therefore record occupancy and the presence of the pinned source, not a permutation assigning identical incoming mass elements to outgoing streams.

To state this precisely, let i and j denote two fine stream descriptions of one source that differ only by which past occupancy is declared to continue into which present occupancy. The exported environmental record is a function of the occupancy history $\{b_c(n)\}$. Under the identity-free source convention the two descriptions are the same physical history, so

$$|e_i\rangle = |e_j\rangle \implies \gamma_{ij} = \langle e_j | e_i \rangle = 1.$$

Consequently

$$C_A^{\text{rec}} = \sum_{ij} p_i p_j |\gamma_{ij}|^2 = 1, \quad f_A^{\text{rec}} = 0$$

for every fold and at every well depth. A density record can determine the net flux, but not a multivalued velocity whose branches differ only by an exchange of identity-free mass. Thus the committed capacity sector cannot write a which-stream record. This is an exact consequence of the declared source ontology and the one-register update; it would fail if an additional persistent identity label were added.

O.8 Cosmology excludes a persistent stream identity on this branch

The alternative is not innocuous. Suppose a committed allocation carried a persistent identity that neighboring records could resolve. Two streams with relative velocity Δv separate by one primitive length after

$$t_{\text{sep}} = \frac{L_*}{\Delta v} = 1.62 \times 10^{-39} \text{ s} \left(\frac{10 \text{ km s}^{-1}}{\Delta v} \right).$$

A pinned face changes the neighboring distribution by $D_{\text{pin}} = 0.1499$ nats per slot per tick (P.3). Using the fidelity-to-amplitude bound $|\gamma| \lesssim e^{-n D_{\text{pin}}/2}$, the time to $|\gamma| < e^{-5}$ is

$$t_{\gamma < e^{-5}} = \frac{10}{D_{\text{pin}}} \tau_* \simeq 3.6 \times 10^{-42} \text{ s}.$$

Persistent identity would therefore make every microscopic fold effectively fully recorded.

That limit fails the early-release bound. At $z = 15$, the stated first-crossing estimate for $M > 10^6 M_\odot$ gives $\delta F \simeq 5.5 \times 10^{-3}$, whereas requiring the induced change in the cosmological density to remain below 0.01 gives

$$\delta F < \frac{0.01}{0.00783(1+z)^3} = 3.1 \times 10^{-4} \quad (z = 15).$$

The identity-carrying branch overshoots by a factor of about 18. This condition excludes the identity-carrying cosmological branch, given the displayed first-crossing estimate and the absence of a separate compensating suppression. Within the minimal branch it selects identity-free committed allocations, after which O.7 applies.

The completed 48^3 phase-relaxation harness is retained only as a protocol check. It sets $\gamma_{ij} = 1$, heals a smooth phase bump by construction, and types the proposed gate into its marked output. It does not test stream recording, the trigger, or the post-refresh ledger. Those become tests only in an ungated multibranch source simulation.

O.9 The trigger cannot be a local capacity gate

The displayed capacity action has one closure register per cell. When allocations from different sources $A \neq A'$ occupy the same cell, its local register can either preserve the source labels, release an allocation without a potential selector, or merge the coarse labels. The last operation is Many-Pasts coarsening: it removes source distinguishability rather than writing a new which-stream bit. The potential enters the existing vertex smoothly through

$$\delta S = -2S_\infty \frac{\Phi}{c^2};$$

the action contains no local source-resolving operator whose spectrum produces the $O(10^{-9})$ photoheating threshold. Therefore a sharp retention switch is not generated inside the present one-register capacity action. The baryonic carrier is not an optional decoration of this minimal completion: it is where the supporting condition must live.

The two roles sometimes assigned to a_0 must remain separate. The running relation

$$a_0(z) = \epsilon c H(z)$$

governs the transverse thermal response and its redshift test; source-clock release is governed by loss of the baryonic carrier and contains no a_0 .

Three candidate triggers are thereby excluded.

1. A generic three-stream fold is not recorded by identity-free capacity. The former cell-wise and source-wide multipliers, 0.588 and 0.625, are retired; neither is an active cosmological multiplier.
2. A single carrier atom or ionization state cannot be the supporting condition. If reionization rerouted the intergalactic allocation, the formal increments would be

$$\Delta\Omega_\Lambda \simeq 0.00783(1+z_{\text{re}})^3 x_{\text{ion}} = 2.2, 4.4, 6.6$$

for $z_{\text{re}} = 6, 7.7, 9$, respectively, already excluding that reading by factors of several.

3. Ordinary entropy production, star formation, metallicity change, and environmental measurement records do not by themselves invoke the marked drain. P.7a restricts it to loss of the particular baryonic source-clock carrier.

The retired fixed-present- a_0 threshold remains excluded for the independent reason stated earlier: this branch predicts $a_0(z) \propto H(z)$, so a threshold frozen to $a_0(0)$ is not covariant.

O.10 Collective trigger and the information observable

What survives is collective. The clock θ_A belongs to source A as a distinct Lagrangian baryonic system. Its pin is supported while matter records continue to distinguish that progenitor. The supporting condition fails when gas from several sources is irreversibly mixed into one retained bound object and the baryonic record no longer resolves progenitor membership. The committed allocation is still labelled A , is collisionless, and did not itself mix; it is therefore orphaned when its clock carrier ceases to exist as a distinguishable matter subsystem.

This makes the required astrophysical quantity precise. Label each gas resolution element before assembly by its progenitor A . At a later time divide the bound gas into resolved regions Y_t , and let $x_A(y) = P(A | Y_t = y)$. Define

$$H(A) = - \sum_A p_A \ln p_A, \quad H(A | Y_t) = - \int dP(y) \sum_A x_A(y) \ln x_A(y),$$

and

$$\epsilon_{\text{mix}}(t) := \frac{H(A | Y_t)}{H(A)} = 1 - \frac{I(A; Y_t)}{H(A)}.$$

Thus $\epsilon_{\text{mix}} = 0$ when the resolved baryonic state still determines the progenitor, and $\epsilon_{\text{mix}} = 1$ when it retains no recoverable progenitor information. This is not a metallicity-mixing fraction or a star-formation efficiency.

Mixing becomes durable only in a common well that retains the participating gas. The sharp astrophysical proxy is

$$V_{\text{thr}}(z) \simeq \begin{cases} 17 \text{ km s}^{-1}, & \text{before reionization,} \\ 30 \text{ km s}^{-1}, & \text{after reionization,} \end{cases}$$

representing internal atomic-cooling heating before reionization and the photoheated ultraviolet background afterward. These velocities are matter-sector inputs, not capacity constants [135, 136].

Let $dF_Z(t_f; V_{\text{thr}})$ be the mass-weighted first-assembly measure for systems satisfying that collective retention condition. First assembly starts the common-well mixing clock; it is not itself the carrier-failure event. Define the cumulative mixing kernel

$$K_{\text{mix}}(\tau; M, z) := P[\text{recoverable progenitor identity has been lost by delay } \tau].$$

The release history is then the delayed convolution

$$F_{\text{rel}}(t) = \int_0^t K_{\text{mix}}(t - t_f; M, z) dF_Z(t_f; V_{\text{thr}}), \quad \omega_{\Lambda,0} = \eta_* \frac{\omega_b}{\epsilon} I_{\text{rel}},$$

with

$$I_{\text{rel}} := \int_0^1 a^{-3} dF_{\text{rel}}(a).$$

The earlier instantaneous product $dF_{\text{rel}} = \epsilon_{\text{mix}}(a) dF_Z$ is recovered only if the mixing kernel is effectively local in assembly time. It is therefore retained as a short-delay approximation, not as the defining law. The distinction is important because finite-time turbulent mixing shifts release to later, less strongly a^{-3} -weighted epochs.

This also changes the interpretation of the former $\epsilon_{\text{mix}}^{\text{cov}} = 0.8529$ normalization. The Poisson expression measures the probability that a parcel is covered by at least one nominal mixing volume. It does not measure $1 - I(A; Y)/H(A)$: contact with one progenitor's mixing region may preserve rather than erase its label. The cosmological normalization must therefore be computed from the tracer-derived K_{mix} and cannot be inferred from coverage alone.

O.11 Full rerouting at carrier failure

The nine-state Gram matrix fixes the marked amplitude $1/3 = 1/\sqrt{9}$; it does not fix how much of a failed source allocation changes stock. That coefficient follows instead from renewal. For the stationary memory family

$$K_a(b, b') = a \delta_{bb'} + (1 - a)p_*(b'),$$

faithful full-support resolution gives

$$H(B' | B) = g_{\text{share,eff}} - I(B; B') < g_{\text{share,eff}} \quad (a > 0),$$

so H.4 admits only $a = 0$. A stable mark obeys

$$\nabla_\mu J_{c,A}^\mu = \Gamma_{\text{form},A} - \Gamma_{\text{rel},A} = 0$$

until its supporting baryonic condition fails. At such a failure the only renewal-compatible stock update is a complete fresh draw:

$$\boxed{\begin{aligned} \mathbf{E}_A &= \mathbf{1}[\text{carrier of } A \text{ has lost progenitor identity}], \\ f_{\text{reroute}} &= 1 \quad \text{for each failed carrier.} \end{aligned}}$$

The coefficient is therefore unity within the selected renewal branch. All remaining uncertainty lies in the astrophysical probability of carrier failure, ϵ_{mix} , rather than in a microscopic routing fraction.

O.12 First-entry assembly history and hydrodynamic mixing kernel

The carrier-loss observable is

$$\epsilon_{\text{mix}}(t) = 1 - \frac{I(A; Y_t)}{H(A)}.$$

The former Poisson estimate $1 - \exp[-N_{\text{eff}} M_{\text{mix}} / (f_b M_h)] = 0.8529$ measures geometrical coverage only. A parcel covered by one progenitor's mixing region can still retain essentially complete progenitor information, so coverage is not the observable used in the release law.

Absorbing first-entry history. The assembly history is fixed by an excursion-set first-entry construction rather than by taking a running maximum of an instantaneous collapsed fraction. The linear variance $S(M) = \sigma^2(M)$ is computed from the smooth Eisenstein–Hu baryon-suppressed transfer function [125] normalized to $\sigma_8 = 0.811$. The retained-well threshold is

$$V_{\text{crit}}(z) = \begin{cases} 17 \text{ km s}^{-1}, & z > z_{\text{re}}, \\ 30 \text{ km s}^{-1}, & z < z_{\text{re}}, \end{cases} \quad z_{\text{re}} = 7.66,$$

with the standard virial relation [131] used to obtain $M_{\text{crit}}(z)$; the two thresholds are the atomic-cooling scale before reionization [136, 135] and the photoheated-gas retention scale found in ultraviolet-background simulations afterwards [85, 86]. Define

$$B(z) = \frac{\delta_c}{D(z)}, \quad S_{\text{crit}}(z) = \sigma^2[M_{\text{crit}}(z)],$$

and for a Markov excursion-set trajectory let $\mathcal{M}(S) = \max_{0 < s < S} \delta(s)$. The cumulative fraction that has ever entered an eligible retained well by redshift z is

$$\boxed{F_Z(z) = P \left[\max_{z' \geq z} \{ \mathcal{M}[S_{\text{crit}}(z')] - B(z') \} \geq 0 \right].}$$

The crossing is absorbing: the rise of the retention threshold at reionization cannot “unassemble” a trajectory that entered an eligible well earlier. A 3×10^5 -trajectory implementation with exact Brownian-bridge maxima between the required variance points gives

z	10	8	7	6	5	3	1	0
F_Z	0.0138	0.0388	0.0522	0.0704	0.104	0.243	0.509	0.671

for the reference background. This executable F_Z is the assembly input used below; it is a specified excursion-set completion, not a substitute for a future hydrodynamic merger tree.

Finite mixing delay. First entry starts the common-well mixing clock. It does not cause instantaneous release. The correct release history is the convolution

$$F_{\text{rel}}(t) = \int^t K_{\text{mix}}(t - t_f; M, z) dF_Z(t_f).$$

This replaces the former product law $dF_{\text{rel}} = \epsilon_{\text{mix}} dF_Z$. The kernel is a cumulative loss of progenitor mutual information after first eligible assembly.

Three-dimensional hydrodynamic surrogate. A controlled surrogate evolves isothermal compressible Euler flow in a periodic box of side $2R_{\text{vir}} = 2.6 \text{ kpc}$ with ten passive progenitor scalars, using the halo mass, virial radius, and turbulent diffusivity of the atomic-cooling model of Ji et al. [137] and the ten-progenitor count of Greif et al. [135]. The fiducial gas has

$$c_s = 11.7 \text{ km s}^{-1}, \quad v_{\text{rms}} = 8 \text{ km s}^{-1}, \quad D_{\text{sgs}} = 10^{-4} \text{ kpc}^2 \text{ Myr}^{-1},$$

with turbulent power on 0.26–0.43 kpc scales. The cumulative mutual-information loss is measured inside R_{vir} . Varying only the initial radial extent of the ten progenitor centres gives

$\tau [\text{Myr}]$	$1.0R_{\text{vir}}$	$0.5R_{\text{vir}}$	$0.3R_{\text{vir}}$	$0.25R_{\text{vir}}$	$0.20R_{\text{vir}}$
70	0.199	0.399	0.585	0.668	0.761
150	0.367	0.579	0.756	0.817	0.876
250	0.519	0.708	0.846	0.888	0.926
500	0.768	0.875	0.938	0.956	0.971

The full-halo result is stable at the tens-of-percent level under resolution, turbulent-velocity, diffusivity, and random-seed controls and never approaches 0.85 at 70 Myr. Initial progenitor concentration is the dominant surrogate variable.

This is not the decisive cosmological hydrodynamic simulation. It omits self-gravity, cosmological filamentary infall, radiative cooling and chemistry, star formation and feedback, unequal progenitor masses, and adaptive refinement. Its role is to replace the coverage proxy by an executable information-loss kernel and to expose the astrophysical quantity the full zoom must determine.

Background normalization with the fixed first-entry history. Convolving the measured kernels with the absorbing F_Z above, without a multiplicative mixing parameter, gives

$R_{\text{prog}}/R_{\text{vir}}$	1.0	0.5	0.3	0.25	0.20	0.12
I_{rel}	57.59	69.62	79.26	82.95	87.22	91.48
h	0.586	0.622	0.649	0.659	0.670	0.681
$\Omega_{\Lambda,0}$	0.595	0.640	0.670	0.680	0.690	0.700

for the reference physical baryon density and the theory-fixed η_* . The $0.12R_{\text{vir}}$ run is an exploratory concentration control, not a prediction. These numbers are normalization diagnostics only: the earlier compressed CMB+BAO table used a superseded assembly history and is not carried forward as evidence for the completed branch. The final distance and growth likelihood must be evaluated only after the cosmological zoom fixes the physical K_{mix} .

The gate and the kernel are not separately identified. The retained-well threshold and the mixing delay enter the normalization through one integral, and they trade against each other. With the earlier running-maximum Zel’dovich first-crossing history [124] and a single-timescale kernel $K = 1 - e^{-\tau/\tau_{\text{mix}}}$, the abundance is

V_{crit}	$\tau_{\text{mix}} = 0$	0.1 Gyr	0.3 Gyr	0.5 Gyr
17 km s ⁻¹	1.17	0.95	0.72	0.60
25 km s ⁻¹	0.83	0.69	0.54	0.46
30 km s ⁻¹	0.70	0.59	0.47	0.40

A pre-reionization gate with the slow full-halo kernel ($\tau_{\text{mix}} \simeq 0.3$ Gyr) and a post-reionization gate with a fast concentrated kernel both return the observed value. The abundance is monotonic in mixing speed at fixed gate, so a one-parameter family of kernels always crosses the preferred I_{rel} once; the crossing point is not a prediction. The quantity the zoom must fix is the pair $(V_{\text{crit}}(z), K_{\text{mix}}(\tau; M, z))$, and the falsifiable statement is that the pair nature produces gives $I_{\text{rel}} \simeq 85\text{--}95$ together with negligible identity loss before first eligible assembly.

Large-scale release bias. The same first-entry construction fixes the response of release locations to a long-wavelength density mode. Peak-background split shifts the excursion barrier as $B(z) \rightarrow B(z) - \Delta_L$. Propagating the shifted first-entry history through the same frozen mixing kernel gives

$$\Gamma = \frac{\dot{F}_{\text{rel}}}{1 - F_{\text{rel}}}, \quad \mathcal{R}_{\Gamma}(a) = \frac{1}{D(a)} \left. \frac{\partial \Gamma(a|\Delta_L)}{\partial \Delta_L} \right|_0, \quad \delta\Gamma = \mathcal{R}_{\Gamma}(a)\Delta_m.$$

No free assembly-bias coefficient is introduced. By the volume-singlet and constitutive results of O.2–O.3, however, this spatial hazard response only redistributes the internal source-resolved and reallocated capacity ledgers; it cancels from the leading total metric-visible capacity perturbation for $k \neq 0$.

Decisive cosmological zoom. The remaining astrophysical calculation is a self-gravitating cosmological hydrodynamic zoom with passive Lagrangian progenitor tags. It must label retained gas before atomic-cooling assembly, include the relevant infall, cooling, feedback, and reionization physics, state the partition Y_t on which the carrier is resolved (finer regions retain more progenitor information, and the theory has not yet fixed that scale), and report

$$\epsilon_{\text{mix}}(t, M, z) = 1 - \frac{I(A; Y_t)}{H(A)}, \quad K_{\text{mix}}(\tau; M, z).$$

That kernel is to be frozen before the full CMB, DESI, Dovekie, lensing, ISW, and growth likelihood is evaluated. A slow full-halo-like kernel, substantial identity loss in minihaloes, or excessive early release would falsify this dark-energy extension without changing the static gravity sector.

O.13 Marked dilation, unit event coefficient, and energy ledger

Let E_A be the carrier-failure effect in O.11. Let $|\alpha\rangle_M$, $\alpha = 1, \dots, 9$, be the orthonormal marked states of $V_1^P \otimes V_1^H$, and let $W_{\alpha A}$ be mutually orthogonal history embeddings. The release isometry is

$$V_A^{\text{rel}}|\psi\rangle = \sqrt{I - E_A}|\psi\rangle|0\rangle_M + \frac{1}{3} \sum_{\alpha=1}^9 |\alpha\rangle_M W_{\alpha A} \sqrt{E_A}|\psi\rangle.$$

Because $W_{\alpha A}^\dagger W_{\beta A} = \delta_{\alpha\beta} I$,

$$(V_A^{\text{rel}})^\dagger V_A^{\text{rel}} = I - \mathbf{E}_A + \frac{1}{9}(9\mathbf{E}_A) = I.$$

Thus the event reroutes the failed allocation in full, while the $1/3$ amplitude only normalizes its nine equal marked outputs. The charged recurrence fugacity ζ_* is not an event probability.

H.11 gives exact reference decoupling after renewal,

$$(\text{id}_R \otimes \mathcal{E}_*)(\rho_{R,\text{ren}}) = \rho_R \otimes \rho_*.$$

If the energetic ledger reads the closure budget after that refresh, conditioning on a carrier failure leaves

$$\rho_{\text{closure}|\text{rel}} = \rho_*, \quad \langle K^2 \rangle_{\text{rel}} = \langle K^2 \rangle_{\eta_*},$$

and therefore

$$\eta_{\text{drain}} = \frac{3/2}{\langle K^2 \rangle_{\eta_*}} = \eta_* = 0.0298668443935.$$

The event coefficient $f_{\text{reroute}} = 1$ and the energy share η_* do different jobs. The former is fixed by faithful renewal at carrier failure; the latter still inherits the named post-refresh ledger premise.

O.14 Conditional normalization and status

The normalization is now expressed only through the delayed, physically measured release history,

$$\omega_{\Lambda,0}^{\text{pred}} = \eta_* \frac{\omega_b}{\epsilon} I_{\text{rel}}, \quad I_{\text{rel}} = \int_0^1 a^{-3} dF_{\text{rel}}(a), \quad \Omega_{\Lambda,0}^{\text{pred}} = \frac{\omega_{\Lambda,0}^{\text{pred}}}{h^2}.$$

The value of h is solved simultaneously from the O.3 flatness equation. The former arithmetic $0.80 \times 0.8529 = 0.682$ is retired: its assembly history was instantaneous and 0.8529 was a coverage probability rather than progenitor mutual-information loss.

The logical grade is now:

- Identity-free capacity cannot record stream assignment, and faithful renewal makes rerouting complete when the baryonic progenitor carrier fails.
- The fixed-event history space has one volume-conjugate singlet. Only $\eta_* \langle \Gamma_{\text{rel}} \rho_u \rangle_\Sigma$ changes the homogeneous cosmological term, and $\delta \rho_\Lambda(k \neq 0) = 0$.
- Carrier loss creates no new material or scalar component. The metric-visible remainder is reallocated source-associated capacity of the retained coarse carrier; the orthogonal marked-history output has no independent zeroth-derivative metric stress.
- The total source-associated capacity obeys $\bar{\rho}_C = \rho_{c,0}^{\text{wb}} a^{-3} (1 - \eta_* F_{\text{rel}})$. For nonzero spatial modes, internal release-hazard fluctuations cancel between the source-resolved and reallocated ledgers, leaving no new dark-energy sound speed, anisotropic stress, or local-vacuum gradient force.
- The assembly input F_Z is fixed by the absorbing first-entry excursion-set algorithm of O.12. The completed three-dimensional compressible-flow surrogate directly measures the delayed mutual-information kernel and demonstrates strong dependence on progenitor geometry.

- The retention gate and the mixing kernel are degenerate in the normalization (O.12): a slow full-halo kernel with the pre-reionization gate and a fast concentrated kernel with the post-reionization gate give the same abundance, with effective equations of state that agree to 10^{-3} below $z = 1$. The zoom must fix the pair; the abundance cannot select it.
- The decisive remaining astrophysical input is the self-gravitating cosmological progenitor-tagged zoom. Its $K_{\text{mix}}(\tau; M, z)$ must be frozen before the final CMB+DESI+Dovekie and growth/lensing/ISW likelihood.
- The post-refresh energy ledger, fixed-measure continuum premise, volume-conjugacy lemma, and initial condition $\Lambda_i = 0$ retain their previous conditional grades.

Once the physical mixing kernel is frozen, convolving it with the fixed first-entry history and evolving the homogeneous drain and one-metric perturbations determines the likelihood without a phenomenological release amplitude or perturbation-bias parameter. A slow full-halo-like kernel, substantial pre-gate identity loss, excessive early release, a persistent effective $w > -1$ branch, or a frozen history inconsistent with cosmological data falsifies this extension without changing the static gravity sector.

Appendix P: Record Dynamics, Organization, Computation, and the Growth of Complexity

Organized complexity accumulates through three distinct processes. Renewal exports each previous state into history, beginning from a minimal-record boundary. For the canonical defect, the source cost also measures how quickly its history becomes distinguishable from vacuum. Gravitational growth builds correlations among retained records. The primitive orientation-matching interaction generates arbitrary local unitaries and composes across a connected network. Finally, collective loss of a baryonic source-clock carrier gives a one-signed release law for the marked record share of committed capacity.

The grades differ across this chain. Long-term storage still requires fresh, nonrecycled history capacity. Realized computation requires the physical dynamics to sample the available gates nondegenerately. Interpreting marked release as coarse-grained record loss also inherits the source-clock premise, progenitor-mixing history, post-refresh ledger, and history orientation of Appendix O.13 and Section 20.

Here a *record* normally means a state written to the history register by the export isometry in Appendix G's Stinespring dilation. P.3 also uses a weaker definition: any persistent physical distinction that separates a defect history from vacuum history.

P.1 Grading conventions

The status terms follow Section 26. A *closed* result is derived from the stated postulates and ensemble. A *conditional* result also requires a named physical reading or an unfinished microscopic completion. Numerical verification is tracked separately. Section P.8 distinguishes reproduced calculations, single-instance results awaiting a fresh check, blind calculations against sealed targets, and exploratory calculations that carry no evidential weight. These results refine the theory's internal structure; they do not change any of the probability ledgers defined elsewhere in the paper.

P.2 The reversible-record theorem pair

One-step preservation. The local renewal channel replaces the current cell state by the fixed state ρ_* ,

$$\mathcal{E}_*(\rho) = \rho_* \text{Tr } \rho.$$

Viewed only on the renewed cell, this map appears to erase the input. Its reversible dilation transfers the input to the complementary history channel $\mathcal{C} : A \rightarrow H$. That channel is an isometric embedding, $\mathcal{C}(\rho) = V\rho V^\dagger$, and therefore has an exact recovery map satisfying $\mathcal{R} \circ \mathcal{C} = \text{id}_A$. Trace distance, fidelity, relative entropy, and every other isometry-invariant distinguishability measure are preserved in the history register. One renewal step transfers the old state; it does not destroy its information.

Cumulative capacity. Suppose update n writes into a fresh independent factor of dimension d_n . After N updates,

$$\dim \mathcal{H}_{\text{hist},N} = \dim \mathcal{H}_{\text{hist},0} \prod_{n=1}^N d_n.$$

The capacity is therefore at least exponential when every $d_n \geq 2$. For the nine-state marked register it is 9^N before gauge constraints. H.11 and P.6 show that, when these are simultaneously retained independent factors and their projected bridges survive the host constraints, this count is realized as the quantum composition $\mathcal{H}_N = (\mathbb{C}^9)^{\otimes N}$ rather than merely as a storage dimension. This cumulative statement requires fresh factors and forbids destructive recycling of earlier records. That requirement remains an open completion.

P.3 The visibility–source identity

The fixed-type identity. The canonical elementary defect excludes one channel label m from one face f . Let P_* be the vacuum distribution on boundary states and let $A_m = \{b : b_f \neq m\}$ be the allowed subset after the exclusion. The defect distribution is the conditioned distribution $P_{\text{def}} = P_*(\cdot | A_m)$. Direct substitution gives

$$D(P_{\text{def}} \| P_*) = -\ln P_*(A_m) = \Delta F_m.$$

The three expressions describe the same likelihood ratio. Spatially, ΔF_m is the defect’s free-energy insertion into the vacuum ensemble and hence the source cost used in Appendix C.5. Temporally, the relative entropy is the Stein exponent: for many independent records, it is the asymptotic error exponent for distinguishing defect history from vacuum history. This interpretation uses the memoryless dressing of Postulate III. On the certified 1680-state ensemble at η_* , the seven channel values lie between 0.1493 and 0.1582 nats.

Isotropic benchmark and Jensen structure. Write $p_m = P_*(b_f = m)$ for the one-face marginal. Since $\sum_m p_m = 1$, averaging the allowed probabilities before taking the logarithm gives $-\ln[\frac{1}{7} \sum_m (1 - p_m)] = \ln \frac{7}{6}$, which is the isotropic source insertion in C.5. Averaging the seven log costs instead gives

$$\bar{D} = -\frac{1}{7} \sum_m \ln(1 - p_m) \geq \ln \frac{7}{6},$$

by Jensen’s inequality, with equality only for a uniform marginal. On the closed ensemble, $\bar{D} - \ln(7/6) = 5.7 \times 10^{-6}$ nats. Admissibility closure keeps the face marginal within 10^{-5} of isotropy, so the source cost and the record-discrimination rate agree to that accuracy. The exact identity

$$H(P_*) - H(P_*(\cdot | A)) = D(P_*(\cdot | A) \| P_*) + \eta_* [\langle K^2 \rangle_{P_*} - \langle K^2 \rangle_{P_*(\cdot | A)}]$$

shows that the small variation among channels comes from closure-energy anisotropy rather than numerical error.

Sufficiency. The likelihood ratio depends only on the indicator $X(b) = \mathbf{1}_{A_m}(b)$, so the one-bit defect statistic is sufficient: collapsing the 1680-state configuration to the scalar “constraint satisfied or not” loses no relative information about this defect. The scalar source projection in C.5 is therefore lossless for the canonical exclusion defect.

Projective form of the same source cost. Embed a probability distribution by its square-root vector,

$$|\Omega(P)\rangle = \sum_b \sqrt{P(b)} |b\rangle.$$

For the conditioned defect $P_{\text{def}} = P_*(\cdot | A_m)$,

$$\langle \Omega_{\text{def}} | \Omega_* \rangle = \sqrt{P_*(A_m)}.$$

The visibility–source identity therefore has the equivalent projective form

$$\boxed{\Delta F_m = -\ln |\langle \Omega_{\text{def}} | \Omega_* \rangle|^2 = -2 \ln \cos \theta_m,} \quad \theta_m = d_{\text{FS}}(\Omega_{\text{def}}, \Omega_*).$$

For the isotropic exclusion $P_*(A_m) = 6/7$, this gives $\theta_m = \arccos \sqrt{6/7} = 0.38760$ radians. For N independent records, the squared overlap is $e^{-N\Delta F_m}$. The gravitational source cost is therefore also the exact history-orthogonalization exponent for this canonical defect family.

Contrapositive and scope. Full support implies $P_*(A) < 1$ for every genuine exclusion, so the relative entropy is positive. Conversely, $D = 0$ implies $P_{\text{def}} = P_*$ and hence no exclusion defect. Within this defect class, a record-invisible excitation has no relative defect information, no scalar source insertion, and no elementary defect mass. A coherent defect outside the exclusion family could evade the one-bit sufficiency result. Its exclusion still depends on the existing GFT singlet-gap gate.

A committed cell that is exempt from refresh leaves its record on the perimeter. Joint face admissibility changes the neighboring marginals by 0.0020 nats per adjacent slot for an exclusion and by 0.1499 nats for a pinned face. The same neighbor coupling both writes this perimeter record and produces the capacity strain that gravitates. A defect that left no record in any neighbor would therefore have no gravitational source, contrary to Postulate II.

For the canonical defect, the source strength in C.5 and the rate at which its history separates from vacuum history are the same relative entropy. The weak-field source calculation and Postulate III use one shared constant.

P.4 Boundary, arrow, and well-foundedness

The minimal-record boundary. Many-Pasts contains neither a second time coordinate nor an archive outside the physical records. If no record survives, the statements “an earlier era occurred and every trace vanished” and “no earlier era occurred” describe the same physical state. The initial boundary is therefore the state with the least record content. A state already containing many records cannot serve as that boundary because the records themselves specify a past. Unrecorded prehistory is not a separate variable in this ontology.

The boundary measure in G.7. Without a boundary condition, the counting formula in G.7 favors high-entropy pasts and encounters the usual fluctuation objection. The minimal-record identification supplies the missing domain: the boundary measure μ_i is defined on the first physically distinguishable record-bearing state. The arrow of time is then a typicality statement conditioned on that state. For the canonical defect class, the contrapositive in P.3 also gives a simple matter sector at the boundary. This part of G.7 now has a named ontological condition in place of an unspecified low-entropy assumption.

The well-foundedness bound. Non-veridical durable records require spontaneous re-entry into the committed phase. At the conditional grade of Section 20, the weighting that orients decommitment suppresses such re-entry. The logical order is anchor $\rightarrow$ counting $\rightarrow$ orientation $\rightarrow$ one-signedness $\rightarrow$ veridicality; an audit confirms that no step assumes its own conclusion. The suppression is exponential. Taking the substrate tick to be the induced Planck time gives a total trial budget $\ln N_{\text{trials}} \simeq 5.6 \times 10^2$ nats (convention spread a few nats), giving

$$P(\text{false durable record of content } K) \lesssim e^{\ln N_{\text{trials}} - K}, \quad K_{\text{crit}} \sim 5.6 \times 10^2 \text{ nats} \approx 8 \times 10^2 \text{ bits}.$$

This threshold corresponds to roughly 0.4 GeV of committed mass-entropy. Macroscopic records ($K \gtrsim 10^{25}$) are veridical at odds indistinguishable from certainty. The result remains conditional on the pinned/synchrony reading of Section 20.

P.5 Organization: what actually grows

Conserved production. For fixed total mass, the identity in P.3 fixes total record production per tick. Complexity growth cannot come from an increasing production rate in this bookkeeping. It must appear as changing correlations among the records already being produced. This separates record amount from record organization, as in the information-growth literature [129].

Native locality. An exploratory exact-matching model measures how quickly the native injectivity channel loses record-stream correlation. Its mutual information is 1.95 nats per tick for adjacent cells, 0.155 with one intervening cell, and 0.0023 with two. The corresponding channel has $\lambda_2 = 0.1723$ and correlation length $\xi_{\text{hist}} = 0.57$ cells. This is a single-instance calculation and is excluded from evidential claims.

The calculation isolates the question for a completed model. The capacity/history channel is short-ranged by itself, whereas the gapless strain mode carries correlations over long distances. Gravity can therefore correlate records written in separated regions. Ordinary matter interactions and shared causes can also correlate them; the result is specific to the history sector's gravitational channel, not exclusive to gravity.

The monotonicity theorem. Assume that the one-stream marginals are stationary to leading order. Write the connected joint distortion of a pair of retained record streams as

$$p_{ij}(x, y; a) = p_i(x)p_j(y) [1 + D_+(a) \varepsilon_{ij}(x, y)],$$

with the marginal constraints $\sum_y p_j(y) \varepsilon_{ij}(x, y) = \sum_x p_i(x) \varepsilon_{ij}(x, y) = 0$ and $|D_+ \varepsilon_{ij}| \ll 1$. This is the precise sense in which the long-range strain mode changes the cross-correlation while the local record alphabet and its vacuum marginals remain fixed. Expanding the relative entropy of the joint state from the product state gives

$$I_{ij}(a) = D_{\text{KL}}(p_{ij} \| p_i p_j) = \frac{D_+^2(a)}{2} \sum_{x, y} p_i(x) p_j(y) \varepsilon_{ij}(x, y)^2 + O(\varepsilon^3).$$

Therefore, for the pairwise organization statistic $\mathcal{I}_H(a) = \sum_{i < j} I_{ij}(a)$,

$$\boxed{\mathcal{I}_H(a) = D_+^2(a) \mathcal{I}_H^{(2)} + O(\varepsilon^3), \quad \frac{d\mathcal{I}_H}{da} = 2D_+ D_+' \mathcal{I}_H^{(2)} + O(\varepsilon^3) \geq 0}$$

through every epoch in which $D_+'(a) \geq 0$. The coefficient $\mathcal{I}_H^{(2)}$ is a sum of squares and is nonnegative for every spectral shape. In a translationally invariant representation the same fact appears as $\widetilde{C}_S^2 = \widetilde{C}_S * \widetilde{C}_S$, a convolution of the nonnegative strain power with itself. Thus the sign does not depend on a special smooth or scale-free spectrum. This is a theorem about the history register, not the thermodynamic second law. Its physical hypothesis is the linear growing-mode scaling of the connected history distortion; the rooted-propagation calculation retains the registered consistency targets $G(r) \sim 1/r$ and $\mathcal{I}_{\text{hist}}(r) \sim G(r)^2 \sim 1/r^2$.

Under the square-root embedding of the joint record distribution, the same expansion gives

$$d_{\text{FS}}^2(p_{ij}, p_i p_j) = \frac{D_+^2}{4} \sum_{x,y} p_i(x) p_j(y) \varepsilon_{ij}(x,y)^2 + O(\varepsilon^3),$$

and hence

$$\boxed{I_{ij} = 2d_{\text{FS}}^2 + O(\varepsilon^3)}.$$

Thus the growing gravitational mode moves the classical record ray away from the product-state (Segre) submanifold; the increase in mutual information is twice the squared projective distance at leading order.

Within the stated weak-coupling regime, P.3 fixes the production rate and the result above fixes the growth of organization. Both statements describe the gravitational sector in history variables.

P.6 Realized complexity

Circuit complexity is the number of elementary gates needed to prepare the realized history state. Sustained growth requires a gate set that is both controllable and sampled from a non-degenerate measure. The fixed routing factors of H.9 commute, but they are not the complete one-mark dynamics. Once the marked fiber is identified with the retained fusion complement, its moving-frame transport and inherited Lorentzian mismatch spectrum generate a full local algebra. Algebraic capacity and physical sampling remain separate questions.

The primitive geometric interaction passes the algebraic controllability test exactly. Complexifying the spin-3/2 pair generators $\{iJ_i \otimes J_j\}$ replaces each one-slot Cartesian basis by the rational $\mathfrak{sl}(2)$ basis $\{E, F, H\}$. Closing the resulting nine pair generators under commutators gives rank 255 over both $\mathbb{F}_{1009}$ and $\mathbb{F}_{10007}$. Full rank modulo either prime certifies rank at least 255 over $\mathbb{Q}$; traceless 16×16 matrices have dimension at most 255. The complex closure is therefore $\mathfrak{sl}(16)$, and the original compact real closure is $\mathfrak{su}(16)$. The smaller $\mathfrak{so}(16)$ and $\mathfrak{sp}(16)$ alternatives are excluded by the same dimension certificate.

Primitive gluing generators. For $j_0 = 3/2$ the finite-width matching operator of Appendix B may be written, after transporting the right face into the left relational frame,

$$Q_f(g) = \frac{9}{4}I - \mathbf{J}_L \cdot \widetilde{\mathbf{J}}_R(g).$$

As the relative holonomy g varies, the transported Cartesian components span the nine bilinears $J_{L,i} \otimes J_{R,j}$ (the fixed orientation sign is already contained in $\widetilde{\mathbf{J}}_R$). The rank-255 certificate above

therefore applies to the orientation-matching interaction itself, without an auxiliary computational Hamiltonian. Full Lie rank is locally stable: a finite list of nested commutators contains a nonzero 255×255 minor, and that minor remains nonzero under sufficiently small perturbations of the generator coefficients. The universal algebra therefore persists in a neighborhood of the stated coefficients. Its theory-specific input is the capacity-selected representation $j_0 = 3/2$ and the associated $16 \rightarrow 7 + 9$ fusion structure.

The retained one-mark algebra. Let $L : Q \rightarrow \mathcal{H}_{\text{mark}}$ be the incidence isometry of H.9. Moving the relative face frame compresses the primitive rotations to

$$A_i = P_Q(I \otimes J_i)P_Q|_Q,$$

whose exact closure is $\mathfrak{so}(9)$. The primitive mismatch operator restricts to

$$Q_f|_Q = 6P_0 + 5P_1 + 3P_2.$$

After subtracting its common part, any nonzero relative Lorentzian phase is a real symmetric traceless generator. The $\mathfrak{so}(9)$ orbit of that generator spans $\text{Sym}_0(9)$, so on the real canonical phase space

$$\boxed{\text{Lie}(\mathfrak{so}(9), iQ_f|_Q) = \mathfrak{su}(9).}$$

Appendix H.11 derives the positive complex structure preserved by this 80-dimensional algebra and proves that the physical one-mark state space is $\mathbb{CP}^8$. Thus the local controllability statement no longer assumes a complex marked Hilbert space in order to obtain a complex unitary algebra.

Composition across a network. Pair controllability alone does not guarantee network controllability. A diagonal bridge shows the obstruction. Start with the full $\mathfrak{sl}(16)$ algebra on slots (1, 2) and couple slot 2 to a third spin-3/2 slot only through a diagonal bridge $H_2 \otimes H_3$. Every generated operator preserves the four eigenspaces of H_3 , so the closure is contained in the traceless block-diagonal algebra

$$\mathfrak{s}(\mathfrak{gl}(16)^{\oplus 4}), \quad \dim = 4(16^2) - 1 = 1023.$$

The exact modular calculation saturates this bound. A diagonal bridge can transport correlation while remaining incapable of rotating the new slot.

The physical network supplies the missing local algebra without a new interaction: slot 3 belongs to another face pair, and that pair's certified $\mathfrak{sl}(16)$ contains $I \otimes \mathfrak{sl}(4)_3$ as a subalgebra. The following tensor-product lemma then closes the composability question. Let $A \in \mathfrak{sl}(m)$ and $B \in \mathfrak{sl}(n)$ be nonzero. Then

$$\text{Lie}(\mathfrak{sl}(m) \otimes I, I \otimes \mathfrak{sl}(n), A \otimes B) = \mathfrak{sl}(mn).$$

Indeed, the adjoint representation of the simple algebra $\mathfrak{sl}(m)$ spans all of $\mathfrak{sl}(m)$ from any nonzero A , and similarly for B . Nested commutators therefore generate $\mathfrak{sl}(m) \otimes \mathfrak{sl}(n)$; together with the two local summands this is the standard vector-space decomposition of $\mathfrak{sl}(mn)$. With $m = 16$, $n = 4$ the three-slot algebra is thus

$$\boxed{\mathfrak{sl}(64)}, \quad \dim = 64^2 - 1 = 4095.$$

A deterministic exact modular breadth-first closure using the physical set $\{\mathfrak{sl}(16)_{12}, \mathfrak{sl}(4)_3, H_2 \otimes H_3\}$ independently reaches rank 4095 over $\mathbb{F}_{1009}$. This calculation checks the analytic tensor-product lemma. A second-prime implementation would strengthen its computational provenance.

The same lemma gives the network theorem by induction on a spanning tree. If a connected cluster of r spin-3/2 slots already carries $\mathfrak{sl}(4^r)$ and an adjacent physical face supplies a universal pair algebra on one cluster slot and a new slot, the inherited local $\mathfrak{sl}(4)$ and any nonzero entangling bilinear generate $\mathfrak{sl}(4^{r+1})$. Hence on any finite connected face network, provided every physical edge realizes the same two-slot controllability family,

$$\boxed{\mathfrak{g}_{\text{network}} = \mathfrak{su}(4^N)}$$

on the compact real form. Every connected slot participates in a certified pair algebra, so the result propagates over the spanning tree. This is an algebraic composability theorem; it does not determine which gates the physical path measure samples.

Composition of retained marks. The primitive bridge also survives the projection onto two neighboring marked complements. For pairs (1, 2) and (3, 4),

$$\begin{aligned} H_{QQ} &= (P_Q^{12} \otimes P_Q^{34})(H_2 \otimes H_3)(P_Q^{12} \otimes P_Q^{34}) \\ &= A_R \otimes A_L, \end{aligned}$$

with

$$A_R = P_Q(I \otimes H)P_Q|_Q, \quad A_L = P_Q(H \otimes I)P_Q|_Q.$$

Exact rational spin-3/2 evaluation gives

$$\text{rank } A_R = \text{rank } A_L = 8, \quad \text{Tr } A_R = \text{Tr } A_L = 0, \quad \text{Tr } A_R^2 = \text{Tr } A_L^2 = 38.$$

The interaction is therefore nonzero and cannot be reduced to a sum of local terms. Applying the tensor-product lemma with $m = n = 9$ gives

$$\boxed{\text{Lie}(\mathfrak{su}(9)_1, \mathfrak{su}(9)_2, iA_R \otimes A_L) = \mathfrak{su}(81)}.$$

By induction, a connected marked network has

$$\boxed{\mathcal{H}_N = (\mathbb{C}^9)^{\otimes N}, \quad \mathfrak{g}_N = \mathfrak{su}(9^N)},$$

provided adjacent retained marks coexist and the projected bridge survives the complete geometric gauge/intertwiner projection. The algebraic entangling mechanism is exact before that final host audit.

Commitment and the gate measure. Algebraic universality establishes computational capacity. A realized computation also requires a structured, non-equilibrium gate measure. The framework contains a parameter-free candidate for this bias: commitment persistently conditions the local capacity draws. Let $M(dg \mid b)$ be the as-yet-uncomputed microscopic kernel taking a local capacity state b to the relative holonomy/gate variable g . The equilibrium and canonical-defect gate measures are the push-forwards

$$\mu_*(dg) = \sum_b P_*(b)M(dg \mid b), \quad \mu_{\text{def}}(dg) = \sum_b P_{\text{def}}(b)M(dg \mid b).$$

The data-processing inequality and P.3 give the exact bound

$$\boxed{0 \leq D(\mu_{\text{def}} \parallel \mu_*) \leq D(P_{\text{def}} \parallel P_*) = \Delta F_m}.$$

Thus the same likelihood ratio that prices source strength and record visibility is the maximum information available to bias the local gate measure. The remaining test is sharp: one must

derive M and show $\mu_{\text{def}} \neq \mu_*$ (equivalently, nonzero information between the defect indicator and the gate variable). If that test passes, committed matter acts both as a record and as a persistent bias on the universal gluing interaction. If it fails, the controllability theorem remains valid but commitment does not program the gate distribution. Universality alone makes no claim of purposeful computation.

The primitive pair interaction can approximate an arbitrary two-slot unitary when its coefficients vary independently or are sampled in a sufficiently broad sequence. This supplies the algebraic capacity for general information processing. A derivation of chemistry would also need the low-energy spectrum, stable composites, gauge interactions, and physical control supplied by the continuum matter completion.

The remaining physical questions concern the gate measure and its control. The completed GFT and spin-foam dynamics must induce an open-support measure on the controllable family, and the commitment kernel above must be shown to deform that measure rather than being statistically invisible to it. Existing circuit-complexity bounds would then constrain realized growth, but algebraic controllability alone does not prove exact linear complexity growth or identify a useful algorithm.

At the native-cell level, the pinned initial configuration has an $O(N)$ preparation circuit and is therefore low-complexity. The preparation complexity of the full geometric condensate remains open with the physical gate-measure problem. P.3 derives record writing by canonical defects, P.5 derives organization by a specified growing strain mode, and P.6 derives the available computational algebra. Its physical sampling measure remains open.

P.7 The self-limiting loop and collective carrier loss

P.5 shows how the growing gravitational mode organizes retained records: to leading nontrivial order their mutual information grows as D_+^2 . The cosmological feedback uses a different operation: irreversible loss of the particular baryonic record that supports a Lagrangian source clock.

Identity-free committed capacity cannot record which stream it occupies. Two descriptions that differ only by a permutation of identical stream assignments export the same occupancy record,

$$|e_i\rangle = |e_j\rangle \implies \gamma_{ij} = 1, \quad C_A^{\text{rec}} = 1.$$

Thus neither shell crossing nor the old fold-record conventions supply a marked event. The release effect is instead

$$\boxed{E_A = \mathbf{1}[\text{carrier of } A \text{ has lost progenitor identity}].}$$

When gas from several progenitors becomes irreversibly mixed in one retained bound object, matter no longer distinguishes the source label while the collisionless committed allocation still carries it. The orphaned allocation is rerouted through the O.13 isometry.

Faithful full-support renewal admits no partial-memory stock update at that event, so $f_{\text{reroute}} = 1$. The history dependence is entirely in

$$\epsilon_{\text{mix}}(t) = 1 - \frac{I(A; Y_t)}{H(A)},$$

the fraction of progenitor information absent from the resolved baryonic state. The marked accumulation law is now delayed rather than instantaneous,

$$\boxed{F_{\text{rel}}(t) = \int_0^t K_{\text{mix}}(t - t_f) dF_Z(t_f), \quad \omega_{\Lambda,0} = \eta_* \frac{\omega_b}{\epsilon} \int_0^1 a^{-3} dF_{\text{rel}}(a), \quad \Omega_{\Lambda,0} = \frac{\omega_{\Lambda,0}}{h^2}.$$

The former 0.8529 coverage estimate is not the mutual-information loss. The completed three-dimensional compressible-flow passive-tracer surrogate gives about 0.20 information loss after 70 Myr for full-halo progenitor placement and demonstrates a strong dependence on central concentration. Its full-halo delayed background fails the compressed CMB+BAO diagnostic; the self-gravitating cosmological zoom must now replace the surrogate.

This also fixes the self-limiting direction without confusing the two a_0 roles. The relation $a_0(z) = \epsilon cH(z)$ controls the transverse thermal response. Release contains no a_0 : structure formation builds retained, turbulently mixed common wells; accumulated Λ then suppresses later growth and hence further carrier loss.

P.7a Three laws of the retained history register and the Landauer reading

The following “laws” are bookkeeping statements internal to the history sector. They are not replacements for the thermodynamic first, second, or third laws. Their value is that production, organization, and irreversible coarse-grained loss are now represented by three different quantities rather than by one ambiguous word “information.”

Law I: record production is priced by the same quantity that sources mass. For the canonical exclusion defect, P.3 proves exactly

$$D(P_{\text{def}} || P_*) = \Delta F_m.$$

The left side is the asymptotic per-record distinguishability rate and the right side is the source insertion. Through Mass–Entropy Equivalence the same committed entropy is read inertially as mass. Thus, at fixed committed defect inventory and refresh cadence, the record-production budget is fixed by the mass/source budget; complexity growth cannot be obtained merely by postulating an ever-increasing rate of record creation. The statement is exact for the canonical exclusion class and does not assert a universal rate for every possible composite matter state.

Law II: gravitational growth monotonically organizes retained records. Under the fixed-marginal weak-correlation hypothesis of P.5, the connected record distortion is linear in the growing strain mode whereas mutual information begins quadratically. Hence

$$\boxed{\mathcal{I}_H(a) = D_+^2(a) \mathcal{I}_H^{(2)} + O(\epsilon^3), \quad D'_+(a) \geq 0 \implies \frac{d\mathcal{I}_H}{da} \geq 0}$$

to leading nontrivial order. Gravity is not claimed to be the only source of correlations in matter; the theorem identifies the history sector’s own long-range strain channel and the sign of the organization it builds.

Law III: loss of a source-clock carrier has a one-signed energetic residue. Appendix O.13 supplies the operator identity behind the third statement. Let

$$\hat{H}_{\text{rel}} = \hat{K}^2, \quad \langle \hat{H}_{\text{rel}} \rangle_* = \langle K^2 \rangle_{\eta_*} = 50.2229154254,$$

and let the normalized marked record strand be generated by

$$\hat{H}_{\text{rec}} = \frac{1}{2} \sum_{a=1}^3 \hat{\xi}_a^2, \quad \langle \hat{H}_{\text{rec}} \rangle = \frac{3}{2}.$$

The dark-energy branch uses three stated conditions: (i) common conversion normalization $\varepsilon_K = \varepsilon_\xi$; (ii) a post-refresh energy ledger, which makes the closure state ρ_* independent of the

slow carrier-failure effect; and (iii) one-signed routing of the marked share out of the retained coarse stress budget. For an event selected by E_A , these give

$$\eta_{\text{drain}} = \frac{\langle \widehat{H}_{\text{rec}} \rangle}{\langle \widehat{H}_{\text{rel}} \rangle_*} = \frac{3/2}{\langle K^2 \rangle_{\eta_*}} = \eta_* = 0.0298668443935.$$

Faithful renewal separately fixes the stock coefficient to $f_{\text{reroute}} = 1$. Therefore, for physical released energy $E_{\text{rel}} \geq 0$,

$$E_{\text{drain}} = \eta_* E_{\text{rel}} \geq 0, \quad E_{\Lambda}(N) = E_{\Lambda}(0) + \sum_{j=1}^N \eta_* E_{\text{rel},j}$$

is monotone. Microscopically the $16 \rightarrow 7 + 9$ update remains isometric. “Erasure” means that progenitor identity is no longer recoverable from the retained baryonic algebra and the marked complement is absent from the local stress bookkeeping; it is not fundamental nonunitarity.

Landauer interpretation, with the scale dependence explicit. The energetic statement admits a Landauer-like interpretation but does not by itself prove a thermal Landauer process. Mass–Entropy Equivalence already supplies

$$\epsilon_{\text{rec}}(\ell) = \kappa_m(\ell) c^2$$

at the scale ℓ of the committed record. At the electron anchor,

$$\epsilon_{\text{rec}}(\lambda_e) = \frac{m_e c^2}{\ln 2} \simeq 0.737 \text{ MeV nat}^{-1}.$$

A literal thermodynamic identification $Q = k_B T \Delta S$ [130] would additionally require a reservoir and a fluctuation–dissipation relation; neither is used in the cosmological normalization.

Three ledgers and the scope falsifier. Ordinary thermal reset, quantum measurement, and particle reactions do not source the cosmological drain merely because records or entropy change. Only the marked source-clock channel selected by

$$E_A = \mathbf{1}[\text{the baryonic carrier has lost progenitor identity}]$$

enters O.3. This prevents the false inference that computation, annihilation, ionization, or generic environmental decoherence sources dark energy.

Decisive test. O.7 proves that the identity-free capacity register cannot carry primitive which-stream information. A three-dimensional compressible-flow tracer surrogate has now been completed and provides an intermediate audit: it measures the correct mutual-information observable, rejects the old coverage identification, and shows that progenitor concentration dominates the mixing rate. It is not the decisive astrophysical calculation because it lacks self-gravity, cosmological infall, cooling, feedback, and an actual merger tree. The registered test is therefore the cosmological zoom: tag gas by its Lagrangian progenitor, measure $K_{\text{mix}}(\tau; M, z)$ through first-galaxy assembly, freeze that output, and only then evaluate its a^{-3} -weighted release history against cosmology. The completed 48^3 phase harness remains a protocol control only; its $\gamma_{ij} = 1$, programmed mark, and independent K^2 draws test none of the carrier-loss dynamics.

The three laws govern different quantities: record production, gravitational organization, and coarse-grained loss. The first two are exact in their stated defect and weak-correlation domains. The third has an exact renewal/isometry core and remains conditional on the baryonic source clock, the progenitor-mixing history, the post-refresh ledger, and the one-signed history orientation.

P.8 Verification register

- *Reproduced twice by independent code, with all printed digits matched:* the 1680-state spectrum and multiplicities; η_* , $g_{\text{share,eff}}$, $\langle K^2 \rangle$, and $\text{Var}(K^2)$; and the rooted 140/69 reduction.
- *Checked by an independent fresh instance:* the P.3 relative-entropy identity, Jensen excess, sufficiency result, and exact decomposition.
- *Exact pair controllability certificate:* the P.6 spin-3/2 pair-generator closure has rank 255 over both $\mathbb{F}_{1009}$ and $\mathbb{F}_{10007}$, certifying $\mathfrak{su}(16)$ controllability. The calculation uses integer $\mathfrak{sl}(2)$ generators and exact finite-field elimination; an independent implementation would provide a further check.
- *Network composability:* the tensor-product Lie-algebra lemma proves the three-slot and connected-network result analytically once the edge $\mathfrak{su}(16)$ certificate is granted. A deterministic modular BFS on the first nontrivial triple reaches rank 4095 over $\mathbb{F}_{1009}$. This numerical check is not a premise of the analytic theorem. The diagonal-bridge control saturates the exact block-diagonal bound 1023.
- *Analytic weak-correlation theorem:* the P.5 monotonicity result has been reduced to the displayed fixed-marginal KL expansion; no numerical sign check is required. An independent proof read remains desirable. The P.4 trial budget remains a single-instance result requiring a fresh check before ledger entry.
- *Superseded and excluded from claims:* the constant-mass release proxy, the fixed-present- a_0 trigger, the single-ionization carrier, and the 0.588/0.625 fold multipliers. None implements collective loss of a baryonic progenitor record.
- *Coverage arithmetic reproduced but reclassified:* with $f_b = 0.1565$, $M_h = 10^8 M_\odot$, $M_{\text{mix}} = 3 \times 10^6 M_\odot$, and $N_{\text{eff}} = 10$, the Poisson formula gives $1 - e^{-1.9168} = 0.8529$. This is a coverage probability, not $1 - I(A; Y)/H(A)$, and the former $0.80 \times 0.8529 = 0.682$ normalization is excluded from prediction claims.
- *Completed three-dimensional tracer surrogate:* isothermal compressible Euler flow with ten passive progenitor labels gives full-halo $\epsilon_{\text{mix}}(70 \text{ Myr}) = 0.199$ at 24^3 , with resolution values 0.215 at 32^3 and 0.181 at 40^3 . Full-halo turbulence, subgrid- diffusion, and seed controls remain below 0.32 at 70 Myr. Concentrating the initial progenitors inside $0.3R_{\text{vir}}$ raises the fiducial result to 0.585, demonstrating that geometry rather than the old coverage arithmetic is the dominant surrogate uncertainty. This is an executable hydro control, not a cosmological zoom.
- *Completed protocol control, not a carrier-loss test:* the rebuilt 48^3 harness reproduces the quoted phase-healing and additivity numbers. Because $\gamma_{ij} = 1$, its record purity remains one; the programmed mark and independent K^2 draws do not test progenitor mixing or ledger ordering.
- *Registered and sealed:* the conjecture relating lepton-ladder records. For generation N , it predicts mutual information between channels equal to the ladder's quadratic term, $N^2 \ln(7/2)$ per pass. The muon supplies a zero-freedom pass/fail test.

The remaining complexity claims have been reduced to two existing open tasks: the GFT gate measure and the register-support scale. None of these results changes the inside, reviewer, or outside probability books.

Appendix Q: Gauge Invariance and the Geometry–Capacity Completion

The ordinary gravitational branch uses a microscopic geometry–capacity completion with two representation factors. The finite capacity alphabet cannot serve as a geometric Gauss index, so geometry and capacity occupy separate factors of the same substrate. An equivariant lock relates the selected V_3 capacity channel to geometric spin and propagates under maximal blocking.

The construction also supplies the finite matching and marked-transfer operators that set the physical scale. Its coefficient-free dilation handles collective loss of a baryonic source-clock carrier. Identity-free capacity cannot write a which-stream record, and faithful renewal fixes unit rerouting after carrier failure. The progenitor-mixing history remains an astrophysical input to be measured. In the equilibrium vacuum, the capacity factor adds no graviton polarization. On a boundary cut, it fixes the conversion between one capacity channel and geometric area.

The chain is

$$\begin{array}{c}
 V_{3/2}^L \otimes V_{3/2}^R \xrightarrow{Q_f} V_3^{\text{cap}} \oplus (V_0 \oplus V_1 \oplus V_2) \\
 \text{capacity data} \quad \neq \quad \text{geometric Gauss data} \\
 \mathcal{H}_{\text{geom}} \otimes \mathcal{H}_{\text{cap}} \otimes \mathbb{C}_{\text{orient}}^2 \xrightarrow{L_3} \text{one relationally locked geometry–capacity description} \\
 V_3^{\otimes N} \xrightarrow{F_N} V_{3N}, \quad F_N^{\text{geom}} L_3^{\otimes N} = e^{i\phi_N} L_{3N} F_N^{\text{cap}} \\
 \text{large-spin geometric witness} \longrightarrow \text{Einstein/TT semiclassical branch,}
 \end{array}$$

On the selected metric-Regge branch, this chain has a closed massless-TT infrared theorem and an exact perfect-action primitive realization. The host-underdetermination theorem shows that the capacity premises cannot fix the independent geometric gluing tensor or measure. Factorization and the one-cell lock therefore define a selected action-level realization of Postulate I. The underdetermination result, Schur uniqueness, maximal-channel propagation, equilibrium rigging-map lift, and the conditional area normalization are established below or by the cited exact lemmas.

Q.1 Two different meanings of closure

Two structures called “closure” appear in the microscopic theory and must be kept separate.

Gauss closure is the exact constraint associated with the local gauge redundancy of the geometric factor. Its generator annihilates gauge-invariant geometric states. It removes redundant descriptions and carries the standard spin-network/spin-foam representation data.

Capacity closure, or admissibility closure, is the physical statistical weighting of the finite routed capacity alphabet defined in Section 6. Its invariant K^2 measures the mismatch of a routed channel configuration from the regular tetrahedral closure pattern. It is not a constraint: every state in the nonempty ensemble has strictly positive K^2 , and different allowed states receive different Gibbs weights $e^{-\eta K^2}$.

Their roles are

$$\boxed{\text{Gauss closure: redundancy/constraint,} \quad \text{capacity closure: physical weighting.}}$$

Identifying them would erase the nonuniform capacity ensemble and the stationarity equation that fixes η_* . Q.4 gives the selected minimal separation.

Q.2 Primitive face matching and the selected seven-state channel

The capacity factor entering the completion is not an arbitrary seven-state flavor. It is the transmitted multiplet already selected by the primitive oriented face-matching problem of Appendix B. Two descriptions of one shared face each carry a primitive spin

$$j_0 = \frac{3}{2}, \quad \mathcal{H}_f = V_{3/2} \otimes V_{3/2} = V_0 \oplus V_1 \oplus V_2 \oplus V_3.$$

After transporting the two local frames into one relative orientation, the positive mismatch operator is

$$Q_f = \frac{1}{2} \left[(\mathbf{J}_L - \tilde{\mathbf{J}}_R)^2 - 2j_0 I \right].$$

On the coupled irrep V_J ,

$$Q_f|_{V_J} = \frac{1}{2} [12 - J(J+1)] I_{V_J},$$

so for $j_0 = 3/2$,

$$\boxed{Q_f = 6P_0 + 5P_1 + 3P_2, \quad \ker Q_f = V_3.}$$

Equivalently the finite-width matching transfer

$$T_s = e^{-sQ_f} = e^{-6s}P_0 + e^{-5s}P_1 + e^{-3s}P_2 + P_3$$

converges to P_3 as $s \rightarrow \infty$. Thus the transmitted capacity channel is

$$\boxed{V_3^{\text{cap}}, \quad \dim V_3 = 7,}$$

while the complete nonmaximal information is

$$\boxed{Q_{\text{cap}} = V_0 \oplus V_1 \oplus V_2, \quad \dim Q_{\text{cap}} = 9.}$$

The independent marked present/history construction of Appendix H.9 produces exactly the same multiplicity-free representation

$$\mathcal{H}_{\text{mark}} = V_1^P \otimes V_1^H = V_0 \oplus V_1 \oplus V_2.$$

This representation identity accounts for the recurring seven-state transmitted channel and nine-state marked complement. It does not identify either object with geometric magnetic data.

The value $j_0 = 3/2$ has the grade stated in Appendix B: conditional on identifying the complete nonmaximal primitive-fusion information with the independently fixed marked register, it is uniquely forced; without that incidence identification it remains the unique surviving point of the stated discrete audit. No smallest-spin rule is used here.

Q.3 Same-factor impossibility theorem

Suppose one tries to reuse the geometric Gauss representation factor itself as the capacity alphabet by identifying the capacity operator with the Casimir of the Gauss generator. The Gauss generator vanishes on gauge-invariant physical states, so its Casimir must vanish there. The certified capacity spectrum instead satisfies

$$K^2 \geq \frac{122}{3} > 0$$

on every state of the nonempty ensemble. Hence

$$\boxed{K_{\text{admissibility}}^2 \neq C_{\text{Gauss}}^2.}$$

Applying the geometric gauge projector only after inserting the Gibbs weight does not evade the contradiction: on physical states a Gauss-Casimir factor would reduce to unity, erasing the nonuniform distribution and removing the stationarity equation that defines η_* . The numerical spine therefore requires capacity data that are physical but gauge inert with respect to the local geometric redundancy.

This theorem rules out the earlier literal reading in which the capacity labels were identified with the open Peter–Weyl magnetic index of the geometric vertex. It does *not* by itself dictate a unique replacement. The factorized construction below is the selected minimal realization because it leaves standard geometric gluing intact while preserving every finite capacity calculation.

Q.4 Selected factorization: one substrate, two bookkeeping factors

As the selected action-level realization of Postulate I, the one-cell bookkeeping space is

$$\boxed{\mathcal{H}_{\text{cell}} = \mathcal{H}_{\text{geom}} \otimes \mathcal{H}_{\text{cap}} \otimes \mathbb{C}_{\text{orient}}^2.}$$

Here $\mathcal{H}_{\text{geom}}$ carries the ordinary gauge-reduced simplicial GFT/spin-foam data: geometric spins, magnetic indices, intertwiners, and the variables required by the geometric vertex. The factor $\mathcal{H}_{\text{cap}}$ carries the finite routed capacity alphabet, its admissibility operator, marks, and renewal variables. The final doublet carries the two microscopic orientations.

The tensor product separates operator roles within one substrate. Postulate I is implemented by the equivariant lock on the two relational frames in Q.5. The coupled theory must also pass the no-extra-mode and Ward-identity audits of Appendix H.10. The capacity factor is not a second freely propagating geometry.

The four capacity ports draw without replacement from the seven transmitted weights,

$$b_i \in \{-3, -2, -1, 0, 1, 2, 3\}, \quad b_i \neq b_j \quad (i \neq j),$$

so

$$\dim \mathcal{B} = P(7, 4) = 840, \quad \boxed{\Omega = 2P(7, 4) = 1680}$$

after orientation doubling. No geometric intertwiner multiplicity enters this count. Consequently the EPRL/GFT factor is free to retain the full intertwiner structure required by its dynamics; the finite 1680 count belongs only to the capacity/orientation code.

The local geometric gauge action is

$$U_{\text{gauge}}(h) = U_{\text{geom}}(h) \otimes I_{\text{cap}} \otimes I_{\text{orient}}.$$

Every capacity observable therefore has the form

$$O_{\text{cap}} = I_{\text{geom}} \otimes O \otimes I_{\text{orient}},$$

and obeys the strong commutation relation

$$\boxed{[O_{\text{cap}}, U_{\text{gauge}}(h)] = 0.}$$

Thus the injectivity predicate, K^2 , Z , $\langle K^2 \rangle$, $\text{Var}(K^2)$, the Gibbs weights, and η_* survive geometric gauge projection unchanged. The factorization therefore leaves the finite numerical spine unchanged.

Q.5 The one-cell equivariant lock and what it means physically

The two bookkeeping factors must describe the same relational orientation on the physical branch. Positive primitive fusion supplies V_3^{cap} ; the selected geometric branch carries the corresponding $j = 3$ rotational representation V_3^{geom} . The action-level lock is a nonzero $SU(2)$ -equivariant isometry

$$L_3 : V_3^{\text{cap}} \longrightarrow V_3^{\text{geom}}$$

satisfying

$$\boxed{L_3 U_{\text{cap}}(g) = U_{\text{geom}}(g) L_3 \quad (g \in SU(2)).}$$

The *existence* of this identification is the selected realization choice. Its form is then rigid. Since both sides are irreducible copies of V_3 ,

$$\text{Hom}_{SU(2)}(V_3, V_3) \cong \mathbb{C},$$

and Schur's lemma gives, after choosing a common relational basis,

$$\boxed{L_3 = e^{i\phi} I.}$$

The common phase is conventional unless another microscopic operator compares the two factors coherently. Therefore the lock introduces no continuous physical coupling.

The lock acts on the relational representation data entering the coupled vertex. It does not project the whole tensor product onto a diagonal span $|m\rangle_{\text{geom}}|m\rangle_{\text{cap}}$. The dynamical test is whether the locked interaction preserves the metric constraints without creating an independent light capacity pole. Appendix H.10 states those audits.

The marked complement has the same multiplicity-free structure,

$$Q_{\text{cap}} = V_0 \oplus V_1 \oplus V_2.$$

Each irrep occurs once, so any equivariant lock to the corresponding spatial-rotation content of the geometric response is unique blockwise up to phases. The matching of representations is exact; the stronger statement that the nine-state marked fiber is literally the ADM metric is neither required nor claimed.

Q.6 Maximal blocking propagates the lock uniquely

Appendix B supplies exact canonical maximal-channel maps that preserve the one-cell lock under coarse graining:

$$F_N^{\text{cap}} : (V_3^{\text{cap}})^{\otimes N} \rightarrow V_{3N}^{\text{cap}}, \quad F_N^{\text{geom}} : (V_3^{\text{geom}})^{\otimes N} \rightarrow V_{3N}^{\text{geom}}.$$

The highest-spin representation V_{3N} occurs with multiplicity one in $V_3^{\otimes N}$. Therefore the one-cell lock induces a unique block lock.

Lemma (equivariant lock under maximal blocking). There exists a normalized equivariant map

$$L_{3N} : V_{3N}^{\text{cap}} \rightarrow V_{3N}^{\text{geom}}$$

and a phase $e^{i\phi_N}$ such that

$$\boxed{F_N^{\text{geom}} L_3^{\otimes N} = e^{i\phi_N} L_{3N} F_N^{\text{cap}}.}$$

Proof. Both sides are normalized $SU(2)$ -equivariant maps from $(V_3^{\text{cap}})^{\otimes N}$ into the unique highest-spin copy V_{3N}^{geom} . Multiplicity one implies that their restrictions to the maximal irrep are proportional. Normalization makes the proportionality constant a phase. $\square$

Thus

$$V_3^{\text{cap}} \xleftrightarrow{L_3} V_3^{\text{geom}}$$

propagates automatically to

$$\boxed{V_{3N}^{\text{cap}} \xleftrightarrow{L_{3N}} V_{3N}^{\text{geom}}}$$

on the selected maximal-channel ray. For spin coherent states,

$$F_N|3, \mathbf{n}\rangle^{\otimes N} = |3N, \mathbf{n}\rangle,$$

so the same relational normal is preserved by coherent blocking.

This representation-theory result does not show that every interacting RG trajectory projects dynamically onto the maximal channel. The primitive positive matching and the diagnostic gaps of Appendix B select that ray in the stated homogeneous/coherent model. It is therefore an exact canonical maximal-channel blocking ray, not a unique full RG flow.

Q.7 Relational-frame covariance and the 840/1680 bookkeeping

A fixed channel basis is a relational-frame convention, not an absolute spatial frame and not an additional geometric Gauss redundancy. Let $R \in SU(2)$ denote the relational orientation of a cell and define

$$P_{\text{cap}}(R) = U_{\text{cap}}(R)P_{\text{cap}}U_{\text{cap}}(R)^\dagger, \quad K^2(R) = U_{\text{cap}}(R)K^2U_{\text{cap}}(R)^\dagger.$$

Under a common global rotation G ,

$$R \mapsto GR, \quad K^2(GR) = U_{\text{cap}}(G)K^2(R)U_{\text{cap}}(G)^\dagger.$$

Rank, spectrum, traces, and all finite state-counting invariants are therefore frame independent. On the capacity factor alone,

$$\text{rank } P_{\text{cap}}(R) = 840, \quad \text{spec } K^2(R) = \text{spec } K^2.$$

Including the independent orientation doublet,

$$P_{\text{code}}(R) = P_{\text{cap}}(R) \otimes I_{\text{orient}}, \quad \boxed{\text{rank } P_{\text{code}}(R) = 1680.}$$

The weighted trace satisfies

$$\text{Tr} \left[P_{\text{code}}(R) e^{-\eta K^2(R)} \right] = \text{Tr} \left[P_{\text{code}} e^{-\eta K^2} \right],$$

where K^2 acts trivially on orientation. Choosing $R = I$ in the finite enumeration therefore fixes a relational frame without breaking physical rotational covariance. Neighboring cells compare those frames through the transported face holonomy g_{LR} already used in Appendix B.

The integer tetrahedral witness makes the same point without representation language. For

$$v_1 = (1, 1, 1), \quad v_2 = (1, -1, -1), \quad v_3 = (-1, 1, -1), \quad v_4 = (-1, -1, 1),$$

one has

$$\left| \sum_i b_i v_i \right|^2 = 4 \sum_i b_i^2 - \left(\sum_i b_i \right)^2,$$

which is exactly the audited capacity-closure polynomial. It is a relational capacity invariant, not a geometric Gauss generator.

Q.8 Division of labor between geometry and capacity

The factorization is easiest to read as a division of operator roles. The two columns are not two substances; they are two sets of variables used by the selected microscopic realization of one substrate.

Geometric factor	Capacity/record factor
local Gauss redundancy and gauge projection	physical admissibility weights $e^{-\eta K^2}$
spin/intertwiner data and simplicial gluing	seven transmitted channel labels and four-port injectivity
Regge phase, metric constraints, and TT propagation	defects, exclusion, renewal, marks, and record bookkeeping
geometric area/flux observable	cut entropy and finite capacity weights
all-state geometric rigging-map problem	exact stationary renewal projector on the empty register

The equilibrium capacity sector has an exact consequence for the physical vacuum support. Its renewed quantum state is the faithful mixed state

$$\rho_* = \sum_b p_{\eta_*}(b) |b\rangle\langle b|,$$

not a pure rank-one state. On the ρ_* -GNS/operator Hilbert space define

$$\omega_*(A) = \text{Tr}(\rho_* A), \quad \Pi_*(A) = \omega_*(A)I.$$

Then

$$\boxed{\Pi_*^2 = \Pi_*, \quad \Pi_*^\dagger = \Pi_*, \quad \text{Ran } \Pi_* = \mathbb{C}I.}$$

Thus the replaceable equilibrium closure register contains exactly one stationary transfer direction. This does not erase the full-support mixed state ρ_* and does not exclude separately retained or conservation-protected collective capacity fields.

If

$$\eta_g : D_g \rightarrow D_g^*$$

is any positive geometric rigging map, the empty capacity decoration lifts it exactly by

$$\boxed{\eta_{\text{full}}(\phi \otimes A)[\psi \otimes B] = \eta_g(\phi)[\psi] \langle A, \Pi_* B \rangle_{\omega_*}.$$

After quotienting null states and completing,

$$\boxed{\mathcal{H}_{\text{phys}}^{\text{full,vac}} \cong \mathcal{H}_{\text{phys}}^g.}$$

So the factorized capacity decoration does not create an additional vacuum constraint solution or graviton polarization once a geometric rigging map exists. Persistent defect/marked sectors are outside this vacuum theorem and remain subject to the coupled interaction audits.

The same distinction applies to vacuum amplitudes. After homogeneous centering, the normalized reference amplitude obeys

$$\mathcal{A}_{\text{full}}^{\text{ref}} = \mathcal{A}_{\text{geom}}^{\text{ref}},$$

but for a generic empty geometry T ,

$$\mathcal{A}_{\text{full}}[T] = \mathcal{A}_{\text{geom}}[T] \hat{Z}_{\text{cap}}[T].$$

Centering removes the source-independent homogeneous extensive term; it does not force every geometry-dependent finite capacity contribution to unity.

Q.9 Action-level realization: from matching operator to physical scale

The capacity/record factor has an explicit action-level realization. The finite ensemble fixes the admissible states, the matching operator fixes their relative cost, and a local field action selects the homogeneous phase. The marked vertex then specifies the microscopic update. Its positive transfer spectrum supplies an energy gap, and the electron anchor converts the dimensionless dynamics into L_* and G_* . In schematic form,

$$\begin{array}{c} \text{finite ensemble} \rightarrow \text{operator} \rightarrow \text{action} \rightarrow \text{selected phase} \\ \rightarrow \text{transfer spectrum} \rightarrow \text{physical scale} \end{array}$$

Appendices B and H contain the derivations and countervertex audits. The equations needed for the action-level chain are collected here.

Phase selection from the primitive matching operator. For $j_0 = 3/2$, the positive face mismatch derived in Q.2 is

$$Q_f = 6P_0 + 5P_1 + 3P_2, \quad \ker Q_f = V_3.$$

Promote the complete primitive fusion space to a field Φ and consider the minimal homogeneous action

$$\Gamma_f = \int d^d x \left[Z |\partial \Phi|^2 + r \Phi^\dagger \Phi + \kappa \Phi^\dagger Q_f \Phi + \frac{u}{2} (\Phi^\dagger \Phi)^2 \right], \quad Z, \kappa, u > 0.$$

Writing ρ_J for the occupation of the coupled irrep V_J and $\rho = \sum_J \rho_J$, its homogeneous potential is

$$V = r\rho + \frac{u}{2}\rho^2 + 6\kappa\rho_0 + 5\kappa\rho_1 + 3\kappa\rho_2.$$

For $r < 0$ the unique support of the homogeneous minimum is therefore

$$\rho_3 = -\frac{r}{u}, \quad \rho_0 = \rho_1 = \rho_2 = 0,$$

with positive relative masses

$$m_{0|3}^2 = 6\kappa, \quad m_{1|3}^2 = 5\kappa, \quad m_{2|3}^2 = 3\kappa.$$

Within this minimal action, the same positive mismatch operator that defines coherent face matching selects the seven-state V_3 sector as the unique homogeneous phase. The displayed action controls phase selection in the representation-space/GFT sector. It does not supply the complete interacting Lorentzian geometric GFT parent.

Fresh-state preparation and the marked record fiber. The stationary capacity measure fixes the fresh-state amplitude

$$A_*(b) = Z^{-1/2} \exp\left[-\frac{\eta_*}{2} K^2(b)\right], \quad |A_*(b)|^2 = p_*(b).$$

Its Gaussian linearization supplies the normalized three-component closure strand used by the marked present/history register. The nine marked states are represented by the bilinear incidence

$$V_Q = \sum_{a,b=1}^3 d_{ab}^\dagger \xi_a^P \xi_b^H + \text{h.c.}$$

with $V_1^P \otimes V_1^H = V_0 \oplus V_1 \oplus V_2$. The reduced Gram matrix is I_9 , so equal marked weights follow from the two-strand contraction rather than from the dimension count alone. This is the explicit dynamical realization of the same nine-state complement that appeared representation-theoretically in Q.2.

Inherited one-mark quadratic action. Let $L : Q \rightarrow \mathcal{H}_{\text{mark}}$ be the exact incidence isometry and set $d = L(P_Q \Phi)$. Because the mark is the retained complement itself, the quadratic marked Hessian is the push-forward of the Q -restricted Hessian,

$$\mathcal{H}_D = L\mathcal{H}_Q L^\dagger, \quad \mathcal{H}_Q = \mathcal{K}_{\text{kin}} + M_c^2 I_Q + \kappa(6P_0 + 5P_1 + 3P_2).$$

This follows by changing variables in $\delta^2 \Gamma_f$; an independent defect-only quadratic term would be a different, nonminimal action. For a local relative frame the same substitution gives

$$A_\mu = L \partial_\mu L^\dagger, \quad Z |\partial(P_Q \Phi)|^2 = Z (D_\mu d)^\dagger D^\mu d.$$

The connection generates the real complement rotations, while the inherited 6:5:3 spectrum gives non-common Lorentzian phase frequencies. Appendix H.11 proves that these two parts generate $\mathfrak{su}(9)$ on the one-mark sector and force its $\mathbb{C}^9/\mathbb{CP}^8$ state geometry.

For each occupied unordered channel pair $e = (m, m')$, the two directed scalar returns have contraction norm

$$u = 2\eta_* \left(\frac{2}{7}\right)^2 = \frac{8\eta_*}{49}, \quad \alpha = \sqrt{1-u}.$$

The square root is fixed at amplitude level by the canonical reversible dilation; using $1-u$ in the amplitude would double count the no-event probability. On the 21 unordered pairs of the seven-channel branch this produces the exact internal factor α^{21} .

Finite marked-transfer action and one-step vertex. Using hard-core marked occupation n_d and auxiliary present/history Grassmann strands $\Gamma_{mm'}$, Appendix H.9 gives the finite marked action

$$\begin{aligned} S_{\text{mark}} = & g_{\text{share,eff}} n_d + \Lambda_d n_d (n_d - 1) \\ & + \frac{1}{2} \sum_{m < m'} \Gamma_{mm'}^T [1 + n_d n_m n_{m'} (\alpha - 1)] J \Gamma_{mm'}, \end{aligned} \quad \Lambda_d \rightarrow +\infty.$$

The auxiliary Majorana variables, equivalently represented by Grassmann integration, encode the antisymmetric present/history return; they are not additional propagating particles. Let W_* prepare the fresh closure state and reversibly export the previous replaceable register. Let Π_Q project onto the unmarked plus nine marked response states, and let $\tilde{U}_e$ be the controlled unitary dilation on pair e . The complete one-step finite transfer is

$$\mathcal{A}_v = \mathcal{G}_v W_* \Pi_Q \exp[-g_{\text{share,eff}} n_d - \Lambda_d n_d (n_d - 1)] \prod_{m < m'} \tilde{U}_{mm'}.$$

Here $\mathcal{G}_v$ denotes the geometric gluing tensor supplied by the selected host. The factors following $\mathcal{G}_v$ are the explicitly specified finite capacity/record decoration. Their internal trace gives

$$\zeta_* = 9e^{-g_{\text{share,eff}}} \left(1 - \frac{8\eta_*}{49}\right)^{21/2} = 0.005123584484947 \dots$$

The quantity ζ_* is the finite marked weight of this reversible vertex, not a phenomenological correction added after the calculation. The complete edge-Hessian audit in Appendix H.9 gives a conditioned kernel proportional to I_{21} on the internal incidence labels. Alternative contractions split the spectrum and define distinct countertheories.

Positive transfer spectrum and the physical scale. Faithful full-support renewal gives the state-weighted seven-channel determinant

$$r = e^{-7g_{\text{share,eff}}},$$

and the positive one-bit survival transfer

$$T_{\text{surv}} = 1 - r.$$

Its Euclidean transfer Hamiltonian therefore has the exact raw positive gap

$$E_{\text{raw}} = -\frac{\hbar}{\tau_*} \ln(1 - r).$$

The marked routing graph dresses the charged mode by

$$Z_e = (1 + \zeta_*)(1 + 7\zeta_*^2).$$

The transverse charged identification is

$$E_e = \frac{3}{2} Z_e E_{\text{raw}}.$$

Equating this lowest charged transfer gap with the electron rest energy $m_e c^2$ fixes the cadence,

$$\tau_* = -\frac{3}{2} Z_e \frac{\hbar}{m_e c^2} \ln(1 - e^{-7g_{\text{share,eff}}}),$$

and hence

$$L_* = c\tau_* = -\frac{3}{2} Z_e \lambda_e \ln(1 - e^{-7g_{\text{share,eff}}}).$$

The ordinary gravitational normalization then follows from the same length,

$$G_* \equiv \frac{c^3 L_*^2}{\hbar}.$$

The logical chain is therefore explicit:

$$Q_f \Rightarrow \Gamma_f \Rightarrow V_3 \text{ phase} \oplus Q \text{ retained dynamics} \Rightarrow \mathcal{A}_v \Rightarrow \zeta_*, Z_e \Rightarrow T_{\text{surv}}, E_e \Rightarrow \tau_*, L_* \Rightarrow G_*.$$

No additional continuous coefficient enters between the finite marked vertex and the electron-anchored scale.

Scope and remaining condition. The action-level chain specifies the finite capacity/record microdynamics used for scale setting: stationary renewal, marked response, reversible routing, the positive charged transfer gap, and the conversion to L_* . The remaining problem lies in the coupled host. Its geometric tensor $\mathcal{G}_v$ must preserve the metric constraints, Ward identity, physical inner product, and causal structure without adding a source-coupled light pole. Appendix H.10 states these audits. The status is

finite capacity/record action and transfer: specified and audited , V_3 homogeneous phase of the minimal action: proved , Q one-mark $\mathbb{C}^9/\mathbb{CP}^8$ reconstruction: proved in the quadratic sector , full coupled Lorentzian geometric parent: conditional/open .
--

Q.9a Marked GFT dilation and collective carrier loss

The GFT action already contains the finite nine-state marked complement and the exact memoryless renewal map. What it cannot contain on the identity-free capacity factor is a which-stream trigger. A permutation of identical stream assignments leaves the occupancy history unchanged, so the capacity environment exports the same state and

$$|e_i\rangle = |e_j\rangle, \quad \gamma_{ij} = 1.$$

Thus neither the prepared-ray complement nor a fold-purity factor is a cosmological control.

The slow source factor must instead include the retained baryonic state that carries the Lagrangian label A . On the enlarged geometry–capacity–matter space define

$$\mathbf{E}_A = \mathbf{1}[\text{the retained baryonic state no longer resolves progenitor } A].$$

The effect is collective and present-tense. Atomic-cooling and photoheated-gas retention enter the matter dynamics because irreversible mixing requires a common retained well; they are not local step functions of the primitive capacity register. Ionization alone, shell crossing, and ordinary environmental record production leave $\mathbf{E}_A = 0$.

Faithful full-support renewal fixes the stock change once $\mathbf{E}_A = 1$. The memory family

$$K_a(b, b') = a\delta_{bb'} + (1 - a)p_*(b')$$

has conditional entropy below $g_{\text{share,eff}}$ for every $a > 0$; hence the only admissible failed-carrier update is $a = 0$, a complete fresh draw. The corresponding event coefficient is

$$f_{\text{reroute}} = 1.$$

Let $|\alpha\rangle_M$, $\alpha = 1, \dots, 9$, denote the orthonormal marked basis and let $W_{\alpha A}$ be orthogonal history embeddings. The normalized GFT/source dilation is

$$V_A^{\text{rel}}|\psi\rangle = \sqrt{I - \mathbf{E}_A}|\psi\rangle|0\rangle_M + \frac{1}{3}\sum_{\alpha=1}^9|\alpha\rangle_M W_{\alpha A}\sqrt{\mathbf{E}_A}|\psi\rangle.$$

It obeys

$$(V_A^{\text{rel}})^\dagger V_A^{\text{rel}} = I - \mathbf{E}_A + \frac{1}{9}\sum_{\alpha=1}^9 \mathbf{E}_A = I.$$

The factor $1/3$ normalizes nine marked outputs; it is not the released stock fraction.

Under refresh-first ordering, H.11 gives

$$(\text{id}_R \otimes \mathcal{E}_*)(\rho_{R,\text{ren}}) = \rho_R \otimes \rho_*,$$

so conditioning on carrier failure leaves

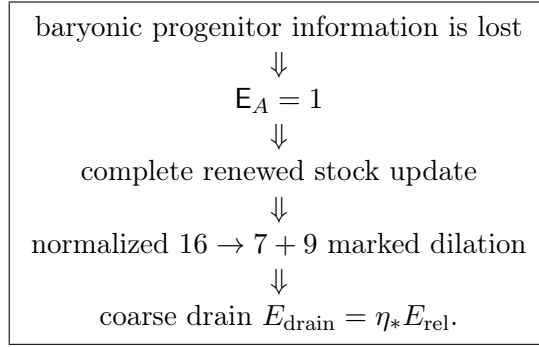
$$\langle K^2 \rangle_{\text{rel}} = \langle K^2 \rangle_{\eta_*}$$

and the energy share is

$$\eta_{\text{drain}} = \frac{3/2}{\langle K^2 \rangle_{\eta_*}} = \eta_*.$$

The post-refresh ledger remains a premise. It is logically separate from the exact unit rerouting coefficient.

The action-level sequence is therefore



The GFT action fixes the last three arrows. The first is an astrophysical matter-state transition measured by

$$\epsilon_{\text{mix}} = 1 - \frac{I(A; Y)}{H(A)}.$$

Appendix O.12 replaces the coverage proxy by an absorbing first-entry assembly history and a direct passive-tracer mutual-information kernel. Appendix O.2–O.3 then projects the universal marked drain onto the unique volume singlet: only its homogeneous component changes the cosmological term, while the nonzero-mode bookkeeping remains in source-associated capacity. This division is the strongest statement the present factorized action supports: it closes the routing and infrared bookkeeping without pretending that a local capacity operator derives galaxy-scale turbulent mixing.

Q.10 From the locked block to the Einstein witness

The representation-theory results above explain how the fixed capacity representation reaches the large-spin geometric regime without running the microscopic spin. On the maximal ray,

$$J_{\text{block}} = 3N \longrightarrow \infty,$$

while the primitive face remains $j = 3$. The block-lock lemma ensures that the capacity and geometric coarse representations continue to carry the same relational orientation.

Three external results support different parts of the geometric branch. Proper-EPRL constructions remove the unwanted asymptotic sectors at the vertex level and retain the Regge exponential in the controlled nondegenerate sector [99, 100]. Han-type spin-foam constructions provide a coupled large-spin/refinement witness approaching Einstein–Regge dynamics. In the controlled linearized construction, the low-energy excitations exhaust the smooth solutions of linearized Einstein gravity and give the two helicity-2 graviton modes [97, 98]. Area-Regge continuum analysis gives the same leading graviton dynamics, with the first length-metric correction at order $a^2 C^2$ [101]. These are separate compatibility results, not one theorem about a decorated vertex.

The selected metric-Regge branch therefore has controlled semiclassical Einstein witnesses at block scale. H.9a gives the full finite-cell-to-Einstein derivation. H.10 proves the strong-resolvent TT theorem (H_{TT}) and its exact gapped fixed- $j = 3$ perfect-action pullback. It also proves the complementary underdetermination result: because the selected host supplies $\mathcal{G}_v$ independently, the finite capacity premises cannot determine its geometric measure or nonperturbative phase. Q.8 shows that the equilibrium capacity factor creates no separate vacuum obstruction.

The retained side of the same $7+9$ split is now equally explicit. The maximal $V_3 = \ker Q_f$ channel carries the Einstein TT witness, whereas the positive-mismatch complement $Q = V_0 \oplus V_1 \oplus V_2$ carries the marked record whose inherited dynamics gives $\mathbb{C}^9/\mathbb{CP}^8$ in H.11. The two results are

complementary consequences of one fusion event, not two competing accounts of the infrared degrees of freedom.

Schematically, on the controlled linearized witness branch,

$$\mathbb{K}_{\text{phys}}(k) \sim Z_G k^2 P_{\text{TT}} + O(a^2 k^4) + O(\Lambda_{\text{sf}}^{-1} k^2),$$

This expression summarizes the common low-energy content of the separate continuum results. It is not a theorem for one decorated proper-EPRL model.

Q.11 Boundary cut and the channel–area normalization

The factorization also resolves the previously open channel–area conversion. On an open boundary cut, the capacity partner across the face is absent, producing the elementary binary exclusion increment

$$\boxed{\Delta S_f = \ln 2.}$$

The geometric boundary datum is not removed by that absence: the open face still carries V_3^{geom} . Thus a primitive horizon puncture in this selected realization is

$$\boxed{\text{geometric } j = 3 \text{ boundary face} \quad + \quad \text{one missing capacity partner.}}$$

Its entropy is the capacity entropy conditioned on the boundary geometry,

$$S_{\text{hor}} = S_{\text{cap}}(\text{cut} \mid g_{\partial}),$$

so multiplying an independent sevenfold geometric degeneracy into the same count would double count the geometry–capacity degree of freedom in this completion.

For a standard LQG puncture the kinematical area operator has eigenvalue [89]

$$A_{\text{LQG}}(J) = 8\pi\gamma_{\text{BI}} L_*^2 \sqrt{J(J+1)}.$$

The present coherent maximal-fusion branch represents a coarse face by the additive classical/Regge flux. For N aligned primitive faces,

$$J_N = 3N,$$

and additivity requires

$$A_N = 8\pi\gamma_{\text{BI}} L_*^2 J_N = 24\pi\gamma_{\text{BI}} N L_*^2.$$

Equivalently the physical coarse area operator on the maximal ray is

$$A_{\text{phys}}^{(N)} = Z_A(N) A_{\text{LQG}}(3N),$$

with

$$\boxed{Z_A(N) = \frac{3N}{\sqrt{3N(3N+1)}} = \sqrt{\frac{3N}{3N+1}}.}$$

This finite renormalization acts on the coarse observable; the microscopic spin and couplings remain fixed. It satisfies

$$Z_A(1) = \frac{\sqrt{3}}{2}, \quad Z_A(N) \rightarrow 1$$

as the block enters the large-spin Regge regime.

A patch containing N cut capacity channels carries

$$\frac{S_N}{k_B} = N \ln 2.$$

On the matched Einstein branch $L_P(G_*) = L_*$, and consistency with the already-established Bekenstein–Hawking normalization requires

$$N \ln 2 = \frac{A_N}{4L_*^2} = 6\pi\gamma_{\text{BI}}N.$$

Therefore

$$\gamma_{\text{BI}} = \frac{\ln 2}{6\pi} = 0.0367726000\dots,$$

and one primitive capacity cut corresponds to

$$A_{\text{ch}} = 4 \ln 2 L_*^2, \quad \rho_{\text{ch}} = \frac{1}{4 \ln 2 L_*^2}.$$

These relations fix the microscopic channel–area embedding within the selected factorized $j = 3$ realization and the coherent/Regge additive-area prescription. The Bekenstein–Hawking law supplies the continuum normalization to which the microscopic cut is matched; this calculation does not derive that law independently. The separate bulk identity $n_{\text{hor}} S_{\infty}^{\text{cell}} = 1/4$ remains a response-normalization cross-check, and n_{hor} is not a literal puncture count.

Q.12 Microscopic acceptance conditions

A complete decorated geometric vertex must satisfy all of the following conditions.

1. *Geometric branch.* The geometric factor retains its ordinary Gauss/intertwiner structure and admits the controlled EPRL/Regge semiclassical branch.
2. *Capacity branch.* The finite factor carries the selected V_3 seven-state multiplet, four single-copy relational ports, the same K^2 , and the same stationary measure used by the numerical spine.
3. *Equivariant lock.* Geometry–capacity interactions enforce the selected relational lock and its block propagation; they do not identify K^2 with a Gauss Casimir.
4. *Decoration locality.* Exclusion, renewal, marks, and record return act on the capacity/record factor, while geometric gluing retains its standard magnetic/intertwiner data.
5. *Hard-core occupation.* The four ports draw from one single-copy capacity multiplet, preserving the injective $P(7, 4)$ count.
6. *Orientation/parity.* The two orientation sectors are exchanged by an exact microscopic $\mathbb{Z}_2$ symmetry.
7. *Measure normalization.* The equilibrium capacity measure factorizes from gauge-orbit volume and is centered by the complete homogeneous capacity free energy of Section 25.
8. *Source control.* The slow baryonic source factor must carry a distinguishable Lagrangian progenitor label. Irreversible loss of that label in the retained baryonic state defines $\mathbf{E}_A$ and enters the normalized nine-state dilation. Prepared-ray phase failure, stream permutation, ionization alone, and generic environmental records are not sufficient controls.
9. *Coupled spectrum and Ward structure.* The full decorated action introduces no extra source-coupled zero pole, preserves the metric constraints and diffeomorphism Ward identity, and retains the G_* normalization.
10. *Physical-inner-product/causal structure.* A Euclidean transfer realization must satisfy the relevant complete-transfer positivity/reflection condition; a directly Lorentzian realization must instead possess a positive rigging form, consistent causal gluing, and healthy retarded poles.

The first eight conditions specify the selected microscopic completion. The first of them is an independent geometric-host acceptance condition, not a consequence of the finite capacity construction; treating it as a derivation of Einstein gravity would assume the missing input. The last two are the interacting audits that prevent the bookkeeping factorization from turning into an unintended second physical sector.

Q.13 Status ledger

The geometry–capacity chain has the following logical grades:

Primitive maximal-channel selection. Proved for the stated Q_f ; the $j_0 = 3/2$ identification carries the Appendix B grade.

Same-factor Gauss identification. Excluded.

Tensor-factor realization. Selected microscopic completion.

Existence of the one-cell relational lock. Selected realization condition.

Form and uniqueness of a nonzero lock. Proved by Schur’s lemma.

Maximal block-lock propagation. Proved.

Gauge-invariant capacity count. Proved: 840 capacity states and 1680 after orientation doubling.

Empty capacity rigging-map lift. Proved exactly.

Selected-branch Einstein TT theorem. Closed; the metric-Regge symbol converges to the massless two-helicity TT operator and the perfect-action pullback gives an exact gapped fixed- $j = 3$ realization.

Channel–area normalization. Closed within the selected completion.

Finite capacity/record marked-transfer vertex. Specified and finitely audited.

Source-controlled marked dilation. Specified and isometric; faithful renewal fixes unit rerouting at baryonic carrier failure, while the progenitor-mixing history and ledger ordering remain open.

Factorized vertex. Capacity-side spectral and causal audits closed; alternative nonfactorized realizations must preserve them.

Unique geometric host from capacity premises. Excluded; $\mathcal{G}_v$ and its measure are independent host data.

Derived host phase. Not yet well posed; a new geometric principle must first fix $\mathcal{G}_v$, after which its nonperturbative Einstein-phase membership can be tested.

The accompanying Lean results for the same-factor Gauss contradiction and the finite $1680/K^2$ arithmetic remain valid because their hypotheses concern the finite capacity factor. Earlier lemmas that explicitly name the open Peter–Weyl factor prove a sufficient kinematic witness, not a unique microscopic realization. The selected factorized completion has explicit representation theory, numerical preservation, block propagation, empty-sector rigging lift, and boundary-area map. Neither uniqueness nor an exact interacting Lorentzian host continuum is claimed.

Appendix R: Reproduction Code

This terminal appendix contains the executable reproduction block after all conceptual, provenance, and action-level appendices. It recomputes the manuscript's numerical spine from the definitions stated in the paper; it does not add a new derivation.

The script below recomputes the numerical consequences of the stated definitions: the 1680-state spectrum, closure solution, sharing entropy, marked vertex, dressed scale, Joyce constant, edge-kernel chain, corrected charged-lepton ratios, slot mutual information, closed routing covariance, primitive route spectrum, adjoint channel, causal color-blocking factor, and alphabet-independent generation cutoff. It also verifies the algebraic normalization identity $n_{\text{hor}} S_{\infty}^{\text{cell}} = 1/4$. CODATA constants are comparison inputs; no observational target is fitted inside the script. It runs under Python 3 with `numpy` and `scipy`. The complete Hessian and graph enumeration are supplied in the accompanying audit script.

```
# =====
# Reproduction script for the Entropic Scalar EFT manuscript.
# Recomputes the full numerical spine from stated definitions:
# the 1680-state K^2 spectrum, eta*, g_share, eff, marked
# vertex, dressed L* and G*, the
# Joyce constant, the 1/4 horizon identity, the edge-kernel
# chain, epsilon, sigma*, the lepton ladder, slot MI, and
# the conditional color-route algebra.
# Dependencies: Python 3, numpy, scipy.
# Every printed value is compared against the manuscript.
# =====
import itertools, math
from math import log, exp, pi, gamma, sqrt
from collections import Counter
import numpy as np
from scipy.optimize import brentq

# 1. Enumerate 1680 states: injective 4-of-7 labels m in -3..3, x2 parity
labels = range(-3,4)
states = list(itertools.permutations(labels,4))
print("P(7,4) =", len(states), " total with parity:", 2*len(states))

def K2(ms):
    S = sum(ms); Sig2 = sum(m*m for m in ms)
    return 48 - (S*S - Sig2)/3

spec = Counter()
for st in states:
    spec[round(K2(st)*3)] += 2 # parity doubling; key = 3*K2 to keep exact
print("Distinct K2 values and multiplicities (K2 as fraction /3):")
for k in sorted(spec): print(f" K2={k}/3 = {k/3:.4f} mult={spec[k]}")
print("Total multiplicity:", sum(spec.values()))

# 2. Solve closure condition <K2> = 3/(2 eta)
Ks = np.array([k/3 for k in sorted(spec)])
ns = np.array([spec[k] for k in sorted(spec)], dtype=float)
def avgK2(eta):
    w = ns*np.exp(-eta*Ks)
    return (w*Ks).sum()/w.sum()
f = lambda eta: avgK2(eta) - 3/(2*eta)
eta_star = brentq(f, 1e-4, 1.0, xtol=1e-16)
print("\neta* =", eta_star, " (paper: 0.02986684439352237)")

w = ns*np.exp(-eta_star*Ks); Z = w.sum(); p_class = w/Z
p_micro = np.exp(-eta_star*Ks)/Z
H = -(ns*p_micro*np.log(p_micro)).sum()
```

```

print("g_share,eff =", H, " (paper: 7.419800023570903)")
avg = avgK2(eta_star)
var = (p_class*(Ks-avg)**2).sum()
print("<K2> =", avg, " (paper 50.2229154254) 3/(2eta*) =", 3/(2*eta_star))
print("Var(K2) =", var, " (paper 15.6889750078) a_UV = 1/Var =", 1/var)
print("C_cl = eta* <K2> =", eta_star*avg)

# 3. Decorated marked vertex, L*, G*
hbar=1.054571817e-34; c=299792458.0; me=9.1093837139e-31 # CODATA 2022
lam_e = hbar/(me*c)
print("\nlambda_e =", lam_e, " (paper 3.861592671986303e-13)")
r = exp(-7*H)
L0 = -1.5*lam_e*math.log1p(-r)
q = 2/7
zeta = 9*exp(-H)*(1-eta_star*8/49)**(21/2)
Zmu = 1+zeta
Ze = (1+zeta)*(1+7*zeta*zeta)
Ztau2 = 1+q*zeta
Lstar = Ze*L0
print("e^{7g} =", exp(7*H), " (paper 3.602860521062804e22)")
print("(2/3)e^{7g} =", (2/3)*exp(7*H), " (paper 2.401907014041869e22)")
print("zeta* =", zeta, " (paper 0.005123584484947)")
print("Z_mu, Z_e, Z_tau2 =", Zmu, Ze, Ztau2)
print("L0 =", L0, " (paper 1.607719470158885e-35)")
print("L* =", Lstar, " (paper 1.616253701392569e-35)")
Gstar = c**3*Lstar**2/hbar
print("G* =", Gstar, " (paper 6.674289077220912e-11)")
G_codata = 6.67430e-11
LP = sqrt(hbar*G_codata/c**3)
print("L* vs Planck length:", (LP-Lstar)/LP*100, "%")
print("G* vs CODATA:", (G_codata-Gstar)/G_codata*100, "%")

# 4. Joyce constant
Gtet = 3*gamma(1/3)**6/(2**(14/3)*pi**4)
print("\nG_tet(0) =", Gtet, " (paper 0.4482203943883814)")
print("ln(7/6)/(pi*Gtet) =", log(7/6)/(pi*Gtet), " (paper 0.109472228)")
print("S_inf_cell = 3ln2/(32 pi Gtet) =", 3*log(2)/(32*pi*Gtet), " (paper 0.0461482516)")
print("n_hor*S_inf_cell =", (8*pi*Gtet/(3*log(2))) * (3*log(2)/(32*pi*Gtet)))
# Factorized j=3 horizon embedding: coherent/Regge channel-area normalization
gamma_BI = log(2)/(6*pi)
a_ch_over_L2 = 24*pi*gamma_BI/6 # equivalently 4 ln 2
ZA1 = sqrt(3/4)
print("gamma_BI = ln2/(6pi) =", gamma_BI, " (paper 0.0367726000254)")
print("a_ch/L^2 =", 4*log(2), " (paper 2.77258872224)")
print("Z_A(N=1) = sqrt(3/4) =", ZA1)
for N in (1, 2, 10, 1000):
    ZA = sqrt((3*N)/(3*N+1))
    Aphys_over_L2 = ZA * 8*pi*gamma_BI*sqrt((3*N)*(3*N+1))
    assert abs(Aphys_over_L2 - 4*N*log(2)) < 1e-10
print("area-blocking identity checked for N=1,2,10,1000")

# 5. Edge kernel chain
Jbare = 2/3*eta_star
Jtree = Jbare/3
Sig = 7+2/9
cloop = 1/(1+Jtree*Sig)
Jren = Jtree*cloop
print("\nJ_bare =", Jbare, " (paper 0.0199112296)")
print("J_tree =", Jtree, " (paper 0.0066370765)")
print("c_loop =", cloop, " (paper 0.95426)")
print("J_ren =", Jren, " (paper 0.00633348)")
print("gamma coeff 4 J_ren/(3 pi^2) =", 4*Jren/(3*pi**2), " (paper 8.556e-4)")

```

```

# 6. epsilon and cluster numbers
epsv = H/(4*pi**2)
print("\nepsilon = g/(4pi^2) =", epsv, " 1-eps =", 1-epsv, " ceiling =", 2-epsv)
print("sigma* = pi/g =", pi/H, " (paper 0.42340665)")

# 7. Corrected charged-lepton ladder
mmu0 = 720*q; mtau0 = 720**2*q**4
mmu = mmu0*Zmu; mtau = mtau0*Zmu*Ztau2
print("\nbaseline m_mu/m_e =", mmu0, " baseline m_tau/m_e =", mtau0)
print("m_mu/m_e =", mmu, " (paper 206.768280237)")
print("m_tau/m_e =", mtau, " (paper 3477.343310)")
print("alpha_mu =", log(206.7683)/log(mmu), " alpha_tau =", log(3477.23)/log(mtau))

# 8. mixing-time ratio
tau_star = Lstar/c; tau_e = hbar/(me*c**2)
print("\ntau* =", tau_star, " tau_e =", tau_e, " ratio =", tau_e/tau_star)

# 9. slot mutual information (marginal of slot under admissibility)
joint = Counter()
for st in states:
    wgt = exp(-eta_star*K2(st))
    joint[(st[0],st[1])] += 2*wgt
tot = sum(joint.values())
pj = {k:v/tot for k,v in joint.items()}
p0 = Counter(); p1 = Counter()
for (a,b),v in pj.items(): p0[a]+=v; p1[b]+=v
I = sum(v*log(v/(p0[a]*p1[b])) for (a,b),v in pj.items())
H0 = -sum(v*log(v) for v in p0.values())
print("\nslot MI =", I, " (paper 0.1545)   H(slot) =", H0, " I/H =", I/H0, " (paper 0.079)")

# 10. Conditional color-route checks
route_rows = []
route_weights = []
for st in states:
    m1,m2,m3,m4 = st
    route_rows.append([
        m1*m2 + m3*m4,
        m1*m3 + m2*m4,
        m1*m4 + m2*m3,
    ])
    route_weights.append(2*exp(-eta_star*K2(st))) # parity doubling
route_rows = np.asarray(route_rows, dtype=float)
route_weights = np.asarray(route_weights, dtype=float)
route_weights /= route_weights.sum()
route_mean = (route_weights[:,None]*route_rows).sum(axis=0)
centered = route_rows-route_mean
Croute = (route_weights[:,None,None]
    * centered[:,None]*centered[:,None,:]).sum(axis=0)
P1 = np.ones((3,3))/3
P2 = np.eye(3)-P1
lam1 = np.trace(P1@Croute)
lam2 = np.trace(P2@Croute)/2
print("\nroute covariance =\n", Croute)
print("route singlet/doublet eigenvalues =", lam1, lam2)
print("(3/4) Var(K2) =", 0.75*var)
assert np.allclose(Croute, lam1*P1+lam2*P2, atol=1e-10)
assert abs(lam1-0.75*var) < 1e-10

Groute = np.eye(3) - (np.ones((3,3))-np.eye(3))/20
Graw = np.linalg.eigvalsh(Groute)
Kroute = (20/21)*Groute
Lroute = sqrt(13)/7*P1
print("primitive route Gram eigenvalues =", Graw)

```

```

print("lambda2/lambda1 =", Graw[-1]/Graw[0], " (paper 7/6)")
print("dilation residual =",
      np.linalg.norm(Kroute.T@Kroute + Lroute.T@Lroute-np.eye(3)))

Utest = np.diag(np.exp(1j*np.array([0.2, -0.5, 0.3])))
trU = np.trace(Utest)
return_projective = 1.5*(1-abs(trU)**2/9)
return_adjoint = (4/3)*(1-(abs(trU)**2-1)/8)
phase = np.exp(0.37j)
return_rephased = 1.5*(1-abs(np.trace(phase*Utest))**2/9)
print("projective/adjoint return costs =",
      return_projective, return_adjoint, return_rephased)
assert abs(return_projective-return_adjoint) < 1e-14
assert abs(return_projective-return_rephased) < 1e-14

t8 = 13/14
g_step = (2/3)*log(14/13)
block = 2*sqrt(6)
g_graph = block*g_step
print("t8 =", t8, " -ln(t8) =", -log(t8))
print("g_HK,step^2 =", g_step)
print("diamond blocking =", block, " g_graph,HK^2 =", g_graph)

# 11. Alphabet-independent generation cutoff
for excluded_count in (1, 2, 3):
    exclusion_variances = []
    for excluded in itertools.combinations(labels, excluded_count):
        reduced = [m for m in labels if m not in excluded]
        reduced_K2 = [K2(st) for st in itertools.permutations(reduced, 4)]
        exclusion_variances.append(float(np.var(reduced_K2)))
    nonzero = sum(value > 1e-12 for value in exclusion_variances)
    zero = sum(abs(value) <= 1e-12 for value in exclusion_variances)
    print(
        f"excluded labels = {excluded_count}: cases = {len(exclusion_variances)}, "
        f"nonzero = {nonzero}, zero = {zero}, "
        f"variance range = [{min(exclusion_variances)}, {max(exclusion_variances)}]"
    )
    if excluded_count < 3:
        assert nonzero == len(exclusion_variances)
    else:
        assert zero == len(exclusion_variances)

```

References

- [1] S. S. McGaugh, F. Lelli, and J. M. Schombert, “Radial acceleration relation in rotationally supported galaxies,” *Physical Review Letters* **117**, 201101 (2016).
- [2] D. Clowe et al., “A direct empirical proof of the existence of dark matter,” *The Astrophysical Journal Letters* **648**, L109–L113 (2006).
- [3] B. Bertotti, L. Iess, and P. Tortora, “A test of general relativity using radio links with the Cassini spacecraft,” *Nature* **425**, 374–376 (2003).
- [4] Y. Tian et al., “The radial acceleration relation in CLASH galaxy clusters,” arXiv:2001.08340 (2020).
- [5] A. J. Guttmann, “Lattice Green functions in all dimensions,” *Journal of Physics A: Mathematical and Theoretical* **43**, 305205 (2010).
- [6] K. G. Wilson, “Confinement of quarks,” *Physical Review D* **10**, 2445–2459 (1974).

- [7] G. S. Joyce, “On the cubic lattice Green functions,” *Proceedings of the Royal Society of London A* **445**, 463–477 (1994).
- [8] R. Arnowitt, S. Deser, and C. W. Misner, “The dynamics of general relativity,” in *Gravitation: An Introduction to Current Research*, ed. L. Witten (Wiley, New York, 1962).
- [9] J. D. Bekenstein, “Black holes and entropy,” *Physical Review D* **7**, 2333–2346 (1973).
- [10] S. W. Hawking, “Particle creation by black holes,” *Communications in Mathematical Physics* **43**, 199–220 (1975).
- [11] G. W. Gibbons and S. W. Hawking, “Action integrals and partition functions in quantum gravity,” *Physical Review D* **15**, 2752–2756 (1977).
- [12] T. Regge and J. A. Wheeler, “Stability of a Schwarzschild singularity,” *Physical Review* **108**, 1063–1069 (1957).
- [13] F. J. Zerilli, “Effective potential for even-parity Regge–Wheeler gravitational perturbation equations,” *Physical Review Letters* **24**, 737–738 (1970).
- [14] A. J. R. Sanderson, T. J. Ponman, A. Finoguenov, E. J. Lloyd-Davies, and M. Markevitch, “The Birmingham–CfA cluster scaling project — I. Gas fraction and the M – T_X relation,” *Monthly Notices of the Royal Astronomical Society* **340**, 989–1010 (2003).
- [15] A. Vikhlinin, A. Kravtsov, W. Forman, C. Jones, M. Markevitch, S. S. Murray, and L. Van Speybroeck, “Chandra sample of nearby relaxed galaxy clusters: mass, gas fraction, and mass–temperature relation,” *The Astrophysical Journal* **640**, 691–709 (2006).
- [16] D. Eckert, S. Ettori, E. Pointecouteau, R. F. J. van der Burg, and S. I. Loubser, “The gravitational field of X-COP galaxy clusters,” *Astronomy & Astrophysics* **662**, A123 (2022).
- [17] P. Li, Y. Tian, M. P. Júlio, M. S. Pawlowski, F. Lelli, S. S. McGaugh, J. M. Schombert, J. I. Read, P.-C. Yu, and C.-M. Ko, “Measuring galaxy cluster mass profiles into the low-acceleration regime with galaxy kinematics,” *Astronomy & Astrophysics* **677**, A24 (2023).
- [18] M. Milgrom, “A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis,” *Astrophysical Journal* **270**, 365 (1983).
- [19] M. Milgrom, “Quasi-linear formulation of MOND,” *Monthly Notices of the Royal Astronomical Society* **403**, 886 (2010), doi:10.1111/j.1365-2966.2009.16184.x.
- [20] J. D. Bekenstein, “Relativistic gravitation theory for the modified Newtonian dynamics paradigm,” *Physical Review D* **70**, 083509 (2004).
- [21] B. Famaey and S. S. McGaugh, “Modified Newtonian dynamics (MOND): observational phenomenology and relativistic extensions,” *Living Reviews in Relativity* **15**, 10 (2012).
- [22] T. Jacobson, “Thermodynamics of spacetime: the Einstein equation of state,” *Physical Review Letters* **75**, 1260 (1995).
- [23] P. Dorau and A. Much, “From quantum relative entropy to the semiclassical Einstein equations,” *Physical Review Letters* **136**, 091602 (2026).
- [24] V. Chandrasekaran, R. Longo, G. Penington, and E. Witten, “An algebra of observables for de Sitter space,” *Journal of High Energy Physics* **2023**(2), 082 (2023), doi:10.1007/JHEP02(2023)082.

- [25] G. Bianconi, “Thermodynamics of the gravity from entropy theory,” *Physical Review D* **114**, 024042 (2026), doi:10.1103/26kn-thgp.
- [26] A. Caticha, “Quantum mechanics as Hamilton–Killing flows on a statistical manifold,” *Physical Sciences Forum* **3**, 12 (2021), doi:10.3390/psf2021003012.
- [27] A. Caticha, “Entropic dynamics approach to quantum electrodynamics,” *Entropy* **27**, 1247 (2025), doi:10.3390/e27121247.
- [28] A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki, and H. Ezawa, “Demonstration of single-electron buildup of an interference pattern,” *American Journal of Physics* **57**, 117–120 (1989), doi:10.1119/1.16104.
- [29] E. Verlinde, “On the origin of gravity and the laws of Newton,” *Journal of High Energy Physics* **2011**(4), 29 (2011).
- [30] E. Verlinde, “Emergent gravity and the dark universe,” *SciPost Physics* **2**, 016 (2017).
- [31] C. Skordis and T. Złóśnik, “New relativistic theory for modified Newtonian dynamics,” *Physical Review Letters* **127**, 161302 (2021).
- [32] J. Collins, A. Perez, D. Sudarsky, L. Urrutia, and H. Vucetich, “Lorentz invariance and quantum gravity: an additional fine-tuning problem?,” *Physical Review Letters* **93**, 191301 (2004).
- [33] L. Bombelli, J. Lee, D. Meyer, and R. D. Sorkin, “Space-time as a causal set,” *Physical Review Letters* **59**, 521 (1987).
- [34] F. Dowker, J. Henson, and R. D. Sorkin, “Quantum gravity phenomenology, Lorentz invariance and discreteness,” *Modern Physics Letters A* **19**, 1829 (2004).
- [35] C. Pittordis, W. Sutherland, and P. Shepherd, “Wide binaries from Gaia DR3: testing GR versus MOND with realistic triple modelling,” *The Open Journal of Astrophysics* **8** (2025), doi:10.33232/001c.142887.
- [36] K.-H. Chae, “Low-acceleration gravitational anomaly from Bayesian 3D modeling of wide binary orbits: methodology and results with Gaia Data Release 3,” *Astrophysical Journal* **985**, 210 (2025), doi:10.3847/1538-4357/adce09.
- [37] K.-H. Chae and Y. Yoon, “Revisiting data quality control and multiple-star modeling in wide binary gravity tests: confirmation of MOND-type gravitational anomaly at low acceleration,” arXiv:2607.14450 (2026).
- [38] R. S. Park, A. Hees, B. Famaey, H. Desmond, and A. Durakovic, “Improved constraints on modified Newtonian gravity from Cassini radio tracking data,” *Physical Review D* **114**, 024066 (2026), doi:10.1103/r7n8-kw38.
- [39] M. P. Júlio, J. I. Read, M. S. Pawlowski, P. Li, D. Vaz, J. Brinchmann, M. P. Rey, O. Agertz, and T. Holmes, “The radial acceleration relation at the EDGE of galaxy formation: testing its universality in low-mass dwarf galaxies,” *Astronomy & Astrophysics* **704**, A330 (2025), doi:10.1051/0004-6361/202557106.
- [40] R. Genzel et al., “Strongly baryon-dominated disk galaxies at the peak of galaxy formation ten billion years ago,” *Nature* **543**, 397 (2017).
- [41] H. Prince and J. Dunkley, “Data compression in cosmology: A compressed likelihood for Planck data,” *Physical Review D* **100**, 083502 (2019).

- [42] B. I. Ciocan, N. F. Bouché, J. Fensch, D. Krajnović, J. Freundlich, H. Desmond, B. Famaey, and R. Tychi, “MUSE-DARK III: The evolution of the radial acceleration relation at intermediate redshifts,” *Astronomy & Astrophysics* **709**, L16 (2026).
- [43] A. A. Vărășteanu, M. J. Jarvis, A. A. Ponomareva, H. Desmond, I. Heywood, T. Yasin, N. Maddox, M. Glowacki, M. Maksymowicz-Maciata, P. E. M. Mancera Piña, and H. Pan, “MIGHTEE-HI: The radial acceleration relation with resolved stellar mass measurements,” *Monthly Notices of the Royal Astronomical Society* **541**, 2366 (2025).
- [44] P. Lang et al., “Falling rotation curves of star-forming galaxies at $0.6 < z < 2.6$,” *Astrophysical Journal* **840**, 92 (2017).
- [45] F. Lelli, S. S. McGaugh, and J. M. Schombert, “SPARC: mass models for 175 disk galaxies with Spitzer photometry and accurate rotation curves,” *Astronomical Journal* **152**, 157 (2016).
- [46] M. M. Brouwer et al., “The weak lensing radial acceleration relation: measuring the dark matter law with KiDS-1000,” *Astronomy & Astrophysics* **650**, A113 (2021).
- [47] T. Mistele, S. S. McGaugh, F. Lelli, J. M. Schombert, and P. Li, “Radial acceleration relation of galaxies with joint kinematic and weak-lensing data,” *Journal of Cosmology and Astroparticle Physics* **2024**(04), 020 (2024), doi:10.1088/1475-7516/2024/04/020.
- [48] W. G. Unruh, “Notes on black-hole evaporation,” *Physical Review D* **14**, 870 (1976).
- [49] R. B. Tully and J. R. Fisher, “A new method of determining distances to galaxies,” *Astronomy & Astrophysics* **54**, 661 (1977).
- [50] S. S. McGaugh, J. M. Schombert, G. D. Bothun, and W. J. G. de Blok, “The baryonic Tully–Fisher relation,” *Astrophysical Journal Letters* **533**, L99 (2000).
- [51] Planck Collaboration, “Planck 2018 results. VI. Cosmological parameters,” *Astronomy & Astrophysics* **641**, A6 (2020).
- [52] A. G. Riess et al., “A comprehensive measurement of the local value of the Hubble constant with $1 \text{ km s}^{-1} \text{ Mpc}^{-1}$ uncertainty from the Hubble Space Telescope and the SH0ES team,” *Astrophysical Journal Letters* **934**, L7 (2022).
- [53] W. L. Freedman, B. F. Madore, I. S. Jang, et al., “Status report on the Chicago–Carnegie Hubble Program (CCHP): measurement of the Hubble constant using the Hubble and James Webb Space Telescopes,” *The Astrophysical Journal* (2025), doi:10.3847/1538-4357/adce78.
- [54] E. Di Valentino et al., “In the realm of the Hubble tension — a review of solutions,” *Classical and Quantum Gravity* **38**, 153001 (2021).
- [55] C. M. Will, “The confrontation between general relativity and experiment,” *Living Reviews in Relativity* **17**, 4 (2014).
- [56] C. Brans and R. H. Dicke, “Mach’s principle and a relativistic theory of gravitation,” *Physical Review* **124**, 925 (1961).
- [57] E. G. Adelberger, B. R. Heckel, and A. E. Nelson, “Tests of the gravitational inverse-square law,” *Annual Review of Nuclear and Particle Science* **53**, 77 (2003).
- [58] T. Padmanabhan, “Thermodynamical aspects of gravity: new insights,” *Reports on Progress in Physics* **73**, 046901 (2010).

- [59] R. H. Sanders and S. S. McGaugh, “Modified Newtonian dynamics as an alternative to dark matter,” *Annual Review of Astronomy and Astrophysics* **40**, 263 (2002).
- [60] D. Oriti, “Group field theory as the second quantization of loop quantum gravity,” *Classical and Quantum Gravity* **33**, 085005 (2016).
- [61] S. Gielen, D. Oriti, and L. Sindoni, “Cosmology from group field theory formalism for quantum gravity,” *Physical Review Letters* **111**, 031301 (2013).
- [62] A. Barbieri, “Quantum tetrahedra and simplicial spin networks,” *Nuclear Physics B* **518**, 714 (1998).
- [63] J. C. Baez and J. W. Barrett, “The quantum tetrahedron in 3 and 4 dimensions,” *Advances in Theoretical and Mathematical Physics* **3**, 815 (1999).
- [64] C. Rovelli and L. Smolin, “Discreteness of area and volume in quantum gravity,” *Nuclear Physics B* **442**, 593 (1995).
- [65] A. M. Gleason, “Measures on the closed subspaces of a Hilbert space,” *Journal of Mathematics and Mechanics* **6**, 885–893 (1957).
- [66] J. B. Hartle, “Decoherent histories quantum mechanics starting with records of what happens,” arXiv:1608.04145 (2016).
- [67] E. T. Jaynes, “The minimum entropy production principle,” *Annual Review of Physical Chemistry* **31**, 579–601 (1980), doi:10.1146/annurev.pc.31.100180.003051.
- [68] W. F. Stinespring, “Positive functions on C^* -algebras,” *Proceedings of the American Mathematical Society* **6**, 211–216 (1955), doi:10.1090/S0002-9939-1955-0069403-4.
- [69] A. D. Wyner and J. Ziv, “Some asymptotic properties of the entropy of a stationary ergodic data source with applications to data compression,” *IEEE Transactions on Information Theory* **35**, 1250–1258 (1989).
- [70] D. S. Ornstein and B. Weiss, “Entropy and data compression schemes,” *IEEE Transactions on Information Theory* **39**, 78–83 (1993).
- [71] J. J. Halliwell, “A review of the decoherent histories approach to quantum mechanics,” arXiv:gr-qc/9407040 (1994).
- [72] D. Eckert et al., “Non-thermal pressure support in X-COP galaxy clusters,” *Astronomy & Astrophysics* **621**, A40 (2019).
- [73] S. Dupourqué, N. Clerc, E. Pointecouteau, D. Eckert, S. Ettori, and F. Vazza, “Investigating the turbulent hot gas in X-COP galaxy clusters,” *Astronomy & Astrophysics* **673**, A91 (2023).
- [74] XRISM Collaboration, “XRISM reveals low nonthermal pressure in the core of the hot, relaxed galaxy cluster Abell 2029,” *Astrophysical Journal Letters* **982**, L5 (2025).
- [75] A. H. Chamseddine and V. Mukhanov, “Mimetic dark matter,” *Journal of High Energy Physics* **1311**, 135 (2013).
- [76] A. Lewis, A. Challinor, and A. Lasenby, “Efficient computation of cosmic microwave background anisotropies in closed Friedmann–Robertson–Walker models,” *Astrophysical Journal* **538**, 473 (2000).

- [77] S. More, B. Diemer, and A. Kravtsov, “The splashback radius as a physical halo boundary and the growth of halo mass,” *Astrophysical Journal* **810**, 36 (2015).
- [78] G. E. Volovik, “Cosmological constant and vacuum energy,” *Annalen der Physik* **14**, 165–176 (2005).
- [79] T. Regge, “General relativity without coordinates,” *Il Nuovo Cimento* **19**, 558–571 (1961).
- [80] J. Ambjørn, J. Jurkiewicz, and R. Loll, “Emergence of a 4D world from causal quantum gravity,” *Physical Review Letters* **93**, 131301 (2004).
- [81] J. Ambjørn, A. Görlich, J. Jurkiewicz, and R. Loll, “Nonperturbative quantum gravity,” *Physics Reports* **519**, 127–210 (2012).
- [82] J. Ambjørn, J. Jurkiewicz, and R. Loll, “Dynamically triangulating Lorentzian quantum gravity,” *Nuclear Physics B* **610**, 347–382 (2001).
- [83] J. Ambjørn, J. Gizbert-Studnicki, A. Görlich, J. Jurkiewicz, N. Klitgaard, and R. Loll, “Characteristics of the new phase in CDT,” *European Physical Journal C* **77**, 152 (2017).
- [84] J. Ambjørn, A. Görlich, J. Jurkiewicz, and R. Loll, “The nonperturbative quantum de Sitter universe,” *Physical Review D* **78**, 063544 (2008).
- [85] A. A. Thoul and D. H. Weinberg, “Hydrodynamic simulations of galaxy formation. II. Photoionization and the formation of low-mass galaxies,” *Astrophysical Journal* **465**, 608 (1996).
- [86] T. Okamoto, L. Gao, and T. Theuns, “Mass loss of galaxies due to an ultraviolet background,” *Monthly Notices of the Royal Astronomical Society* **390**, 920–928 (2008).
- [87] J. Ambjørn, S. Jordan, J. Jurkiewicz, and R. Loll, “A second-order phase transition in CDT,” *Physical Review Letters* **107**, 211303 (2011).
- [88] J. Ambjørn, D. Coumbe, J. Gizbert-Studnicki, A. Görlich, and J. Jurkiewicz, “Critical phenomena in causal dynamical triangulations,” *Classical and Quantum Gravity* **36**, 224001 (2019).
- [89] A. Perez, “The spin-foam approach to quantum gravity,” *Living Reviews in Relativity* **16**, 3 (2013).
- [90] M. Han, “Semiclassical behavior of spinfoam amplitude with small spins and entanglement entropy,” *Physical Review D* **100**, 084049 (2019).
- [91] M. Han, “Summation and triangulation independence of Lorentzian spinfoam amplitudes for all LQG,” *Physical Review D* **113**, 084034 (2026), doi:10.1103/n76f-31gf.
- [92] T. Thiemann and A. Zipfel, “Linking covariant and canonical LQG II: spin foam projector,” *Classical and Quantum Gravity* **31**, 125008 (2014).
- [93] A. Calcinari and S. Gielen, “Relational dynamics and Page–Wootters formalism in group field theory,” *Quantum* **9**, 1610 (2025).
- [94] M. Han and Y. Ma, “Master constraint operators in loop quantum gravity,” *Physics Letters B* **635**, 225–231 (2006).
- [95] M. Han and T. Thiemann, “On the relation between rigging inner product and master constraint direct integral decomposition,” arXiv:0911.3431 (2010).

- [96] W. J. Fairbairn and C. Meusburger, “Quantum deformation of two four-dimensional spin foam models,” *Journal of Mathematical Physics* **53**, 022501 (2012).
- [97] M. Han, “Einstein equation from covariant loop quantum gravity in semiclassical continuum limit,” *Physical Review D* **96**, 024047 (2017).
- [98] M. Han, Z. Huang, and A. Zipfel, “Emergent four-dimensional linearized gravity from a spin foam model,” *Physical Review D* **100**, 024060 (2019).
- [99] J. Engle, “Proposed proper Engle–Pereira–Rovelli–Livine vertex amplitude,” *Physical Review D* **87**, 084048 (2013).
- [100] J. Engle, I. Vilenky, and A. Zipfel, “The Lorentzian proper vertex amplitude: Asymptotics,” *Physical Review D* **94**, 064025 (2016).
- [101] B. Dittrich and A. Kogios, “From spin foams to area metric dynamics to gravitons,” *Classical and Quantum Gravity* **40**, 095011 (2023).
- [102] P. A. Höhn, Canonical linearized Regge calculus: Counting lattice gravitons with Pachner moves, *Physical Review D* **91**, 124034 (2015).
- [103] M. Roček and R. M. Williams, Quantum Regge calculus, *Physics Letters B* **104**, 31–37 (1981).
- [104] B. Dittrich, W. Kamiński, and S. Steinhaus, “Discretization independence implies non-locality in 4D discrete quantum gravity,” *Classical and Quantum Gravity* **31**, 245009 (2014).
- [105] D. Lovelock, “The Einstein tensor and its generalizations,” *Journal of Mathematical Physics* **12**, 498–501 (1971).
- [106] S. Deser, “Self-interaction and gauge invariance,” *General Relativity and Gravitation* **1**, 9–18 (1970).
- [107] J. Borissova, B. Dittrich, A. Eichhorn, and M. Schiffer, “Renormalization group flows in area-metric gravity,” arXiv:2507.02034 [gr-qc] (2025).
- [108] M. Bruno, E. Colafranceschi, F. M. Mele, and C. Rovelli, “The structure of the continuum limit of spin foams,” arXiv:2603.16999 [gr-qc] (2026).
- [109] J. Engle, E. Livine, R. Pereira, and C. Rovelli, “LQG vertex with finite Immirzi parameter,” *Nuclear Physics B* **799**, 136–149 (2008).
- [110] F. Gozzini, “A high-performance code for EPRL spin foam amplitudes,” *Classical and Quantum Gravity* **38**, 225010 (2021).
- [111] W. G. Unruh, “Unimodular theory of canonical quantum gravity,” *Physical Review D* **40**, 1048 (1989).
- [112] M. Henneaux and C. Teitelboim, “The cosmological constant and general covariance,” *Physics Letters B* **222**, 195 (1989).
- [113] S. Weinberg, “The cosmological constant problem,” *Reviews of Modern Physics* **61**, 1 (1989).
- [114] T. Josset, A. Perez, and D. Sudarsky, “Dark energy from violation of energy conservation,” *Physical Review Letters* **118**, 021102 (2017).
- [115] A. Perez and D. Sudarsky, “Dark energy from quantum gravity discreteness,” *Physical Review Letters* **122**, 221302 (2019).

- [116] M. Ahmed, S. Dodelson, P. B. Greene, and R. D. Sorkin, “Everpresent Λ ,” *Physical Review D* **69**, 103523 (2004).
- [117] S. Das, A. Nasiri, and Y. K. Yazdi, “Everpresent Λ . Part II. Structural stability,” *Journal of Cosmology and Astroparticle Physics* **10**, 047 (2023).
- [118] J. D. Barrow, “A strong constraint on ever-present λ ,” *Physical Review D* **75**, 067301 (2007).
- [119] S. D. H. Hsu, “Entropy bounds and dark energy,” *Physics Letters B* **594**, 13 (2004).
- [120] M. Li, “A model of holographic dark energy,” *Physics Letters B* **603**, 1 (2004).
- [121] DESI Collaboration, “DESI DR2 results II: Measurements of baryon acoustic oscillations and cosmological constraints,” arXiv:2503.14738 (2025).
- [122] D. Brout et al., “The Pantheon+ analysis: Cosmological constraints,” *The Astrophysical Journal* **938**, 110 (2022).
- [123] W. H. Press and P. Schechter, “Formation of galaxies and clusters of galaxies by self-similar gravitational condensation,” *The Astrophysical Journal* **187**, 425 (1974).
- [124] A. G. Doroshkevich, “Spatial structure of perturbations and origin of galactic rotation in fluctuation theory,” *Astrophysics* **6**, 320 (1970).
- [125] D. J. Eisenstein and W. Hu, “Baryonic features in the matter transfer function,” *The Astrophysical Journal* **496**, 605 (1998).
- [126] A. N. Ormondroyd, W. J. Handley, M. P. Hobson, A. N. Lasenby, and D. Yallup, “Dynamic or systematic? Bayesian model selection between dark energy and supernova biases,” *Monthly Notices of the Royal Astronomical Society* **548**, stag615 (2026).
- [127] DES Collaboration, “Dark Energy Survey supernova program: A reanalysis of cosmology results and evidence for evolving dark energy with an updated Type Ia supernova calibration,” *Monthly Notices of the Royal Astronomical Society* **548**, stag632 (2026).
- [128] G. F. Lesci, C. Giocoli, F. Marulli, M. Romanello, L. Moscardini, et al., “AMICO galaxy clusters in KiDS-1000: Splashback radius from weak lensing and cluster–galaxy correlation function,” *Astronomy & Astrophysics* **710**, A269 (2026).
- [129] C. Hidalgo, *Why Information Grows: The Evolution of Order, from Atoms to Economies* (Basic Books, New York, 2015).
- [130] R. Landauer, “Irreversibility and heat generation in the computing process,” *IBM Journal of Research and Development* **5**, 183–191 (1961).
- [131] G. L. Bryan and M. L. Norman, “Statistical properties of X-ray clusters: Analytic and numerical comparisons,” *The Astrophysical Journal* **495**, 80 (1998).
- [132] T. Shin et al., “Measurement of the splashback feature around SZ-selected galaxy clusters with DES, SPT and ACT,” *Monthly Notices of the Royal Astronomical Society* **487**, 2900 (2019).
- [133] D. Rana et al., “The eROSITA Final Equatorial-Depth Survey (eFEDS): Splashback radius of X-ray galaxy clusters using galaxies from HSC survey,” *Monthly Notices of the Royal Astronomical Society* **522**, 4181 (2023).

- [134] D. Rubin et al., “Union through UNITY: Cosmology with 2,000 SNe using a unified Bayesian framework,” arXiv:2311.12098 (2023).
- [135] T. H. Greif, J. L. Johnson, R. S. Klessen, and V. Bromm, “The first galaxies: assembly, cooling and the onset of turbulence,” *Monthly Notices of the Royal Astronomical Society* **387**, 1021–1036 (2008).
- [136] R. Barkana and A. Loeb, “The photoevaporation of dwarf galaxies during reionization,” *The Astrophysical Journal* **523**, 54–65 (1999).
- [137] A. P. Ji, A. Frebel, and V. Bromm, “Preserving chemical signatures of primordial star formation in the first low-mass stars,” *Monthly Notices of the Royal Astronomical Society* **454**, 659–674 (2015).
- [138] B. F. Griffen, G. A. Dooley, A. P. Ji, B. W. O’Shea, F. A. Gomez, and A. Frebel, “Tracing the first stars and galaxies of the Milky Way,” *Monthly Notices of the Royal Astronomical Society* **474**, 443–459 (2018).