Entropic Scalar EFT: From Entanglement Microstructure to Gravity and Cosmic Structure

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Abstract

We propose that empty space is not a passive backdrop but a physical medium with a finite budget of quantum entanglement: the linking structure that allows parts of a quantum system to share state. Matter forms when some of that capacity becomes locked into stable, localized defects of the medium. A particle's mass measures how much entanglement is committed to such a defect. Gravity is the surrounding capacity-strain field: near matter, slightly less entanglement capacity is freely available, and in the weak-field limit the fractional shortfall gives the gravitational potential. The excess acceleration seen in galaxies, usually attributed to particle dark matter, is treated here as the large-scale continuation of the same capacity response rather than as a new unseen substance.

The central result is that this picture is not freely adjustable after the fact. Once one accepts the finite-capacity medium, the three founding postulates, and a specific minimal model for the smallest cell of space, finite counting fixes the cell entropy and the ordinary weak-field response. The resulting capacity action is the static scalar sector of the Einstein action written in the capacity variable, so it gives Newton's law and the leading no-slip metric without introducing another gravitational field. A separately identified transverse branch gives the galactic acceleration scale and the observed relation between galaxy rotation and ordinary matter, subject to the microscopic matching conditions stated in the paper.

The electron plays a double role. As the lightest clean charged defect, it fixes the exchange rate between committed entanglement and mass and calibrates the absolute cell scale. Many-Pasts supplies the history-space interpretation of that calibration while preserving ordinary Born-rule statistics and no-signaling. Applying the same faithful-resolution condition used for the cell ensemble makes the local renewal process memoryless. A reversible marked-transfer action then derives the finite charged response and routes it through the electron and the heavier charged-lepton shells. This adds no new founding premise and leaves the original tetrahedral construction intact.

We also test the cell model in a computer simulation of dynamical spacetime. Turning on the medium's weighting orders the microscopic cell states while the background geometry remains stable, and a scrambled control confirms that the ordering follows the closure structure itself. Inserted defects then strain the nearby capacity and measurably deform the local geometry. In a separate transport calculation, a conserved carrier responds to defects of different strength through one common rule, and the disturbance persists without detected screening across the measured range. A predicted shift of the host geometry likewise follows the cell model across a family of simulation settings, while the control follows its own distinct prediction. These tests are limited in scale and do not yet measure Newton's constant, but they connect the proposed medium to dynamical geometry through measured consequences rather than analogy alone.

Beyond ordinary weak gravity, the framework extends to time-dependent transport, clusters, cosmology, the saturated early universe, black holes, and particle structure at explicitly labeled levels of closure. The finite marked-transfer and charged-lepton calculation is closed inside its displayed action. Its embedding in a stable geometric condensate, together with the transverse, cosmological, and strong-field completions, remains conditional or open as stated in the closure table.

Contents

Part I. Physical Idea and Foundations

1. Introduction: The Physical Claim

Space, in this proposal, is a finite medium of entanglement capacity. The particles we call matter are stable defects that lock away part of that capacity, and the surrounding medium responds to the commitment. Seen at large scales, that response is gravity:

- **Matter** is a localized capacity defect of the substrate.
- **Mass** is the entanglement that defect commits, read in mass units.
- **Gravity** is the extended capacity strain — the fractional capacity deficit — the medium carries around the defect.
- **Dark-matter phenomenology** comes from two further regimes of the same medium: the long-range capacity strain on galactic scales and the saturated phase in the early universe.
- **General relativity** is the low-energy geometry of this capacity medium.

Written as a continuum theory, this becomes a scalar EFT for a vacuum-relative entanglement field $S_{\mathrm{ent}}(x)$ and its deficit δS relative to the background capacity. The defect sector is written at continuum scale in ordinary stress-energy variables, but its ontology is unchanged: inertial mass enters through the mass-per-entropy map κ_m, and the weak-field potential is the fractional deficit $\delta S/S_\infty$.

The gravitational response normally attributed to a dark halo is assigned here to the capacity structure of the vacuum itself. Ordinary weak gravity, the galactic excess, and the homogeneous cosmological mode are different regimes of one medium: general relativity supplies its low-energy geometry, and the capacity variable tracks how localized defects deplete and redistribute the available entanglement. Section 29.9 compares this proposal with recent information-theoretic constructions of gravity, after the paper's own development is complete.

The inputs are the finite-capacity substrate, three postulates, and the tetrahedral cell ensemble. From these, one chain of finite calculations follows. Admissibility fixes the sharing entropy, edge transport fixes the tree stiffness, the decorated transfer vertex fixes the charged marked event, source projection fixes the ordinary coupling ratio, and the weak-field bridge relates fractional capacity deficit to gravitational potential. The finite marked-transfer calculation is derived inside the displayed action; its stable geometric embedding and several infrared branches remain conditional.

The most controlled branch is the ordinary static weak-field action and source map

$$\text{microstructure} \longrightarrow \text{coefficient chain} \longrightarrow \text{static capacity EFT} \longrightarrow \{G,\ \text{baseline metric}\}.$$

It recovers the Newtonian point-source limit, the leading no-slip metric, and the parametrized post-Newtonian values of general relativity through its Einstein parent. A specified transverse effective branch produces the galactic acceleration scale a_0 and the radial-acceleration relation without per-system tuning, subject to the microscopic matching conditions stated in Section 15 and Appendix N. The electron anchor, memoryless dressing, and marked vertex fix the substrate length inside the stated support-to-rate branch, and the resulting Newton normalization and corrected charged-lepton ratios agree with current measurements within one standard deviation. The comparison also runs backward: with the marked weight held at its action value, the measured Newton constant selects the vertex's routing integer, and the unique survivor is the same seven fixed by the tetrahedral alphabet (Section 13.5). Appendix L separates this action-level closure from the historical fact that the residuals were already known.

Later parts treat time-dependent transport, galaxy clusters, cosmology, the saturated early phase, strong fields, and particle and gauge extensions. Their derivational status is listed in Part VIII.

Many-Pasts, the third postulate, already does work in this chain: faithful sector resolution selects the memoryless electron-dressing kernel, and Many-Pasts supplies the history space in which that kernel operates. Its consequences for quantum probability, branch realization, and the arrow of time are developed in Section 22 and Appendix G.

1.1 What Is Primitive, and What Is Closed

The word “closure” is used here in a specific sense. The paper does not derive the existence of a finite entanglement substrate or the tetrahedral boundary architecture from a deeper microscopic Hamiltonian; those are theory-defining inputs, and the closure claim begins only after they are fixed. There are five such inputs: finite local entanglement capacity; geometry–capacity equivalence; mass–entropy equivalence, including matter as localized defects of committed capacity; the Many-Pasts ontology of Postulate III, with its operational quantum measure imported as stated below; and the tetrahedral ultraviolet architecture, including positive oriented matching of the two primitive descriptions of a shared face. Appendix B writes that matching rule as a primitive pair operator and separates its exact consequences from the remaining incidence condition. This refines the existing ultraviolet input rather than adding a fourth postulate. Maximum caliber is likewise not a sixth input: it restates, for histories, the same faithful full-support condition already used to select the admissibility ensemble — every pass carries the largest path entropy compatible with the same fixed marginal. The equivalence has to be stated explicitly because finite capacity by itself does not imply renewal.

Given those inputs, the chain closes step by step. Finite counting fixes the admissibility weighting and the effective entropy. Faithful full-support resolution fixes the memoryless replacement kernel, and the decorated native-cell vertex realizes its reversible update. The state-weighted determinant supplies the baseline seven-channel recurrence, and the same closure amplitude, projected through two directed singlet returns and canonically dilated, supplies the marked correction and its 21-edge determinant. The electron anchor fixes the proper-time cadence, and the spatial and temporal readings of the same phase mode give $L_* = c\tau_*$. Edge transport and source projection then determine the ordinary static response. The longitudinal functional turns out to be the Einstein scalar-constraint sector in a different variable, so it yields the Newtonian limit, the baseline no-slip metric, and the PPN values of general relativity. The geometric condensate embedding and the galactic branch remain conditional.

The decorated vertex adds no sixth foundational premise. It specifies the minimal marked field content and gluing that realize the faithful full-support rule, native tetrahedral update, and one-bit fermionic electron anchor already listed above. Those action-level choices are stronger than a numerical ansatz because they determine the response multiplicity, determinant power, and routing together; they are also falsifiable, since the nonminimal and differently routed vertices give the alternative values displayed in Appendix H.9.

In compressed form, the central claim is

$$\boxed{\text{primitive UV capacity hypothesis}} \to \boxed{\text{finite counting + admissibility}} \to \boxed{L_*}$$
$$\to \boxed{\gamma,\ \kappa/\gamma} \to \boxed{\delta S \leftrightarrow \Phi} \to \boxed{G,\ \text{baseline metric}},$$
$$\boxed{a_0,\ \text{RAR}}\ \text{in the conditional transverse EFT.}$$

The microstructure is an input: a finite ultraviolet counting problem from which the weak-field sector is derived.

The absolute length calibration uses the electron, the lightest elementary charged defect, as the dimensional anchor. Faithful full-support resolution forces the replacement kernel to be memoryless: a dressing pass can carry the full admissibility entropy only if it retains no memory of the endpoint it replaces (the one-line proof appears in Section 1.2). The lightest-defect functional selects the fermionic ceiling $k = 7$ with $\Delta_7 = 0$, and the state-weighted determinant of the seven renewed clouds is $r = e^{-7g_{\text{share,eff}}}$. The decorated vertex fixes the residual marked-fiber factor and its electron routing Z_e. Positivity gives the raw survival energy $E_{\text{raw}} = -(\hbar/\tau_*)\ln(1-r)$, and the dressed electron identification $m_e c^2 = (3/2)Z_e E_{\text{raw}}$ then fixes τ_*, and with it $L_* = c\tau_*$, without an independent clock or geometric diameter. Appendix H gives the finite action and both adversarial audits.

The reduced capacity functional can superficially resemble the scalar sector of a Brans–Dicke theory [51], but the resemblance is misleading. In the ordinary static branch it is the Einstein constraint action rewritten through $\delta S = -2S_\infty \Phi/c^2$, not a second scalar–tensor action. The open action questions are narrower and sharper: the transverse thermal influence functional, the generic covariant capacity observable outside controlled reductions, and the saturation-boundary functional.

Several tasks remain: derive the ensemble from a deeper Hamiltonian, embed the decorated transfer vertex in a stable geometric completion, audit the separate finite-loop stiffness return operator, compute the transverse metric kernel, and construct the strong-field boundary action. The concrete geometric routes are a specified GFT condensate and the capacity-decorated CDT critical-surface program of Section 24.9. Each unresolved step is listed explicitly in the closure table.

The construction rests on five commitments.

First, the vacuum is a medium with a bounded local capacity for entanglement.

Second, spacetime geometry and that capacity structure are the same substrate seen at different scales, so that in the weak field gravity is the fractional deficit of locally available capacity.

Third, matter is localized committed capacity: a particle is a stable defect of the medium, and its inertial mass is the entanglement content of that defect read in mass units.

Fourth, a recorded present is supported by many compatible microscopic pasts. The operational branch assigns probabilities only to decoherent record histories through the standard quantum decoherence functional, then conditions them on the realized present. It preserves Born statistics and no-signaling. The reversible renewal dilation supplies a concrete microscopic role for the history degrees of freedom: they receive the previous local state while the present register is renewed.

Fifth, the ultraviolet cell has tetrahedral boundary architecture. Each shared face begins with two fermionic primitive slots and positive oriented matching. The matching operator transmits the maximal coupled multiplet. The marked present/history fiber has the representation content of the discarded complement only for $j_0 = \frac{3}{2}$; conditional on identifying it as that complement, the three-dimensional closure vector fixes the seven-state alphabet internally. Without that incidence identification, $j_0 = \frac{3}{2}$ remains the surviving branch of the discrete audit in Appendix B. Four relational ports, single-copy channel capacity, and the two orientations then give the 1680-state ensemble. The same faithful-resolution standard is applied to its histories: the local process carries the full available path entropy and retains no endpoint memory. “Maximum caliber” names this temporal application of the same standard; it adds no independent commitment to the five listed here.

The finite-capacity substrate is the ultraviolet premise; geometry–capacity equivalence, mass–entropy equivalence, and Many-Pasts are the three postulates; and the tetrahedral ensemble is the ultraviolet architecture. Faithful full-support resolution acts on both states and histories. From these inputs the paper derives the admissibility weighting, $g_{\mathrm{share,eff}}$, the replacement kernel, the one-layer native-vertex update, the marked transfer, the tree edge factor, the weak-field bridge, and the Newtonian metric branch, all inside the displayed decorated action. The separate loop-dressed stiffness, the galactic acceleration scale, and the radial-acceleration relation carry the conditional grades recorded in the closure table.

The architecture therefore contains three postulates, one finite-capacity substrate premise, one minimal ultraviolet ensemble, and a small number of explicitly labeled conditional readings. Applied to the same ensemble, faithful resolution fixes both the state entropy and the maximum-caliber history process.

1.2 Physical Motivation for the Primitives

These commitments are premises, and the derivation begins only after they are fixed. They are nevertheless not arbitrary. Each is motivated by a place where established physics already strains against its own foundations — black-hole thermodynamics, quantum information, the equality of inertial and gravitational mass, and the quantum-mechanical role of records — and each makes a known difficulty look less mysterious once it is adopted. That motivation does not prove the premises; the numerical and structural consequences derived from them provide the tests.

Mainstream gravitational physics has been converging on a finite-capacity substrate for decades. A black hole's entropy scales with the area of its horizon rather than the volume it encloses, as though the contents of a region were written on its boundary; the Bekenstein bound limits the information a bounded region can hold; and entanglement-based reconstructions of geometry tie the shape of spacetime directly to patterns of entanglement. Each of these is usually treated as a deep clue without a mechanism. This paper takes the clue literally: the vacuum is a medium with a finite local budget of entanglement, and geometry is the large-scale description of that budget. Several long-standing puzzles then become the ordinary behavior of a medium that can fill up. A black hole is a region whose capacity is exhausted, so the only live bookkeeping sits at the boundary between exhausted and available capacity — which is why the entropy tracks the area and not the volume. The same finite state space supplies an ultraviolet cutoff: below the cell scale there is no continuum left to diverge, and a finite state space has nothing to renormalize away. The assumption that the empty state maximizes capacity agrees with the thermodynamic direction suggested by gravitational entropy, and three recent continuum results support the same reading from independent directions: the observer-dressed de Sitter algebra makes empty de Sitter the maximum-entropy gravitational state [23], a horizon modular calculation recovers Einstein curvature from the relative information carried by an excitation [22], and a geometric-relative-entropy action yields a local bulk information dynamics with an Einstein limit [24]. None of them supplies the finite cell or its coefficients; Section 29.9 states precisely what they do and do not establish for the present construction.

The mass–entropy identification addresses a coincidence that general relativity encodes but does not explain. General relativity builds in the equality of inertial and gravitational mass geometrically, through the equivalence principle, but gives no microphysical account of why the mass that resists acceleration and the mass that sources attraction should be one and the same. Here both are readings of a defect's committed entanglement, so the equality follows from the construction instead of being imposed by hand. The same identification bears on why gravity is so weak. The induced gravitational scale contains the exact factor $Z_e^2 \ln^2(1 - e^{-7g_{\mathrm{share,eff}}})$, whose dominant hierarchy is $e^{-14g_{\mathrm{share,eff}}}$: a large sharing entropy makes gravity exponentially feeble, and the gap between gravity and the other forces becomes a matter of arithmetic rather than fine-tuning. The galactic extension carries a lower closure grade. In its conditional transverse branch, $a_0 = \epsilon c H_0$ with $\epsilon \equiv g_{\text{share,eff}}/(4\pi^2)$ ties the onset of anomalous rotation to the cosmic horizon, giving concrete form to the old Machian suspicion that local inertia should depend on the universe at large, while the one-capacity-invariant response ties the galactic field directly to the ordinary-matter source — the feature that lets a galaxy's rotation track its visible matter so tightly. The microscopic phase-cell normalization and influence kernel are the tests of that connection.

The scale-setting proposal separates two questions that sound alike: how hard the medium works to keep a defect bound, and how often the defect fully re-forms. Because the medium never holds still — every instant it is re-drawn from all the ways it could be — a particle is a pattern the medium must continually re-form, and the entanglement it commits is a maintained quantity, re-established at every update. Keeping the knot bound means holding a grip on every strand at once, and separate holds add up, so the maintenance is large. Full re-completion, with every strand falling into alignment at the same instant, is a simultaneous coincidence, and coincidences multiply, so it is exponentially rare. The particle's mass follows the second question, not the first: it is fixed by how rarely the binding fully re-closes. This is why a particle can be vastly lighter than the natural substrate scale with no small number inserted anywhere: the electron is deeply bound, and deep binding makes its full coherent recurrence exponentially rare. Quantitatively, seven channel entropies add, so their effective support multiplies; the state-weighted determinant turns that support into a record-conditioned transfer rate; the positive survival operator gives the rest-energy gap; and the decorated marked vertex fixes the finite closure-response correction while the electron anchor fixes the clock. Section 23 and Appendix H give this its full form.

A memoryless update rule posits no hidden machinery carrying information forward from tick to tick, and in this construction it is also forced. For any stationary per-channel kernel with marginal p_{η_*},

$$H(B_{t+1} \mid B_t) = g_{\text{share,eff}} - I(B_t; B_{t+1}) \leq g_{\text{share,eff}},$$

so a dressing pass carries the full admissibility entropy if and only if the endpoint mutual information vanishes, which fixes the refresh kernel $K(b, b') = p_{\eta_*}(b')$. The allowed local single-label dynamics cannot realize this requirement: they freeze into disconnected sectors that never explore the full space (Appendix D.4), so the refresh must act nonlocally on the native cell. The decorated vertex prepares the diagonal fresh amplitude and realizes the charged recurrence; its geometric condensate embedding and durable history capacity remain open. A single universal refresh rate also lets the theory carry a smallest length without conflicting with the experiments that ended earlier discrete-spacetime proposals: the granularity lives in the capacity, not in a preferred spatial lattice, so gravitational waves and light travel at the same speed and there is no frame-dependent dispersion left to detect.

Many-Pasts holds that the present configuration of the entanglement network is supported not by one definite microscopic past but by a conditional ensemble of compatible decoherent histories. It addresses directly the quantum puzzles that interpretations of quantum mechanics were invented to handle. The interference in a double-slit experiment is the persistence of the unrecorded alternative histories in that ensemble; a durable which-path record conditions the ensemble, and the interference goes away. The correlations of an entangled pair come from weighting the histories of the whole joint system, which reproduces the nonclassical statistics with no signal passing between the two wings. Measurement adds no separate collapse law; it lays down a durable record, after which the relevant histories are the ones compatible with it. The operational construction is therefore ordinary quantum mechanics equipped with a history-space ontology, and it leaves laboratory predictions intact: Born-rule statistics and no-signaling both hold. Its arrow-of-time extension still needs a substrate typicality theorem. The postulate also has a concrete job beyond interpretation: faithful sector resolution independently selects the memoryless dressing, and Many-Pasts supplies the history-space setting in which that process operates and in which its exported registers live.

Tetrahedra are the standard building block in several approaches to quantum geometry, so the cell is a familiar object rather than one invented for this paper. What the framework adds is a precise primitive matching rule. After the two descriptions of a shared face are transported into one orientation, its positive mismatch operator has the maximal-spin sector as its unique null space. For the physical branch, the independently constructed nine-state marked register matches the complete nonmaximal fusion sector only at $j_0 = \frac{3}{2}$. Conditional on identifying those two objects dynamically, three-dimensional closure fixes seven face labels; four relational ports, single-copy channel capacity, and two orientations then give exactly 1680 boundary states. Without that identification the same result is the unique surviving point of a stated discrete branch audit, not a consequence of minimality. The closure structure also permits only three nondegenerate charged-lepton shells, a candidate answer to why the Standard Model contains three charged-lepton generations. Appendix L preserves the historical provenance of the construction. Once the branch is fixed, its entropy contains no adjustable continuous parameter.

Several mainstream results already point in the same direction: Einstein's equation derived as a thermodynamic relation of state, horizon entanglement entropy, and reconstructions of spatial connectivity from entanglement. The construction below makes a finite microscopic proposal within that program and derives quantities that can be checked.

Motivation alone carries little evidential weight, because a proposal of this scope can almost always assemble a list of mysteries explained after the fact. The relevant questions are how much freedom remained when the construction was chosen and whether one fixed construction survives measurements it did not anticipate. Appendix L audits the first question. The following sections derive the weak-field consequences and compare them with observation.

2. Canonical Field Content and Definitions

Before the symbols, four plain words recur throughout. *Capacity* is the entanglement support locally available in the medium. A *defect* is a stable, localized commitment of that capacity — what we coarse-grain into a particle. A *deficit* is capacity no longer freely available to the surrounding vacuum because a defect has committed it. *Strain* is the extended profile of that deficit reaching out into the medium, whose fractional size the weak-field potential tracks. The field variables below are the precise versions of these words.

We define the fundamental continuum variable as the vacuum-relative coarse-grained entanglement assigned to a UV probe cell of size L_* centered at x:

$$S_{\text{ent}}(x) \in \mathbb{R},$$

measured in nats and therefore dimensionless. This is not a literal microscopic entropy density at a mathematical point. It is the leading scalar order parameter associated with a vacuum-relative entanglement defect after coarse-graining over a UV cell.

This definition keeps the microscopic and continuum pictures tied together. At continuum level, $S_{\text{ent}}(x)$ is the field that appears in the action and field equations. At the microscopic level it is the coarse variable recording how much local entanglement capacity remains available in the underlying medium after averaging over a UV cell.

The asymptotic vacuum-capacity baseline is denoted S_∞, and the deficit field is

$$\delta S(x) \equiv S_\infty - S_{\text{ent}}(x).$$

Positive δS denotes reduced available vacuum entanglement capacity in the neighborhood of a localized defect or defect distribution. It is the extended capacity-strain field sourced by the defect sector, not an independent medium acted on by matter from outside. For nonlinear work it is useful to define the bounded occupancy fraction

$$q(x) \equiv \frac{S_{\rm ent}(x)}{S_\infty} = 1 - \frac{\delta S}{S_\infty} \in [0,1].$$

The variables $S_{\rm ent}$, δS, and q therefore describe the same local physics in three closely related ways: available capacity, missing capacity relative to vacuum, and surviving-capacity fraction. Each is used where it is most transparent: δS for the weak-field theory, because it maps directly onto the Newtonian potential; q for the nonlinear and strong-field completion, because boundedness is built in from the start; and $S_{\rm ent}$ itself for the covariant EFT, because it is the field that appears in the action. The operational meanings are:

- $q = 1$: vacuum capacity fully available in the absence of local defect-induced capacity strain;
- $0 < q < 1$: partial local capacity reduction around a defect configuration;
- $q = 0$: complete local exhaustion of available capacity on the physical branch.

Fixed-epoch normalization. The absolute normalization of $S_{\rm ent}$ and S_∞ is a convention once an epoch and cell convention have been fixed. Under a constant rescaling

$$S_{\rm ent} \mapsto K S_{\rm ent}, \qquad S_\infty \mapsto K S_\infty, \qquad \delta S \mapsto K\delta S,$$

the observable bridge

$$\frac{\Phi}{c^2} = -\frac{\delta S}{2S_\infty}$$

is unchanged. The source equation is invariant in the same sense: rescaling the entropy field rescales the source coefficient with it, so the observable Newtonian normalization depends on the gauge-invariant combination $\kappa/(\gamma S_\infty)$ rather than on S_∞ alone. A cell-normalized description and a horizon-normalized description can therefore assign different numerical values to S_∞ without changing Φ, G, or the PPN limit. This is not a time-dependent gauge symmetry; it is a fixed-epoch entropy-unit convention. Gravity sees fractional capacity depletion.

Substrate length scale. The canonical UV cell length is not taken to be the conventional Planck length as an input. Faithful full-support resolution fixes the renewal kernel and its history-space factorization. The state-weighted determinant gives the seven-channel recurrence

$$r = e^{-7g_{\rm share,eff}}, \qquad L_*^{(0)} = -\frac{3}{2}\lambda_e \ln(1-r), \qquad \lambda_e = \frac{\hbar}{m_e c}.$$

The decorated marked-transfer vertex derived in Appendix H fixes

$$\zeta_* = 9e^{-g_{\rm share,eff}}\left(1 - \frac{8\eta_*}{49}\right)^{21/2}, \qquad Z_e = (1+\zeta_*)(1+7\zeta_*^2).$$

The physical electron-anchored scale is

$$\boxed{L_* = Z_e L_*^{(0)}} = 1.6162537014 \times 10^{-35} \text{ m}.$$

The corresponding induced gravitational scale is

$$G_* := \frac{c^3 L_*^2}{\hbar} = \frac{9}{4}\frac{\hbar c}{m_e^2} Z_e^2 \ln^2\!\left(1 - e^{-7g_{\rm share,eff}}\right) = 6.6742890772 \times 10^{-11} \text{ m}^3\,\text{kg}^{-1}\,\text{s}^{-2}.$$

The lightest one-bit fermionic defect resolves the seven face sectors once and exports the transverse 2/3 share of that dressing block. The marked vertex accounts for its finite closure-response fiber and label-return loop. The conventional Planck length $L_P = \sqrt{\hbar G/c^3}$ remains useful for comparison and for standard black-hole thermodynamic notation, but it is not the primitive scale-setting input here.

The principal coefficients and derived quantities used throughout are:

$$\gamma: \text{ entanglement-field stiffness,} \tag{1}$$

$$\kappa: \text{ defect–entropy coupling,} \tag{2}$$

$$\kappa_m(\ell): \text{ mass-per-entropy map at scale } \ell, \tag{3}$$

$$L_*: \text{ substrate cell length in the electron-anchored support-to-rate map,} \tag{4}$$

$$G_*: \text{ gravitational scale induced by } L_*, \tag{5}$$

$$g_{\text{share,max}} = \ln(1680), \tag{6}$$

$$g_{\text{share,eff}}: \text{ admissibility-weighted effective sharing entropy,} \tag{7}$$

$$J_{\text{bare}},\, J_{\text{eff}}^{\text{tree}},\, J_{\text{eff}}^{(\text{ren})}: \text{ UV edge-kernel couplings,} \tag{8}$$

$$a_0 = \frac{cH_0 g_{\text{share,eff}}}{4\pi^2} \quad \text{in the conditional compact two-phase normalization.} \tag{9}$$

The gravitational potentials are denoted Φ and Ψ, and the canonical weak-field bridge will be written as

$$\frac{\Phi}{c^2} = -\frac{\delta S}{2S_\infty}.$$

These same symbols reappear in the ultraviolet closure chain, the continuum action, and the phenomenology sections. From this point onward each one keeps the same meaning, so the later derivations build on a single notation rather than shifting between parallel conventions.

3. The Three Postulates

The framework rests on exactly three postulates because they answer three different questions. *What is spacetime geometry?* — Postulate I: geometry is the long-wavelength expression of the capacity substrate itself. *What is matter, and what is mass?* — Postulate II: matter is localized committed capacity, and mass is its inertial reading. *What supports the present, and what fixes the direction of history?* — Postulate III: the present is supported by a history-space ontology over the decoherent pasts compatible with one realized record. Operational probabilities come from the standard decoherence functional, while the proposed arrow of time requires an additional typicality result. Faithful sector resolution, applied to this history space, selects the memoryless dressing kernel used by the decorated scale-setting action.

3.1 Information–Geometry Equivalence

The first postulate states that spacetime geometry is the continuum expression of the capacity substrate. This is stronger than saying entanglement contributes an additional piece of stress-energy inside otherwise standard general relativity: the metric and the scalar capacity sector are two projections of one finite medium, and S_{ent} is not appended to an independent background geometry. In the weak field, gravitational potential is the fractional deficit of available capacity.

Two consequences of this reading should be kept distinct from the start. First, absolute S_{ent} is not itself "the gravitational potential"; the observable weak-field potential comes from the fractional deficit $\delta S/S_\infty$, which is why a fixed-epoch rescaling of entropy units leaves gravity unchanged (Section 2). Second, because geometry and capacity are two descriptions of one response, the deficit is not an extra force appended to an independently existing metric. Section 10 proves that the ordinary reduced capacity functional is the Einstein constraint action in the capacity coordinate. The remaining common-parent problem concerns the transverse thermal and boundary sectors, not the baseline Newtonian response.

3.2 Mass–Entropy Equivalence

The second postulate identifies mass as the inertial reading of localized capacity commitment. At scale ℓ,

$$m(\ell) = \kappa_m(\ell)\,\Delta S.$$

A particle is already a localized defect of the entanglement substrate, so $m = \kappa_m \Delta S$ does not assert an analogy between two independent things; it asserts that the inertial content of the defect *is* its entanglement content, read in mass units.

For elementary fermionic sectors the canonical defect increment is

$$\Delta S_f = \ln 2.$$

The one bit here is not arbitrary. An elementary fermionic exclusion is binary — the face is occupied or unoccupied — and a binary distinction carries exactly $\ln 2$ of missing entanglement. This is the simplest possible defect increment, which is why the lightest such defect, the electron, becomes the cleanest anchor for the mass–entropy map (Section 13). Composite sectors instead require their fully dressed bound-state entanglement budgets.

Two corollaries are used later. First, because mass and entanglement budget are two descriptions of the same defect, and the masses of separated defects add, capacity committed in service of one defect cannot simultaneously serve another: shared service would make the joint budget, and with it the joint mass, sub-additive. Commitment is therefore per-defect — each committed unit carries the label of the defect it serves. Second, the same bookkeeping makes the saturated early phase countable: its abundance follows from the number of committed units, with no double-counting across separated defects (Section 20).

3.3 Many-Pasts Hypothesis

The third postulate concerns the microscopic support of the present entanglement network. A recorded present can be compatible with many coarse-grained histories of the substrate. Many-Pasts takes those alternative pasts seriously while retaining one realized macroscopic present. Its probability theory must distinguish amplitudes for alternatives that still interfere, probabilities for recorded presents, and conditional probabilities for decoherent histories compatible with a given record.

The operational construction uses the decoherent-histories formalism [66]. A coarse history $h = (\alpha_1, \ldots, \alpha_n)$ has class operator

$$C_h = \Pi^{(n)}_{\alpha_n} U_{n,n-1} \cdots \Pi^{(1)}_{\alpha_1} U_{1,0},$$

and decoherence functional

$$\mathcal{D}(h, h') = \mathrm{Tr}\Big(C_h \rho_0 C^{\dagger}_{h'}\Big).$$

When a family decoheres, $\mathcal{D}(h, h') \simeq 0$ for $h \neq h'$, its diagonal entries obey the ordinary probability sum rules. If P denotes a final macroscopic record and $\mathcal{H}_P$ is a decoherent refinement of the histories ending in that record, then

$$p(P) = \sum_{h \in \mathcal{H}_P} \mathcal{D}(h, h) = \mathrm{Tr}(\Pi_P \rho_{\text{now}}), \qquad p(h \mid P) = \frac{\mathcal{D}(h, h)}{p(P)}.$$

The probability of the present is obtained from the common normalized measure over all records before the history distribution is conditioned on P. Unresolved alternatives remain combined at amplitude level; no positive probability is assigned to individual fine-grained paths that have not decohered.

This construction makes the operational branch standard quantum mechanics. Quantum instruments give the Born probabilities of laboratory records, and local trace-preserving instruments give no-signaling marginals. Many-Pasts changes their history-space interpretation without adding a collapse term or a signaling bias.

The ontological reading is record-retentive. The realized present includes both its current macroscopic configuration and the physical records that encode earlier events. Compatible pasts are distinguished to the resolution carried by those present records; they are not additional coexisting spacetimes. When a physical process erases a record distinction, the conditional history measure coarsens by summing the histories that the surviving record can no longer separate. Conditioning on a future record is defined only after that record belongs to a present configuration, so the construction supplies no future-to-past force or retrocausal update law.

Once quantum kinematics are admitted, Gleason's theorem constrains the probability measure. On a Hilbert space of dimension greater than two, a normalized, noncontextual, additive measure on a sufficiently rich lattice of record projectors has the form

$$\mu(\Pi) = \mathrm{Tr}(\rho\Pi)$$

by Gleason's theorem [60]. A medium-decoherent family with a pure initial state admits orthogonal generalized records for its branch state vectors, so each history probability can be represented as the probability of a single-time record projector [61]. The Born form is therefore unique for record-defined decoherent families if record completeness and noncontextual additivity across compatible record refinements are imposed. This is a conditional uniqueness theorem inside quantum kinematics, not a derivation of those kinematics from the substrate. The Hilbert space, unitary dynamics, initial state, and record-projector richness remain imported, and a mixed state requires the corresponding purification or generalized-record construction.

The postulate and the renewal theorem have separate logical roles. Many-Pasts supplies the ontology and the record-conditioned history space. The already-stated faithful full-support condition selects the memoryless dressing kernel when applied to paths at fixed admissibility marginal; "maximum caliber" names this history-space form and is independent of the Born measure on laboratory records. The lightest-defect functional selects the same absence of temporal memory and, within the support-to-length map, selects seven occupied channels with vanishing inter-channel correlation. A reversible dilation exports the old replaceable closure register to history. The decorated transfer vertex supplies the determinant recurrence, finite marked response, and Compton phase readout.

The renewal statement applies to the replaceable closure register, not to every degree of freedom carried by a charged defect. The marked internal fiber and its position degree of freedom belong to the retained system. Appendix H.11 proves the corresponding coherence criterion: after a dilation, a spatial off-diagonal is multiplied by the overlap of the discarded records produced by its two branches. Free marked transport must therefore export no cell address or other which-path label. This is a consistency condition on the microscopic realization of the existing operational quantum postulate, not an additional founding premise.

4. Relativistic Continuum Structure

4.1 Capacity budget and continuum symmetry

The continuum description is expected to be covariant because the substrate itself is finite-capacity, isotropic, and relational — covariance is read off from the substrate's own properties rather than added as a geometric axiom at the outset.

The first ingredient is a finite maximal update rate, denoted by the same constant c that later appears in the transport relation $D/\tau_0 = c^2$. In the present interpretation, c measures the largest rate at which the substrate can propagate and reorganize information. A defect at rest spends that budget entirely on local temporal evolution. A defect in motion must spend part of the same budget on spatial transport within the surrounding network. Because the substrate is isotropic, the cost of motion depends only on the rotational scalar v^2 at leading order, with the temporal rate maximal at $v = 0$ and vanishing when the budget is exhausted at $v = c$. These endpoint conditions alone admit many interpolating functions and so do not fix the form of the time-dilation relation. The form is fixed once the finite update speed is treated as invariant across inertial coarse descriptions: homogeneity, isotropy, and the relativity principle then select the Lorentz group rather than the Galilean one, giving the invariant interval

$$c^2 d\tau^2 = c^2 dt^2 - d\mathbf{x}^2,$$

and hence

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{v^2}{c^2}}.$$

The capacity-budget picture supplies the substrate interpretation of this Lorentzian kinematics: motion allocates part of the finite update budget to spatial transport, leaving the remaining fraction as proper-time evolution.

The same capacity language also unifies motion-induced and gravity-induced clock slowing. In the nonlinear branch the surviving-capacity fraction is

$$q = \frac{S_{\mathrm{ent}}}{S_\infty},$$

so smaller q means that less local update capacity remains available. Motion reduces the temporal share of the budget by consuming part of it in spatial transport; a nearby defect reduces the local budget by depleting available capacity. The two familiar time-dilation effects are therefore interpreted as two regimes of one mechanism.

The second ingredient is the relational character of the substrate. The network is not embedded in a prior physical manifold whose coordinate labels carry independent meaning; its physical content is the pattern of local capacities, defects, and neighborhood relations. Continuum coordinates are descriptive labels imposed on that relational structure, and smooth coordinate changes relabel the same underlying configuration. This is precisely why the low-energy theory must be written in generally covariant form.

The metric sector, then, is not introduced from outside. Lorentzian geometry is the natural coarse description of a finite-capacity, isotropic, relational substrate, and the Einstein sector is its lowest-order continuum gravitational expression, with the entanglement scalar tracking how localized defects redistribute the same capacity geometry. As with any discrete substrate, this continuum claim faces a sharp known obstacle: a discrete structure with a preferred rest frame feeds dimension-four Lorentz-violating operators into the infrared with order-unity coefficients through loops [31], against laboratory bounds many orders of magnitude below unity. The protection here is structural. The tetrahedral ensemble is combinatorial and pre-geometric: it lives in the state counting from which the continuum is constructed, defines no embedding lattice in the emergent spacetime, and imprints on the EFT only through the frame-independent scalars L_*, $g_{\mathrm{share,eff}}$, and η_*. Discreteness of this class is compatible with exact low-energy Lorentz symmetry, as causal-set sprinkling demonstrates by construction [32, 33]. The cosmological bath does select a frame, but only in the environmental sense the CMB does: a state rather than an operator, while the laboratory bounds constrain operators. The supporting calculation this argument calls for — that substrate loops generate no dimension-four Lorentz-violating operators — is still required and is listed in the closure table.

4.2 Dependency Map of the Theory

The logical flow begins with the three foundational postulates — Information–Geometry, Mass–Entropy, and Many-Pasts — with faithful full-support resolution applied to both states and histories, and runs through the static weak-field chain before reaching the conditional sectors:

$$\begin{gathered}\{\text{three postulates}\}\\ \to\ \text{finite-capacity substrate ontology}\ \to\ \text{tetrahedral boundary ensemble}\\ \to\ \text{faithful resolution of states and histories}\ \to\ \text{local replacement / history export}\\ \to\ \text{edge transport / loop dressing / source map}\\ \to\ \text{capacity form of the Einstein constraint} \to\ \text{Newton / baseline no-slip / GR PPN,}\\ \text{with the one-invariant transverse EFT} + \text{horizon matching}\\ \to\ \{a_0,\ \text{RAR}\}\ \text{conditionally,}\\ \text{while transverse-sector lensing remains open.}\end{gathered}$$

Only then come the conditional and frontier sectors — transport, clusters, cosmology, strong field, and the particle/gauge extensions — each developed as a consequence or completion of the same framework.

Two features of this map matter. First, it is a dependency graph, not an equality of closure status: the ordinary static branch is closed more tightly than the transverse, cosmological, or strong-field sectors, and Part VIII makes that difference explicit in a closure-status table. Second, Many-Pasts appears at the top of the map because the scale-setting chain uses it: faithful full-support resolution selects the local renewal process on the history space that Many-Pasts supplies. Their combination gives a reversible present/history exchange; neither the Born history measure nor record conditioning alone selects memorylessness.

Part II. UV Coefficient Chain

Part I fixed what the theory is about. The question now is whether the local capacity-sharing structure can actually be *counted*. If the substrate has finite local capacity, the coefficients that appear in the continuum weak-field theory should not be free continuum parameters; they should descend from a finite local boundary problem. The next five sections follow that problem through: the smallest boundary cell that can carry capacity and close isotropically, the weighting that selects well-closed configurations, the cost of neighboring cells disagreeing, the local returns that dress that cost, and the continuum coefficient they leave behind. The baseline calculation is internal to the microscopic construction. Appendix B separately propagates nearby discrete branches to the Newton scale, and Appendix L records that this comparison is postdictive rather than historically blind.

Several of the ultraviolet choices below may look at first like independent tunings: the tetrahedral cell, the seven labels, the injective assignment, the parity doubling, the admissibility kernel, the transverse export, and the electron anchor. None is a phenomenological knob, and none varies from galaxy to galaxy. Section 5 and Appendix B derive the finite ensemble and its entropy; Appendix C derives the edge projection; Appendices D and H derive the replacement process, the reversible history export, the factorized lightest branch, and the decorated marked-transfer action; Appendix L records the historical fork accounting. The remaining microscopic task is to embed that finite transfer vertex in a stable geometric continuum, through a specified GFT action or a capacity-decorated CDT transfer matrix on a critical trajectory.

5. Why a Tetrahedral Boundary Ensemble

The problem is to find a finite boundary cell that can carry channel entropy, close isotropically, and hand a scalar response to the continuum. In three spatial dimensions the minimal volumetric simplex is a tetrahedron. The construction uses five ingredients:

- a tetrahedral volumetric cell;
- half-integer primitive data on the two sides of each shared face;
- positive matching after the two sides are placed in one orientation;
- four distinguishable relational ports with single-copy channel capacity;
- binary cell orientation.

This package is not presented as the only possible ultraviolet completion. Postulate II assigns a half-integer primitive spin

$$j_0 = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$$

to each side of a shared face. Before gluing, the pair spans

$$V_{j_0} \otimes V_{j_0} = \bigoplus_{J=0}^{2j_0} V_J.$$

After orientation transport, positive coherent matching is generated by a nonnegative mismatch operator whose unique null space is the maximal coupled sector. Its sharp limit is therefore

$$\mathcal{G}_{\text{sharp}} = P_{2j_0},$$

and the transmitted face alphabet has

$$|M| = \dim V_{2j_0} = 4j_0 + 1$$

states. Appendix B derives this operator on the full tensor product, gives its finite-width spectrum, and states the Hessian tests that would falsify the primitive matching rule.

The marked sector provides a second, more selective relation. The independently constructed present/history response is a product of two spatial vectors,

$$\mathcal{H}_{\text{mark}} = V_1^P \otimes V_1^H = V_0 \oplus V_1 \oplus V_2.$$

The complete nonmaximal information left by maximal fusion is

$$Q(j) = (V_j \otimes V_j) \ominus V_{2j} = \bigoplus_{J=0}^{2j-1} V_J.$$

Because both sums are multiplicity-free,

$$\mathcal{H}_{\text{mark}}(s) \cong Q(j) \quad \Longleftrightarrow \quad j = s + \frac{1}{2}.$$

The closure response is a three-component spatial vector, so in $d = 3$ it carries $s = 1$. Conditional on the physical incidence statement that the marked fiber is the faithful record of the nonmaximal fusion channels,

$$j_0 = \frac{3}{2}, \qquad V_{2j_0} = V_3, \qquad |M| = 7.$$

This is a conditional derivation from the already present marked and fusion structures, not a new minimality axiom. The representation equivalence and its complement isometry are exact. What remains to be shown by the complete microscopic vertex is that the two representation spaces are the same dynamical present/history record. If that incidence identification fails, $j_0 = \frac{3}{2}$ remains the branch selected by the discrete audit of Appendix B.5.

The four faces are relationally distinct ports. A displayed letter is the occupation of one mode in a single cell-level channel resource; the fermionic single-copy ceiling therefore forbids the same channel from being routed through two ports at once. This gives an injective assignment without antisymmetrizing away the port labels. The two global orientations remain distinct microscopic states. The resulting count is

$$\Omega_{\text{tet}} = 2P(7, 4) = 1680,$$

and the combinatorial sharing ceiling is

$$g_{\text{share,max}} = \ln(1680) = 7.42654907240.$$

The exact equality

$$16 = 7 + 9$$

has a direct meaning: in the sharp primitive pair map, seven states form the geometric link and the other nine can be retained by the marked record. Within the physical three-dimensional rotation algebra the conditional chain is

$$d = 3 \Longrightarrow V_1^P \otimes V_1^H \Longrightarrow j_0 = \frac{3}{2} \Longrightarrow V_3 \Longrightarrow 7.$$

It is not asserted as a theorem under dimensional continuation to arbitrary $\mathrm{Spin}(d)$.

The exact K^2 spectrum and branch audits are given in Appendix B. The j-labeled tetrahedron used here coincides with the quantum tetrahedron of simplicial spin networks [57, 58], whose discrete geometric spectra [59] arise from the same $SU(2)$ representation theory. The present construction differs in weighting these states by admissibility closure rather than by a spin-foam amplitude, and in routing them to a capacity entropy rather than to area and volume operators.

6. Admissibility Closure

6.1 Minimal isotropic kernel

Not every boundary configuration should count equally. The raw combinatorial ensemble is too permissive to be the complete ultraviolet input: some configurations sit close to the regular closure pattern expected of a smooth local cell, while others are badly distorted. Admissibility closure is the statement, in its mildest form, that more poorly closed configurations contribute less to the coarse ensemble. The minimal rotationally invariant measure of that distortion is a single quadratic closure-defect scalar K^2, and the weighting it induces is

$$p_\eta(b) \propto e^{-\eta K^2(b)}.$$

Normalization, isotropy, and a fixed quadratic closure moment select this maximum-entropy kernel. Higher invariants such as K^4 carry additional ultraviolet information and enter as subleading refinements.

6.2 Closure condition and uniqueness

The admissibility precision η is not chosen externally; it is fixed by maximizing the normalized closure evidence. Tetrahedral closure is the vanishing of the three-component oriented-face sum, so the closure-defect space is three-dimensional, and the quadratic family on it carries a determinant weight $\eta^{3/2}$. The closure-evidence functional is therefore

$$\mathcal{F}(\eta) = \ln Z(\eta) + \tfrac{3}{2} \ln \eta,$$

and its stationary point gives the closure condition

$$\langle K^2 \rangle_\eta = \frac{3}{2\eta},$$

in which the factor 3/2 is the determinant weight of the three independent closure components, while the discreteness and multiplicities of the spectrum stay inside the exact sum $Z(\eta)$. This is the stationary normalized-evidence point of the exact closure spectrum, and it is a maximum rather than a bare root (Appendix B.2).

On that spectrum it is unique,

$$\eta_* = 0.0298668443935.$$

The closed branch is locally stiff: small fractional changes in η produce only small fractional changes in the downstream effective sharing entropy.

6.3 Effective sharing entropy

The admissibility-weighted effective sharing entropy is

$$g_{\text{share,eff}} = 7.41980002357.$$

The gap between $g_{\text{share,max}}$ and $g_{\text{share,eff}}$ is therefore not loss imposed by hand. It is the difference between the raw combinatorial ceiling and the admissibility-closed effective boundary entropy that actually propagates into observable couplings.

The continuum description does not inherit the naive channel-counting ceiling; it inherits the portion of the channel space that survives after closure is imposed. The downstream couplings should therefore be read as consequences of admissibility-closed sharing, not of raw combinatorics alone.

With η_* fixed, the effective sharing entropy carries no remaining freedom; the exact spectrum, multiplicities, and uniqueness proof are given in Appendix B.2.

7. Edge Kernel and Tree-Level Coupling

Admissibility determines the capacity of one cell. The edge kernel determines the cost when neighboring cells differ, and that cost becomes the continuum stiffness: stronger resistance to local disagreement makes capacity deficits spread less readily. The same ultraviolet closure data fix both quantities. The geometric bridge is the tetrahedral identity

$$\sum_{i=1}^{4} \hat{n}_i \hat{n}_i^{\mathsf{T}} = \frac{4}{3} I_3,$$

which implies a channel-averaged transverse fraction of 2/3 and gives the bare edge smoothness coupling

$$J_{\text{bare}} = \frac{2}{3} \eta_*.$$

If adjacent cells disagree strongly the edge pays a larger penalty; if they agree, the penalty is small. The factor 2/3 is the geometric fraction that survives after averaging the four tetrahedral channel directions into the isotropic continuum limit — the part of the disagreement that the scalar sharing channel actually carries.

For a $z = 4$ regular coarse adjacency graph, the tree-to-lattice reduction then yields

$$J_{\rm eff}^{\rm tree} = \frac{J_{\rm bare}}{3} = \frac{2\eta_*}{9}.$$

The division by 3 comes from the branching geometry of the rooted $z = 4$ graph. One neighboring link points back toward the source, while the remaining $z - 1 = 3$ links carry forward transport into the tree. The net long-range transport $J_{\rm eff}^{\rm tree}$ is therefore the portion of the microscopic edge penalty that survives this local branching.

Origin of the horizon target. The horizon target

$$\sigma_* = \frac{\pi}{g_{\rm share,eff}}$$

is the closure-consistency value required by the horizon-normalized field convention. In the admissibility-closed boundary ensemble, one active microscopic sharing unit carries effective entropy $g_{\rm share,eff}$. In the continuum normalization used for the weak-field scalar, the occupancy variable is normalized by

$$S = \pi Q_{\rm occ},$$

so a coarse horizon-normalized channel with occupancy $Q_{\rm occ} = 1$ carries entropy π in the S-field convention. If σ_* denotes the asymptotic conditional-independence weight seen by the rooted shell hierarchy (Appendix B.3), consistency between the boundary entropy count and the horizon-normalized continuum field requires

$$\sigma_* \, g_{\rm share,eff} = \pi,$$

and therefore

$$\sigma_* = \frac{\pi}{g_{\rm share,eff}} = 0.42340665\ldots.$$

The factor σ_* matches the admissibility-closed microscopic entropy normalization to the horizon-normalized scalar-field convention. The rooted shell observable converges rapidly to this closure target, constraining the nonlocal correction at small shell depth. The four tetrahedral channel directions average to the isotropic tensor structure in the continuum limit, so the combinatorial data that fix admissibility also fix tree-level transport. Appendix C gives the shell hierarchy and phase-selection checks.

8. Finite-Loop Renormalization

Tree level is not the end of the ultraviolet chain. The full lattice admits local closed-return motifs that recycle part of the transmitted information before it contributes to net coarse transport. The leading correction is organized as a local Dyson self-energy dressing,

$$J_{\rm eff}^{\rm (ren)} = \frac{J_{\rm eff}^{\rm tree}}{1 + J_{\rm eff}^{\rm tree}\Sigma_{\rm ret}}.$$

A purely tree-like transmission rule would let the relevant amplitude move outward once and never locally return; a real coarse graph is not that simple. Some of the transmitted information cycles back through short closed motifs before contributing to long-distance transport. The renormalized coupling is therefore the true stiffness felt by the coarse field after these local returns have been resummed.

The structure of that self-energy is not a generic loop number. The returns split into seven sector-diagonal channels and one collective mode. The seven are the face-label channels, each returning independently without mixing. The one is the permutation-symmetric combination across channels, which returns as a shared closure-singlet rather than as a channel-specific loop, and it is weighted by the same transverse projection and branch-dilution factors that define the tree edge map,

$$\left(\frac{2}{3}\right)\left(\frac{1}{3}\right) = \frac{2}{9}.$$

The leading local self-energy is therefore

$$\Sigma_{\text{ret}} = 7 + \frac{2}{9} = \frac{65}{9}.$$

Equivalently, on the seven-channel scalar return space,

$$R_{\text{ret}} = I_7 + \frac{2}{9}P_{\text{sing}}, \qquad P_{\text{sing}} = |u\rangle\langle u|, \qquad u = \frac{1}{\sqrt{7}}(1,\ldots,1),$$

with $\Sigma_{\text{ret}} = \text{Tr}(R_{\text{ret}})$. The orthogonal six-dimensional sum-zero sector carries no net scalar charge in the coarse branch and so adds no separate scalar return. The singlet weight is fixed by the tree map, not introduced here, so no new loop parameter appears. Permutation symmetry fixes the existence of the singlet but not the placement of the suppression factors on it alone; that placement is the minimal-return-operator reading whose graph-level derivation Appendix C.3 records as the outstanding audit. The induced uncertainty is bounded: replacing $\Sigma_{\text{ret}} = 65/9$ by 7, $7 + 2/3$, or 8 shifts $J_{\text{eff}}^{(\text{ren})}$ and γ by at most 0.5% and leaves G, a_0, and κ/γ exactly unchanged, since $J_{\text{eff}}^{(\text{ren})}$ cancels in the source-to-stiffness ratio (Appendix C.5).

$$c_{\text{loop}}^{(\text{ren})} \equiv \frac{J_{\text{eff}}^{(\text{ren})}}{J_{\text{eff}}^{\text{tree}}} = \frac{1}{1 + J_{\text{eff}}^{\text{tree}}\Sigma_{\text{ret}}} \approx 0.95426,$$

and

$$J_{\text{eff}}^{(\text{ren})} \approx 0.00633348.$$

This reproduces the shell-target crossing near $J_{\text{bare,cross}} \sim 0.019$ at the 0.05% level.

The loop correction is no longer schematic: the finite renormalization is written as an explicit local self-energy. The remaining audit task is the independent graph-level derivation of the relative diagonal and singlet weights of the same scalar-return operator, not the introduction of any new loop parameter.

9. Continuum Stiffness and SI Normalization

The last UV step reads the lattice weighting as a quantum action rather than a thermal one: the lattice quadratic form is interpreted as a Euclidean action weight,

$$\frac{I_E}{\hbar} = \frac{J_{\text{eff}}^{(\text{ren})}}{2}\sum_{a,i}(Q_a - Q_{a+L_*\hat{n}_i})^2,$$

where the sum runs over one sublattice representative a of each bipartite primitive cell and its four outgoing bonds $\hat{n}_i$, so each undirected nearest-neighbor edge is counted once (the convention of Appendix C.4). The microscopic four-cell is assigned the volume

$$\Delta V_4 = \frac{L_*^4}{c}$$

as a coarse-graining convention: the abstract tetrahedral cell complex has no space-filling regular-tetrahedron Euclidean embedding (Appendix C.5), so cell volumes and face areas enter as normalization conventions of the coarse map, not as geometry supplied by the graph. Up to this point the derivation has determined a dimensionless lattice weighting. The continuum EFT, however, needs a dimensionful coefficient multiplying derivatives of a field in spacetime. The Euclidean-action interpretation upgrades the lattice closure data into a continuum action density with the right units and the right covariant target.

The same tetrahedral identity used in the edge-kernel reduction then yields the continuum coefficient for the occupancy field Q_{occ},

$$\gamma_Q = \frac{4\hbar c}{3L_*^2} J_{\text{eff}}^{(\text{ren})}.$$

Here L_* is the canonical tetrahedral spacing, with one coarse cell carrying volume L_*^3 up to the fixed cell-shape convention, and $J_{\text{eff}}^{(\text{ren})}$ is the loop-dressed edge coupling. The numerical factor $4/3$ is the isotropic projection

$$\sum_i \hat{n}_i \hat{n}_i^T = \frac{4}{3} I_3$$

that turns the tetrahedral edge directions into the continuum gradient tensor.

The field normalization is fixed by horizon capacity:

$$S = \pi Q_{\text{occ}}.$$

Therefore the canonical EFT coefficient in the $\frac{\gamma}{2}(\partial S)^2$ convention is

$$\gamma = \frac{4\hbar c}{3\pi^2 L_*^2} J_{\text{eff}}^{(\text{ren})}.$$

Physically, γ is the continuum stiffness of the entanglement-capacity field. A larger γ makes spatial gradients more costly and suppresses the capacity-deficit response to a given source; a smaller γ allows larger variations of the field. The faithful sector-resolution principle fixes L_* without using G. It is nevertheless useful to define the gravitational scale induced by this length,

$$G_* := \frac{c^3 L_*^2}{\hbar}.$$

Then the stiffness may be written in Einstein-normalized form as

$$\gamma = \frac{4J_{\text{eff}}^{(\text{ren})}}{3\pi^2} \frac{c^4}{G_*}.$$

This is the same algebra as the familiar Planck-cell rewrite, but read in the opposite direction: the substrate cell length induces the gravitational scale rather than being chosen by first inserting the measured value of G. Within the Euclidean-action and cell-volume conventions stated above, the SI-normalized stiffness coefficient is fixed; because the absolute normalization of an isolated scalar functional is conventional, the invariant content of this step is the ratio $\kappa/(\gamma S_\infty)$ that the weak-field matching of Section 11 consumes. In that ratio $J_{\text{eff}}^{(\text{ren})}$ cancels (Appendix C.5), so the loop-dressed coupling carries no content for the Newton normalization; its nontrivial input enters the stiffness itself and the dynamical and galactic sectors.

This completes the micro-to-continuum coefficient chain. The tetrahedral ensemble determines the effective sharing entropy; the edge kernel and loop dressing turn that entropy into a discrete stiffness; and the Euclidean matching turns the discrete stiffness into the continuum coefficient γ of the weak-field EFT.

Closed UV-to-IR chain. The UV coefficient chain can now be summarized as

$$\{\Omega_{\mathrm{tet}},\, K^2,\, \eta_*,\, g_{\mathrm{share,eff}},\, L_*,\, J_{\mathrm{bare}},\, J_{\mathrm{eff}}^{\mathrm{tree}},\, \Sigma_{\mathrm{ret}},\, J_{\mathrm{eff}}^{(\mathrm{ren})},\, \gamma\} \longrightarrow \{\kappa,\, G,\, a_0,\, g_{\mathrm{obs}}(g_{\mathrm{bar}})\}.$$

The first bracket is the micro-to-continuum closure chain; the second collects the weak-field observables it feeds. All later weak-field coefficients come from this chain.

The remaining microscopic question is independent confirmation of the same action-kernel interpretation from fuller inhomogeneous dynamics, not an unresolved normalization constant.

Part III. The Closed Static Branch: Einstein Gravity in the Capacity Variable

10. Einstein Parent Action and the Reduced Capacity Frame

The ordinary longitudinal capacity branch has a covariant parent action, and it is simply the ordinary metric action — no independently varied capacity scalar is added to it:

$$I_0[g,\psi] = \frac{c^3}{16\pi G}\int_{\mathcal{M}} d^4x\,\sqrt{-g}\,(R-2\Lambda) + I_{\mathrm{GHY}}[g] + I_{\mathrm{matter}}[g,\psi],$$

where covariant coordinates use $x^0 = ct$. When the time integral is instead written in seconds, $dx^0 = c\,dt$ and the ADM prefactor is correspondingly $c^4/(16\pi G)$. Matter is coupled once, to one physical metric. The capacity functional is the static scalar-constraint reduction of I_0, expressed in a different field coordinate. This statement can be proved without appealing to the final Poisson equation.

Static scalar reduction of Einstein–Hilbert gravity. Use Newtonian gauge,

$$ds^2 = -\left(1+\frac{2\Phi}{c^2}\right)(dx^0)^2 + \left(1-\frac{2\Psi}{c^2}\right)\delta_{ij}dx^i dx^j, \qquad x^0 = ct,$$

and retain the static scalar sector through quadratic order. In ADM variables [8] the shift and extrinsic curvature vanish, so the Einstein–Hilbert plus Gibbons–Hawking–York action is

$$I_{\mathrm{ADM}}^{\mathrm{static}} = \frac{c^4}{16\pi G}\int dt\,d^3x\,N\sqrt{h}\,{}^{(3)}R + I_{\mathrm{matter}}^{\mathrm{static}} + I_\infty.$$

Expanding $N = 1+\Phi/c^2$ and $h_{ij} = (1-2\Psi/c^2)\delta_{ij}$, cancelling the reference boundary term at infinity, and using $I_{\mathrm{matter}}^{(1)} = -\int dt\,d^3x\,\rho\Phi$, gives

$$I_{0,\mathrm{scal}}^{(2)}[\Phi,\Psi] = \int dt\,d^3x\left[\frac{1}{8\pi G}\left((\nabla\Psi)^2 - 2\nabla\Phi\cdot\nabla\Psi\right) - \rho\Phi\right].$$

The lapse perturbation remains a constraint variable. Its variation and the spatial-scalar variation give, respectively,

$$\nabla^2\Psi = 4\pi G\rho, \qquad \nabla^2(\Phi-\Psi) = 0.$$

Asymptotic flatness removes the harmonic difference, so $\Phi = \Psi$. Eliminating Ψ therefore produces

$$\boxed{I_{\mathrm{Newton}}[\Phi] = \int dt\,d^3x\left[-\frac{(\nabla\Phi)^2}{8\pi G} - \rho\Phi\right].}$$

Thus no independent scalar stress tensor is needed to create the linear potential, and the equality $\Phi = \Psi$ in the baseline branch is a metric constraint equation rather than an anisotropic-stress assumption. Appendix N gives the expansion, boundary bookkeeping, and degree-of-freedom audit in full.

Exact reduced-action identity. On the renormalized static branch, write $S_{\rm ent} = S_\infty - \delta S$. Source-independent extensive terms are removed by the vacuum normalization proved for the regulated joint measure in Section 25. The capacity functional is then

$$I_{\rm cap}^{\rm static}[\delta S;\rho] = \int dt\, d^3x \left[-\frac{\gamma}{2}(\nabla\delta S)^2 + \kappa\rho\,\delta S\right].$$

The field redefinition

$$\delta S = -\frac{2S_\infty}{c^2}\Phi$$

turns it into

$$I_{\rm cap}^{\rm static} = \int dt\, d^3x \left[-\frac{2\gamma S_\infty^2}{c^4}(\nabla\Phi)^2 - \frac{2\kappa S_\infty}{c^2}\rho\Phi\right].$$

Using

$$G = \frac{c^2\kappa}{8\pi\gamma S_\infty}$$

gives the action-level equality

$$\boxed{I_{\rm cap}^{\rm static} = Z_S I_{\rm Newton}, \qquad Z_S \equiv \frac{2\kappa S_\infty}{c^2}.}$$

The field-independent factor Z_S cannot affect the classical reduced equations. It can matter when the UV construction is asked to normalize fluctuations or correlation functions, but it does not represent a second determination of G and it does not license adding $I_{\rm cap}$ to I_0. The equality assumes the same asymptotically flat or Dirichlet boundary data on both sides. At a finite boundary the Newton surface term and its image under $\delta S = -2S_\infty\Phi/c^2$ must be included as well.

Scope of the background-covariant notation. For transport calculations the same reduced equation is packaged as

$$I_{\rm cap}[S_{\rm ent};\chi\,|\,g_{\rm ref}] = \int d^4x\,\sqrt{-g_{\rm ref}}\left[-\frac{\gamma}{2}g_{\rm ref}^{\mu\nu}\partial_\mu S_{\rm ent}\partial_\nu S_{\rm ent} - \lambda S_{\rm ent} - \kappa\chi S_{\rm ent}\right].$$

The vertical bar is essential: $g_{\rm ref}$ and the reduced source projection χ are held fixed while $S_{\rm ent}$ is varied. This notation is useful for extending the reduced response in time, but it is not a covariant scalar–tensor parent action. In the static nonrelativistic sector $\chi \simeq \rho$; covariantly the source is the full stress tensor through $I_{\rm matter}[g,\psi]$.

Capacity coefficients and the source theorem. The UV calculation still fixes how the geometric constraint is coordinatized by the substrate variable. With

$$\sigma_{\rm def} = \frac{\rho}{\kappa_m(L_*)},$$

the Green-matched projection is

$$\nabla^2\delta S = -\frac{3L_*}{4G_{\rm tet}(0)}\,\sigma_{\rm def}, \qquad \frac{\kappa}{\gamma} = \frac{3L_*}{4G_{\rm tet}(0)\kappa_m(L_*)}.$$

This is the microscopic map between defect density and the capacity coordinate on the Einstein constraint surface. It is not an extra matter coupling in the covariant parent theory.

The length backbone of Newton's constant. The weak-field normalization is

$$G = \frac{c^2\kappa}{8\pi\gamma S_\infty}.$$

Faithful full-support resolution and the positive marked-transfer spectrum fix

$$L_* = -\frac{3}{2}\, Z_e\lambda_e \ln\!\left(1 - e^{-7g_{\mathrm{share,eff}}}\right), \qquad \lambda_e = \frac{\hbar}{m_e c},$$

and therefore induces

$$G_* = \frac{c^3 L_*^2}{\hbar} = \frac{9}{4}\,\frac{\hbar c}{m_e^2}\, Z_e^2 \ln^2\!\left(1 - e^{-7g_{\mathrm{share,eff}}}\right).$$

Substituting the source-map identities

$$\frac{\kappa}{\gamma} = \frac{3L_*}{4G_{\mathrm{tet}}(0)\kappa_m(L_*)}, \qquad \kappa_m(L_*) = \frac{\hbar}{cL_*\ln 2}, \qquad S_\infty^{\mathrm{cell}} = \frac{3\ln 2}{32\pi G_{\mathrm{tet}}(0)},$$

into the weak-field expression gives identically

$$G = \frac{c^3 L_*^2}{\hbar} = G_*.$$

Thus there is one scale-setting route to G: electron recurrence fixes L_*, and L_* fixes the gravitational scale. The stiffness, source coefficient, and capacity normalization are a consistent static-EFT representation of that same length backbone, not a second determination that could have disagreed with it. The only numerical comparison in this sector is G_* against the measured Newton constant.

The decorated scale gives $G_* = 6.6742890772 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$, or -0.073σ relative to CODATA. Because both the early entropy construction and the later residual-closing vertex were developed with the discrepancy known, Appendix L treats this as a high-precision postdiction. The nontrivial content is the shared action that also fixes the two charged-lepton corrections.

11. Capacity Variable, Bridge Law, and Variational Status

Varying the reduced capacity-frame functional with respect to S_{ent} gives

$$\gamma\Box S_{\mathrm{ent}} = \lambda + \kappa\chi.$$

On the renormalized static, nonrelativistic branch this becomes

$$\nabla^2\delta S = -\frac{\kappa}{\gamma}\rho.$$

Define the surviving fractional capacity

$$q(x) \equiv \frac{S_{\mathrm{ent}}(x)}{S_\infty} = 1 - \frac{\delta S(x)}{S_\infty}.$$

The action reduction above fixes the weak-field bridge directly,

$$\frac{\Phi}{c^2} = -\frac{\delta S}{2S_\infty}.$$

Equivalently,

$$q = 1 + \frac{2\Phi}{c^2} + O(c^{-4}).$$

The bounded nonlinear rule

$$N^2 = q$$

is the continuous multiplicative completion selected by the capacity-composition rule. Its status must now be stated more precisely. In a static spherical exterior, q is the invariant geometric scalar

$$q = h^{ab}\partial_a R \partial_b R = 1 - \frac{2GM_{\rm MS}}{c^2 R},$$

and in Schwarzschild coordinates it equals N^2. In a generic spacetime the lapse is foliation dependent, so $N^2 = q$ by itself is not a covariant constraint. The metric-only parent therefore treats the weak-field δS and the spherical q as reduced or composite geometric variables; it does not promote either to an unconstrained second gravitational field. Appendix N.6 formulates the remaining target as a diffeomorphism-invariant, potentially quasilocal and state-dependent functional, rather than presuming that an additional fundamental scalar is needed.

Combining the static source equation with the weak-field bridge gives

$$G = \frac{c^2\kappa}{8\pi\gamma S_\infty}.$$

This relation uses only the invariant combination $\kappa/(\gamma S_\infty)$: a fixed-epoch rescaling of the entropy units changes S_∞ and κ together and leaves the observable potential unchanged.

Two coordinates on one response. The metric parent solves the Hamiltonian and spatial constraints for Φ and Ψ; the capacity frame uses δS as a field coordinate on that reduced solution. Around a constant background, a canonical scalar stress would begin as $(\partial\delta S)^2 = O(\rho^2)$ and could not be the source of the observed $O(\rho)$ potential. The action identity removes that mismatch: the linear capacity response is the reduced metric constraint itself.

Matter enters once. For the nonrelativistic static branch, the microscopic source theorem reduces the full metric source to the defect density ρ. Covariantly, matter enters only through $I_{\rm matter}[g,\psi]$, so the full stress tensor gravitates, including trace-free radiation, and the Bianchi identity enforces the usual conservation law. The notation $\chi \simeq \rho$ belongs only to the reduced nonrelativistic source map; an explicit universal term $S_{\rm ent}T^\mu{}_\mu$ is neither required nor adopted.

Parent-action decision. The preferred minimal construction for the ordinary branch is therefore metric-only:

$$I^{\rm long}_{\rm parent} = I_0[g,\psi], \qquad \delta S = \delta S[g,\psi] \quad \text{after constraint reduction.}$$

It propagates the two tensor polarizations of general relativity and no extra scalar. Constrained-clock and scalar–tensor alternatives remain useful control cases, but both add structure and generically add a mode; Appendix N records why neither is selected. The transverse thermal sector responsible for the galactic excess is not included in this closure and requires its own metric influence functional.

12. Newtonian Gravity and the Point-Source Limit

In the renormalized static weak-field sector the scalar equation reduces to

$$\nabla^2 \delta S = -\frac{\kappa}{\gamma}\rho.$$

After the background is renormalized away and the source is taken to be nonrelativistic, the deficit field obeys an ordinary Poisson equation. Its mathematical structure is the one used in standard weak-field gravity, with δS as the field variable.

For a point source M,

$$\delta S(r) = \frac{\kappa M}{4\pi\gamma r}.$$

Using the bridge law,

$$\frac{\Phi}{c^2} = -\frac{\delta S}{2S_\infty},$$

the gravitational acceleration becomes

$$g(r) = \frac{c^2\kappa}{8\pi\gamma S_\infty}\frac{M}{r^2} = \frac{GM}{r^2}.$$

Thus Newtonian gravity is recovered as the weak-field response of the entanglement-capacity medium: the sourced scalar equation and the bridge law together imply the familiar point-mass force law, with nothing further assumed.

Interpretation. A point defect produces a $1/r$ capacity deficit, and the bridge maps its gradient to the Newtonian inverse-square force. Ordinary gravity is the small-deficit, weak-curvature limit of the extended capacity strain around localized defects. Section 24 exhibits the same structure on the discrete substrate: cell-by-cell re-equilibration screens at sub-cell range, whereas a conserved capacity current with maintenance sinks obeys the massless graph-Poisson equation and produces the $1/r$ deficit. The Newtonian form therefore diagnoses the conservation law behind it.

13. Electron Anchor: One-Bit Mass Scale and Seven-Sector Length Scale

The electron supplies the elementary mass anchor $m_e/\ln 2$. Its Compton scale also assigns the length associated with the seven-sector effective support. The decorated marked-transfer action closes the finite correction to that scale and routes the same correction through the heavier charged-lepton shells. These are two readings of one dimensionful datum, not independent measurements, so the weak-field normalization remains a single calibration.

13.1 Why the electron is the anchor

The mass–entropy map needs an elementary anchor because the elementary matter sector is the localized defect sector. The electron is the lightest simple charged fermionic defect, and its mass is not obscured by hadronic or QCD dressing. A single fermionic face-exclusion defect carries the canonical increment

$$\Delta S_f = \ln 2,$$

one bit of missing entanglement, because an excluded face is a binary occupied/unoccupied defect of the local network.

13.2 One-bit mass anchor

At the electron Compton scale $\ell = \lambda_e$ the mass–entropy map reads

$$\kappa_m(\lambda_e) = \frac{m_e}{\ln 2}.$$

Dividing the electron mass by the fixed one-bit increment fixes the mass-per-entropy conversion at the electron's own scale. Run back to the cutoff cell, the conversion is

$$\kappa_{m,\mathrm{UV}} = \frac{\hbar}{cL_*}\frac{1}{\ln 2},$$

with canonical running law

$$\kappa_m(\ell) = \kappa_{m,\mathrm{UV}}\left(\frac{L_*}{\ell}\right)^{1+\alpha_{\mathrm{cl}}}, \qquad \alpha_{\mathrm{cl}} = 0$$

in the closed branch. One bit fixes the electron-scale conversion; the running law carries it to the UV scale; the same conversion then feeds the weak-field source map. This is the only point at which the mass anchor enters gravity.

13.3 Seven-sector length anchor

The same electron anchors the cell length through the marked support-to-rate map. Faithful full-support resolution uniquely selects the memoryless kernel. Within the stated recurrence mass functional, fermionic exclusion and electron lightness jointly select the maximum channel count $k = 7$ and vanishing inter-channel correlation $\Delta_7 = 0$. On the commutative renewed-state algebra, the state-weighted determinant of the likelihood multiplication operator is

$$\Delta_{\tau_p}(\mathsf{R}) = \exp[\tau_p(\ln \mathsf{R})] = e^{-g_{\mathrm{share,eff}}}.$$

Multiplicativity across the seven factorized clouds gives

$$r = \Delta_{\tau_p^{\otimes 7}}(\mathsf{R}^{\otimes 7}) = e^{-7g_{\mathrm{share,eff}}}.$$

This is the record-conditioned geometric transfer rate, not the collision probability of two independently sampled blocks. The positive survival spectrum gives the baseline scale

$$L_*^{(0)} = -\frac{3}{2}\lambda_e \ln(1-r),$$

with 3/2 the transverse export factor of Appendix C.5.

Appendix H realizes this determinant transfer in a finite charged action. The construction separates the baseline recurrence from the defect-bound closure response, so the one-channel mixing projector, seven-channel survival operator, and marked-fiber determinant remain distinct.

This does not mean an electron is a single tetrahedral cell carrying seven simultaneous labels. The local ensemble supplies seven distinguishable dressing layers. Each layer has fermionic occupation at most one, and the recurrence mass minimum occupies all seven once. The electron is the lightest coherent one-bit defect on that selected branch and exports the transverse share of the resulting support. The Compton scale calibrates the substrate length hierarchy; the one-bit mass calibrates the source map. The two uses impose a nontrivial joint requirement on the electron's role in the gravitational normalization without duplicating a single input.

13.4 Decorated marked-transfer vertex

The original Newton normalization was low by about 1.05%. Because $G_* \propto L_*^2$, the missing amplitude in the substrate length was 0.5309%. The baseline muon ratio required a 0.5124% uplift. The tau ratio required 0.6562%, which separates into the same universal uplift and a smaller 0.1431% second-shell factor. Their common sign and scale motivated one small charged response with shell-dependent routing. Appendix L records that this was an action-level postdiction.

That response also had to be additive. The tetrahedral ensemble already fixed the entropy, closure spectrum, edge projection, source map, and the micro-to-macro coefficient chain. Changing its label count, admissibility weight, or transverse export to repair the residuals would move results that did not share the discrepancy. The admissible repair was consequently required to vanish in the unmarked vacuum, reuse the established closure incidence, and act only on the charged transfer graph. This criterion motivates the marked vertex below; it does not determine the answer numerically. The field content, determinant power, and routing still have to follow from the displayed action and survive the alternative-kernel audits.

The renewed closure amplitude has the exact Gaussian representation

$$e^{-\eta_* C^2/2} = \int \frac{d^3\xi}{(2\pi)^{3/2}} \exp\left[-\frac{1}{2}\xi^2 + i\sqrt{\eta_*}\,\xi_a C_a\right].$$

The amplitude-level closure incidence is therefore $\sqrt{\eta_*}$. For each occupied unordered channel pair $e = (m, m')$, the two directed scalar returns have row operator

$$R_e = \frac{2}{7}\big(\langle m \to m'| + \langle m' \to m|\big).$$

The contraction $B_e = \sqrt{\eta_*} R_e$ obeys

$$B_e B_e^\dagger = 2\eta_* \left(\frac{2}{7}\right)^2 = \frac{8\eta_*}{49} \equiv u.$$

Its canonical unitary dilation has no-event amplitude $\sqrt{1-u}$. The seven-channel lightest branch activates all $\binom{7}{2} = 21$ pair records, giving

$$Z_{\text{edge}} = \left(1 - \frac{8\eta_*}{49}\right)^{21/2}.$$

The present and history Gaussian strands each have three normalized first excitations. Their ordered products form nine orthonormal marked states, so the internal trace of one hard-core marked fiber gives

$$\boxed{\zeta_* = 9e^{-g_{\text{share,eff}}} \left(1 - \frac{8\eta_*}{49}\right)^{21/2}} = 0.005123584484947.$$

This internal trace computes a scalar vertex weight at fixed path branch; it is not a measurement that discards the defect's position label. Free transport applies the same internal contraction on every position branch, as required by Appendix H.11. The finite transfer graph has a universal marked alternative, one two-vertex label return, and one second-shell singlet passage. Its exact factors are

$$Z_\mu = 1 + \zeta_*, \qquad Z_e = (1+\zeta_*)(1+7\zeta_*^2), \qquad Z_{\tau,2} = 1 + \frac{2}{7}\zeta_*.$$

The dressed scale is

$$\boxed{L_* = Z_e L_*^{(0)}}.$$

Appendix H writes the controlled vertex, decomposes its complete edge Hessian, and enumerates all 56,800 states of the one-step routing block.

The anchor is a gauge choice. Because counting fixes only dimensionless quantities, exactly one dimensionful measurement must be supplied, and which one is a convention. The physical content of the framework is carried by anchor-invariant statements: the lepton ratios m_μ/m_e and m_τ/m_e; the hierarchy

$$\frac{m_e}{m_P} = -\frac{3}{2} Z_e \ln(1 - e^{-7g_{\text{share,eff}}});$$

the horizon normalization identity 1/4 within the stated cell convention; the abundance ratio Ω_c/Ω_b; and, within its branch assignment, a_0/cH_0. The electron remains the anchor because its mass is measured most precisely. Appendix L records that the high-precision marked correction was constructed after the residuals were known, so anchor invariance does not turn the agreement into a blind prediction.

13.5 Consistency checks

The marked action uses no continuously fitted coefficient. Substitution gives

$$G_* = 6.6742890772 \times 10^{-11}\ \text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2} \qquad (-0.073\sigma),$$

$$\frac{m_\mu}{m_e} = 720\frac{2}{7} Z_\mu = 206.768280237 \qquad (-0.535\sigma),$$

$$\frac{m_\tau}{m_e} = 720^2 \left(\frac{2}{7}\right)^4 Z_\mu Z_{\tau,2} = 3477.343310 \qquad (+0.481\sigma).$$

These three comparisons probe one marked-fiber weight through different graph polynomials. The scalar stiffness, source map, and weak-field bridge reproduce the same dressed G_* after substitution, so they remain one consistency chain rather than an independent determination.

Routing-integer audit. The displayed action makes the coefficient of ζ_*^2 in Z_e equal to the number of persistent charged labels: the second marked return sums over the alphabet, so the factor is $1 + 7\zeta_*^2$ (Appendix H.9). Holding ζ_* fixed and temporarily treating that integer as unknown, each unit step in n moves the induced G_* by 2.34 CODATA standard deviations: $n = 6$ gives -2.41σ, $n = 7$ gives -0.073σ, and $n = 8$ gives $+2.26\sigma$. Within $-200 \le n \le 200$, seven is the only integer within two standard deviations. Dropping the second-order term ($n = 0$) gives -16.4σ. Appendix B now supplies two readings of this result. Conditional on the marked-fusion incidence identification, the three-dimensional closure vector fixes the same seven internally and the Newton comparison checks its routing. Without that identification, the comparison selects the seven-state member of the stated discrete family. Neither reading is historically blind: the target and the residual were already known, as Appendix L records.

Two remarks keep this honest. The resolvability is partly fortunate: $\zeta_*^2 \simeq 2.6 \times 10^{-5}$ places the rung spacing just above the CODATA uncertainty, so the integer is measurable at all; a smaller ζ_* would bury the ladder inside the error bar, and a larger one would leave $n = 7$ one near-miss among many. And since the alphabet is symmetric about zero, one might expect the neutral label $m = 0$ to drop out of a return sum, giving coefficient six; $n = 6$ lies outside the band, so the neutral label demonstrably participates in the second marked return. At the tau the analogous test is weaker: among the forty-three simple rationals with denominator up to eleven, both the derived 2/7 ($+0.49\sigma$) and 3/11 (-0.49σ) survive, so the second-shell routing is supported rather than uniquely resolved.

The same comparison has a dimensionless form. Squaring the hierarchy gives the electron's gravitational coupling,

$$\frac{G_* m_e^2}{\hbar c} = \frac{9}{4}\, Z_e^2 \ln^2\!\left(1 - e^{-7g_{\text{share,eff}}}\right),$$

with 9/4 the square of the transverse export factor and Z_e^2 the marked-transfer dressing. The relation also has an area reading. A horizon stores one bit in area $4 \ln 2\, L_P^2$, while the electron spreads its single bit over the Compton area λ_e^2. The marked factor changes the finite packing correction without altering the dominant hierarchy $e^{-14 g_{\mathrm{share,eff}}}$.

13.6 Composite sectors

For composite hadrons the claim is weaker and different in kind. The relevant quantity is the dressed, vacuum-subtracted bound-state entropy,

$$m_{\mathrm{hadron}} = \kappa_m(\ell_H)\, S_{\mathrm{ent},H}^{\mathrm{dressed}},$$

with the dressed budget generated by confinement, gluonic structure, trace-anomaly dynamics, and chiral vacuum reorganization. A lattice derivation of that dressed entropy is not yet available. The present claim is limited to structural compatibility between the mass–entropy map and the standard QCD mass budget. The elementary-fermion anchor is settled in the simple sectors; the hadronic coefficients remain open.

14. Baseline Metric Closure: No Slip and PPN

For the branch that reproduces Newtonian gravity, the physical metric is already fixed by the Einstein parent action. The scalar constraint reduction of Section 10 gives

$$\nabla^2(\Phi - \Psi) = 0,$$

so asymptotic flatness implies

$$\Phi = \Psi.$$

This is an action-level result. It is not inferred from the canonical stress tensor of δS, because no independently gravitating capacity scalar is present. Since the parent action of this branch is exactly Einstein–Hilbert plus minimally coupled matter, its vacuum post-Newtonian solution has

$$\gamma_{\mathrm{PPN}} = \beta_{\mathrm{PPN}} = 1$$

and the remaining standard PPN parameters vanish, subject to the usual assumptions on the matter sector and boundary conditions. The capacity redefinition does not alter those values. Here *closed* refers only to the baseline contribution. For the full theory one should write schematically

$$\gamma_{\mathrm{PPN}}^{\mathrm{obs}} = 1 + \delta\gamma_\perp, \qquad \beta_{\mathrm{PPN}}^{\mathrm{obs}} = 1 + \delta\beta_\perp.$$

The observed PPN parameters are closed only after the transverse calculation shows that these corrections vanish, are screened, or lie below Solar-System bounds.

The metric response of the conditional galactic branch is a separate question. It is taken up with that branch in Section 16.

Part IV. The Conditional Galactic Branch

15. Galactic Dynamics

The static Einstein branch supplies the baryonic acceleration g_{bar}. The proposed galactic extension adds the equilibrium response of a transverse substrate sector. The calculation below separates the algebra that is fixed inside the effective model from the microscopic statements still awaiting a specified GFT interaction and metric vertex. In particular, the exponential response follows once the transverse gap and normalization are supplied; the present paper does not claim that those inputs have already been derived from a standard simplicial vertex.

Vacuum-state origin of the Bose–Einstein occupancy. For a bosonic mode in a de Sitter static patch, the restricted vacuum state is thermal at

$$k_B T_H = \frac{\hbar c}{2\pi R_A},$$

and its occupation is $n_B(x) = 1/(e^x - 1)$ for the dimensionless thermal argument $x \equiv E/(k_B T_H)$. For the present apparent horizon, $R_A = c/H_0$ in the branch used here, so

$$a_H = cH_0.$$

The transverse doublet supplies a sharper origin for the phase measure than dimension counting alone. For each stable normal mode the two real components Y_1, Y_2 and their conjugate momenta form two canonical oscillator pairs. Passing to action–angle variables (J_i, θ_i) gives

$$\Omega_{\text{symp}} = dJ_1 \wedge d\theta_1 + dJ_2 \wedge d\theta_2, \qquad \theta_i \sim \theta_i + 2\pi.$$

Canonical quantization spaces neighboring actions by $\Delta J_i = \hbar$, so one two-oscillator quantum cell has invariant phase-space volume $(2\pi\hbar)^2$. Restricting to one quantum action cell leaves a dimensionless angular Haar volume $(2\pi)^2$. The factor is therefore fixed by the symplectic two-oscillator structure once that mode and its one-action-cell loading are specified. A real transverse two-vector viewed only as a configuration-space direction would supply one polar angle and would not prove the result.

The proposed transverse normalization assigns one sharing entropy $g_{\text{share,eff}}$ to that one-action-cell angular torus. Its entropy per unit invariant phase volume is

$$\epsilon_\perp = \frac{g_{\text{share,eff}}}{(2\pi)^2}.$$

Equating the reversible horizon work $k_B T_H \epsilon_\perp$ with the Unruh energy $\hbar a_0/(2\pi c)$ [43] gives

$$a_0 = \frac{g_{\text{share,eff}}}{4\pi^2}\, a_H = \frac{cH_0 g_{\text{share,eff}}}{4\pi^2}.$$

The algebra is exact after the loading assignment. The compact angular measure follows from the canonical transverse doublet, while two physical premises remain: the infrared state must load one sharing entropy into one quantum action cell, and that loading must couple reversibly to the horizon state. The 2π factors in the Unruh and Gibbons–Hawking temperatures cancel in their dimensionless ratio, so they do not independently generate the cell normalization. With the Planck value of H_0, the branch gives $a_0 = 1.231 \times 10^{-10}\,\mathrm{m\,s^{-2}}$, compared with the RAR scale $1.20 \times 10^{-10}\,\mathrm{m\,s^{-2}}$, whose quoted uncertainty is ± 0.02 random and ± 0.24 systematic [1, 46]. The displacement is $+2.6\%$: 1.5σ against the random error alone, and about 0.1σ against the full budget, whose systematic part is dominated by the stellar mass-to-light normalization. If the loading and horizon coupling hold, the model also predicts $a_0(z) \propto H(z)$.

$1+2$ **channel decomposition and the radial-acceleration law.** The baryonic gradient selects a local longitudinal direction. The remaining two components form a transverse doublet $Y = (Y_1, Y_2)$, and its source-visible scalar is the radial occupation

$$R^2 = Y_1^2 + Y_2^2.$$

After the refresh-gapped relative modes are integrated out, the minimal one-invariant truncation describes longitudinal commitment X and transverse occupation R^2 through one local capacity mismatch,

$$\Delta C(X, R) = \alpha_C + g_C X + \beta_C R^2.$$

Finite susceptibility gives an effective potential $V = F(\Delta C)$ with $F'(0) = 0$ and $u_C \equiv F''(0) > 0$. Its leading expansion is $V = u_C(\Delta C)^2/2 + O(\Delta C^3)$. This form follows from a Gaussian large-deviation expansion once the low-energy theory has only one total-capacity variable. The existence of that one-invariant infrared truncation is the physical premise; finite capacity alone would not exclude additional light invariants.

Let (X_0, R_0) lie on the stationary surface $\Delta C = 0$. The quadratic Hessian in $(\delta X, \delta R)$ is

$$H = u_C \begin{pmatrix} g_C^2 & 2g_C\beta_C R_0 \\ 2g_C\beta_C R_0 & 4\beta_C^2 R_0^2 \end{pmatrix} = u_C\, \mathbf{c}\mathbf{c}^T, \qquad \mathbf{c} = (g_C, 2\beta_C R_0)^T.$$

Writing its entries as A_L, A_T, and $C_\times$, and choosing the relative field orientation so that $C_\times \geq 0$, gives

$$C_\times^2 = A_L A_T, \qquad \det H = 0.$$

The geometric mean is therefore the cross coefficient of a rank-one capacity Hessian. It is not a generic normal-mode frequency of two coupled oscillators. The zero eigenvector is tangent to $\Delta C = 0$ and transfers occupancy between longitudinal and transverse sectors without changing the total. Positive gradient terms make that redistribution mode dispersive at nonzero wave number.

Two further consequences follow exactly from the same quadratic form. Writing

$$\delta^2 V = \frac{1}{2}\left(A_L \delta X^2 + 2C_\times \delta X \delta R + A_T \delta R^2\right),$$

the transverse coordinate that minimizes the energy at fixed δX is

$$\delta R_*(\delta X) = -\frac{C_\times}{A_T}\,\delta X.$$

Thus $C_\times/A_T$ is the exact static response coefficient in the one-invariant model. With the common kinetic normalization $Z_L = Z_T = Z_0$ derived below, the clamped curvatures define $\omega_L^2 = A_L/Z_0$ and $\omega_T^2 = A_T/Z_0$, and rank one gives

$$\frac{C_\times}{A_T} = \sqrt{\frac{A_L}{A_T}} = \frac{\omega_L}{\omega_T}.$$

These are clamped frequencies, not the eigenfrequencies of the freely relaxing rank-one system; the adiabatic tangent mode remains gapless at zero wave number.

At leading two-derivative order, tetrahedral symmetry gives the three-component capacity representation one kinetic coefficient. Appendix N derives $Z_L = Z_T$ from the decomposition $4 = 1 \oplus 3$. Equal kinetic normalization removes an otherwise arbitrary relative field rescaling. The further effective matching

$$A_L = \frac{g_{\text{bar}}}{cH_0}, \qquad A_T = \frac{a_0}{cH_0}$$

then yields the dimensionless curvature ratio

$$x = \frac{C_\times}{A_T} = \sqrt{\frac{A_L}{A_T}} = \sqrt{\frac{g_{\text{bar}}}{a_0}}.$$

This matching identifies the two canonical inverse susceptibilities with their Unruh-to-horizon energy ratios. The response lemma above reduces the remaining thermal identification to one anchoring condition if the source-driven energy is read from the clamped longitudinal curvature: with $E_\perp \equiv \hbar\omega_L$,

$$\frac{E_\perp}{k_B T_H} = \frac{\omega_L}{\omega_T} \quad \Longleftrightarrow \quad \hbar\omega_T = k_B T_H.$$

The curvature ratio is therefore not an additional free equality on top of the clamped-oscillator reading. The physical question is whether the transverse retarded response is governed by these clamped curvatures and whether its transverse anchor sits at the thermal point. The rank-one Hessian alone does not answer that question: its freely relaxing tangent mode is gapless and its orthogonal eigenvalue is proportional to $A_L + A_T$. The full condensate influence functional must distinguish the clamped response from the adiabatic poles.

The resulting radial-acceleration law is

$$g_{\text{obs}} = g_{\text{bar}}\big(1 + n_B(x)\big) = \frac{g_{\text{bar}}}{1 - \exp\Big(-\sqrt{g_{\text{bar}}/a_0}\Big)},$$

with the asymptotic limits

$$g_{\text{bar}} \gg a_0 \Longrightarrow g_{\text{obs}} \approx g_{\text{bar}}, \tag{10}$$

$$g_{\text{bar}} \ll a_0 \Longrightarrow g_{\text{obs}} \approx \sqrt{a_0 g_{\text{bar}}}. \tag{11}$$

The two endpoint limits follow from the occupancy factor. The exact interpolation is conditional on the effective transverse matching and on the absence of an additional form factor in the microscopic influence kernel. The one-invariant truncation removes extra light field-space directions at zero derivative, but it does not remove the momentum dependence of a propagating field or the spectral density generated by gradient terms and continuum modes. It therefore cannot prove $F(\omega, \mathbf{k}) \equiv 1$ by itself. Empirically, any residual static form factor is bounded below the few-percent level across the five decades of x probed jointly by solar-system precision and the measured flatness of the galactic rotation and lensing relations. The deep-MOND branch gives the baryonic Tully–Fisher law [44, 45]

$$v^4 \approx a_0 G M_b.$$

Division of labor between the action and the state. The ordinary Einstein response supplies the baseline multiplier 1. For a bosonic transverse mode with source-dependent energy $E_\perp$ and $x = E_\perp/(k_B T_H)$, the equilibrium thermal free energy is

$$F_{\text{th}}(E_\perp) = k_B T_H \ln\big(1 - e^{-x}\big)\,.$$

Its derivative is

$$\frac{\partial F_{\text{th}}}{\partial E_\perp} = \frac{1}{e^x - 1} = n_B(x).$$

Forward and reverse thermal processes are already contained in this partition function. No blocked-absorption rule or thermodynamic-arrow argument is needed. If the source-gap coupling converts this derivative into the transverse acceleration response without an additional form factor, the total multiplier is

$$\nu(x) = 1 + n_B(x) = \frac{1}{1 - e^{-x}}, \qquad g_{\text{obs}} = g_{\text{bar}}\,\nu(x).$$

With $x = \sqrt{g_{\text{bar}}/a_0}$ this gives the law above.

The same relation admits a conservative static response potential. Define $W'(g) = \nu(\sqrt{g/a_0})$ and set $x = \sqrt{g/a_0}$. Since $dg = 2a_0 x\, dx$,

$$W(g) = a_0\big[x^2 + 2x\ln(1 - e^{-x}) - 2\,\mathrm{Li}_2(e^{-x})\big] + \text{const},$$

and direct differentiation returns $W'(g) = 1/(1 - e^{-x})$. The existence of this potential shows that the exponential RAR is compatible with equilibrium statistical mechanics. It does not determine the relativistic metric response.

Closure status of the galactic branch. The statements in this branch have different grades. The rank-one identity $C_\times^2 = A_L A_T$, the static response $\delta R_*/\delta X = -C_\times/A_T$, and the clamped-frequency ratio $C_\times/A_T = \omega_L/\omega_T$ follow exactly from a one-invariant capacity potential with equal kinetic normalization. Tetrahedral symmetry fixes $Z_L = Z_T$ at leading two-derivative order. The assignments $A_L = g_{\rm bar}/(cH_0)$ and $A_T = a_0/(cH_0)$ remain conditional and require the horizon state to keep the combination $4u_C\beta_C^2R_0^2$ environment-independent. Once the clamped-oscillator reading is adopted, thermal matching reduces to the single anchor $\hbar\omega_T = k_BT_H$; the influence functional must still show that the physical response uses the clamped rather than adiabatic spectrum, a choice most consequential in the deep-MOND regime where $\omega_L \ll \omega_T$. Canonical quantization fixes the $(2\pi)^2$ angular measure of the two-oscillator cell, while its one-entropy loading and reversible horizon coupling remain conditional. Finally, $g_{\rm obs} = g_{\rm bar}[1 + n_B(x)]$ requires the influence functional to introduce no additional source-dependent vertex or spectral factor. Until those dynamical tests are passed, the RAR is an exact consequence of a specified leading effective completion rather than a closed consequence of the UV ensemble.

16. The Open Galactic Lensing Kernel

The low-acceleration excess is a different action problem. The rate calculation fixes

$$g_{\rm obs} = \frac{g_{\rm bar}}{1 - \exp[-\sqrt{g_{\rm bar}/a_0}]},$$

but a radial force law does not determine the spatial metric. To predict photon deflection one must integrate out the transverse substrate modes in their physical horizon-thermal state and derive the metric response. The appropriate object may be a closed-time-path influence functional,

$$\Gamma_{\rm eff}[g_+, g_-; \psi_+, \psi_-] = I_0[g_+, \psi_+] - I_0[g_-, \psi_-] + \Gamma_\perp[g_+, g_-; \rho_\perp],$$

rather than a local equilibrium action. Its retarded metric kernel must generate the RAR in the static matter channel, its spatial components must determine Ψ, and its Ward identity must enforce covariant conservation. Appendix N specifies the minimal Hessian and correlator needed to decide this.

If that calculation gives equal transverse corrections,

$$\Delta\Phi_\perp = \Delta\Psi_\perp,$$

then the excess may be represented by the effective density

$$\rho_{\rm halo}(r) = \frac{1}{4\pi G r^2}\frac{d}{dr}\Big[r^2(g_{\rm obs} - g_{\rm bar})\Big],$$

and dynamics and lensing share the same kernel. At present this equality is a falsifiable target, not a theorem. It is an observationally viable one: the stacked weak-lensing measurement of Brouwer et al. [42], which assumes GR-like deflection by the modified potential, matches the extrapolated response function two decades below the rotation-curve regime, with the early-type caveat discussed in Section 27.1.

Local tests. The RAR correction is exponentially suppressed when $g_{\rm bar} \gg a_0$, so the closed Einstein branch dominates in the Solar System. This explains why the target PPN limit is the GR one, but it is not a substitute for deriving the transverse kernel and checking that it generates no residual preferred-frame or nonlocal effect. Matter still couples to one physical metric and follows its geodesics; there is no additional test-body scalar charge. Weak-lensing RAR measurements test the desired transverse completion, while the baseline no-slip and PPN statements no longer depend on that unfinished sector.

Part V. Transport, Clusters, and Cosmology

The microstructure-to-static-weak-field chain is the most directly constrained part of the theory. The sectors of this part ask how the same finite-capacity substrate behaves when sources move. Transport asks how the capacity-strain field propagates after a source changes; the cluster sector asks how it couples to matter in distinct dynamical phases; cosmology treats the homogeneous mode; and the saturated phase treats the proposed committed carrier. These sectors share the static weak-field ontology but not its evidential status: the Einstein/capacity action equivalence, ordinary source map, and finite marked-transfer scale are the controlled baseline inside their displayed actions, while the transport, cluster, and cosmological additions carry their stated conditional or open grades.

17. Causal Transport and Telegrapher Dynamics

The static weak-field branch gives the settled capacity-deficit profile and applies to quasi-static galaxies. Clusters and mergers also require its time evolution: propagation from a moving source, lag behind a fast disturbance, saturation, and the response to matter in distinct dynamical phases. A static Poisson law contains none of these effects.

A physical entanglement-capacity medium requires a causal propagation law as well as a settled profile. The time-dependent sector supplies that completion of the static EFT, and the cluster and cosmological sectors use it. This section gives its minimal causal form; Section 18 applies it to clusters and mergers.

The canonical time-dependent completion is written relative to the substrate four-velocity u^μ:

$$\tau_0(u^\mu\nabla_\mu)^2\delta S + u^\mu\nabla_\mu\delta S = Dh^{\mu\nu}\nabla_\mu\nabla_\nu\delta S + A\chi, \qquad h^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu.$$

The vector u^μ is the local rest frame of the entanglement-capacity medium, not an additional ad hoc force carrier. In that frame, $u^\mu = (1,0,0,0)$, the equation reduces to the familiar telegrapher form

$$\tau_0\partial_t^2\delta S + \partial_t\delta S = D\nabla^2\delta S + A\chi(x,t),$$

with static-matching condition

$$\frac{A}{D} = \frac{\kappa}{\gamma}.$$

A changing source requires propagation and relaxation; the static Poisson equation gives only the settled profile. The telegrapher equation is the minimal causal extension that reduces to the same static branch when time dependence becomes negligible. Here "relaxation" means time-dependent settling of the existing capacity-strain field.

Causality requires

$$\frac{D}{\tau_0} = c^2,$$

so the transport sector propagates disturbances at finite speed. In the canonical no-new-IR-scale branch,

$$\tau_0^{-1} = H_0, \qquad D = \frac{c^2}{H_0}.$$

This transport equation separates two roles that must remain distinct. Ordinary galactic support belongs to the near-stationary static branch; the telegrapher sector governs what happens when the source history is no longer quasi-static — propagation delay, relaxation, and merger-era lag.

For galactic modes the Appendix E analysis shows that the long relaxation time does not destroy the static limit. Galactic modes lie deep in the underdamped regime, so the static Poisson branch is recovered as the exact time average relevant to ordinary galactic dynamics. The assumption here is that the source is quasi-static on galactic timescales and supported on wavelengths far shorter than the critical scale $\lambda_c \sim 4\pi c/H_0$; under those conditions the oscillatory transient averages out instead of competing with the static branch.

The transport branch is closed at the level of $D/\tau_0 = c^2$ and the preferred choice $\tau_0^{-1} = H_0$. The cluster and merger phenomenology, which sets the source weights this transport then evolves, is developed in Section 18.

18. Cluster Source Projection and the Diffuse–Decoupled Channel Split

Clusters are where the simple galactic radial-acceleration relation stops being enough, and they fail it in two distinct ways. Relaxed clusters show a hook-shaped residual — near unity in the stellar-dominated center, peaking at intermediate acceleration, converging back in the deep outskirts. Merging clusters show lensing peaks that stay with the collisionless galaxies while the dominant baryonic mass, the X-ray gas, is displaced. A viable account must produce both without granting galaxies hidden baryonic mass and without altering the galaxy law itself.

The proposed long-range entropic-excess channel couples differently to the three baryonic phases. A diffuse, phase-averaged medium carries the transverse projection ϵ inherited from the conditional galactic normalization; a dynamically decoupled collisionless component recovers the full projection; and a virialized coherent bath can be lifted above it. The projection value is fixed once that transverse branch is adopted. The phase-selection rule, lift profile, and resolved-map test remain open.

The cluster residual varies with acceleration. Lensing and kinematic analyses of relaxed clusters [4, 16, 17] find the ratio of observed to galaxy-RAR-predicted acceleration near unity in stellar-dominated centers, rising to roughly 3–5 at intermediate accelerations ($g_{\rm bar} \sim 10^{-11}$–$10^{-10}\,\mathrm{m\,s^{-2}}$), and apparently returning toward the galaxy relation at the lowest probed accelerations, subject to gas-extrapolation caveats [17]. The older integrated value of 1.5–2 from higher-acceleration hydrostatic analyses samples the high-$g_{\rm bar}$ edge of this hook-shaped profile. Merging clusters add a spatial constraint. In Bullet-type systems, ram pressure displaces the intracluster plasma from the collisionless galaxies while the lensing peaks remain with the outgoing collisionless components. A viable cluster sector must explain both observations without hidden baryonic mass in galaxies or a change to the galactic mass anchor.

Transport lag and extra galaxy mass do not supply the required offset. In the canonical $\tau_0^{-1} = H_0$ branch, disturbances propagate at c and cross a megaparsec-scale configuration in roughly 3 Myr, about two and a half orders below a gigayear merger timescale. The field therefore tracks the moving source; an underdamped configuration retains memory of the pre-merger centroid rather than the outgoing collisionless component. Extra galaxy mass is excluded because the galactic acceleration scale and gas-dominated-dwarf RAR fix the deficit per unit baryonic mass. The proposed mechanism is instead a phase-dependent source projection for the long-range entropic-excess channel.

The projection coefficient as the galactic reduction factor. Section 15 fixes the galactic acceleration scale as the transverse reduction of the horizon thermal scale,

$$a_0 = \frac{g_{\rm share,eff}}{4\pi^2}\, a_H, \qquad a_H = cH_0,$$

where $(2\pi)^2$ is the angular Haar volume of one canonical two-oscillator action cell and $g_{\rm share,eff}$ is the admissibility-sharing content conditionally loaded into it. The same loading and horizon-coupling construction gives

$$\epsilon \equiv \frac{a_0}{a_H} = \frac{g_{\text{share,eff}}}{4\pi^2} \simeq 0.188.$$

This introduces no cluster-specific coefficient. It imports the reduction factor of the galactic branch and reads it as a source projection: a source restricted to the transverse static sector couples at strength ϵ relative to a source accessing the full horizon projection.

The diffuse and decoupled source classes. The assignment follows from coherence under coarse-graining. Diffuse matter is a continuum of locally uncorrelated, thermalized source elements; under coarse-graining its off-diagonal source cross-terms average away and only the diagonal transverse static projection survives. It therefore couples at ϵ. This includes shocked or unvirialized intracluster plasma, the warm–hot intergalactic medium, and cold but *diffuse* galactic H I when the galaxy is treated as a single smooth source—which is why gas-dominated dwarfs and low-surface-brightness galaxies remain on the galaxy RAR. The suppressed projection is thus a coherence effect, not a temperature effect; a hot phase that has *virialized* into a coherent bath is the exception, taken up below.

The unsuppressed projection is accessed by matter that is *not* part of the phase-averaged continuum: a collisionless overdensity that is spatially separated from, and dynamically decoupled from, a surrounding diffuse medium. The criterion is relational, not intrinsic compactness. Compactness alone would misclassify: stars in ordinary galaxies, isolated ellipticals, and globular clusters are compact and bound yet must remain on the galaxy RAR, and they do, because none is a collisionless node decoupled from a distinct diffuse continuum. A galaxy in a cluster is different only because it is embedded in, and decoupled from, the intracluster medium. The suppression is the property of participating in the continuum; matter that has decoupled from the continuum escapes it. So decoupled collisionless matter sits at the baseline weight, and incoherent diffuse gas is suppressed to ϵ, with $\epsilon = g_{\text{share,eff}}/4\pi^2$.

A *virialized* bath is the third state. Once the diffuse atmosphere relaxes into a coherent, extended phase it may open a collective response above the decoupled baseline. The proposed ceiling $W_{\text{bath}} = 1/\epsilon \simeq 5.32$ is the reciprocal capacity of the adopted transverse cell. It is fixed within that branch but inherits its microscopic uncertainty. The lift profile between $W_{\text{bath}} = 1$ and $1/\epsilon$ is evaluated against cluster data below.

The effective entropic-channel source χ_{ent} is therefore regime-dependent. In a relaxed cluster the gas is a virialized bath,

$$\boxed{\chi_{\text{rel}} = \rho_{\text{dec}} + W_{\text{bath}}\,\rho_{\text{bath}}}$$

with ρ_{dec} the decoupled collisionless substructure and ρ_{bath} the virialized continuum; in a non-equilibrium merger the central gas is shocked and incoherent while only a residual atmosphere stays virialized,

$$\boxed{\chi_{\text{merge}} = \rho_{\text{dec}} + \epsilon\,\rho_{\text{shock}} + W_{\text{bath}}\,\rho_{\text{vir}}}$$

which in the Bullet limit, where the displaced gas is shocked and little virialized bath remains on the cores, reduces to $\chi_{\text{Bullet}} \simeq \rho_{\text{dec}} + \epsilon\,\rho_{\text{shock}}$. Ordinary matter continues to gravitate through the usual metric coupling; the projection rule concerns only the long-range entropic-excess channel.

The measured hot-atmosphere factor. A component is decoupled only relative to a surrounding medium, so the relevant factor is tied to a measured property of that medium: the fraction of the halo's cosmic baryon allotment that has become an extended virialized hot phase,

$$\boxed{B_{\text{bath}} = \text{clip}_{[0,1]}\left[\frac{M_{\text{hot,vir}}(< r_{500})}{f_{b,\text{cos}}\,M_{500}}\right]} \qquad f_{b,\text{cos}} = \frac{\Omega_b}{\Omega_m} \simeq 0.156,$$

with $f_{b,\mathrm{cos}}$ fixed by Planck values [46] and $M_{\mathrm{hot,vir}}$, M_{500} read from X-ray/SZ and total-mass estimates. Once the transverse branch is adopted, ϵ is shared across all systems, the baryon fraction is fixed cosmologically, and the per-system quantities are measured. The undetermined object is the coherence-growth profile $W_{\mathrm{bath}}(B_{\mathrm{bath}})$. In merging systems $M_{\mathrm{hot,vir}}$ refers to the pre-merger virialized atmosphere.

Relaxed-cluster residual. For a *relaxed* cluster the diffuse gas is a virialized continuum, and the residual is carried by that continuum, not by the decoupled stellar component. Let

$$f_{\mathrm{cont}} = \frac{M_{\mathrm{hot,vir}}}{M_{\mathrm{baryon}}}$$

be the fraction of observed baryons in the virialized diffuse continuum. The residual relative to a galaxy-RAR extrapolation is then

$$\boxed{\mathcal{R}_{\mathrm{rel}} = 1 + \big(W_{\mathrm{bath}} - 1\big)\, f_{\mathrm{cont}}}$$

and the minimal linear candidate for the lift,

$$W_{\mathrm{bath}} = 1 + (1-\epsilon)B_{\mathrm{bath}}, \qquad 1-\epsilon = 1 - \frac{g_{\mathrm{share,eff}}}{4\pi^2} \simeq 0.812,$$

uses the amplitude $(1-\epsilon)$, the part of the full horizon channel that the suppressed transverse branch is missing—not a new coefficient. The physical reading is that as a virialized diffuse bath forms, the continuum itself opens a collective cluster response whose amplitude scales with how complete the bath is (B_{bath}) and how much of the baryon budget sits in it (f_{cont}).

This linear form has the qualitatively correct mass trend: both B_{bath} and f_{cont} *increase* with halo mass—hot-gas fractions rise toward clusters [15, 14] and the atmosphere becomes more fully virialized—so their product rises monotonically from groups to massive clusters, reproducing the observed direction with no fitted parameter. Its amplitude, however, is excluded. For CLASH-scale clusters the measured factor gives $B_{\mathrm{bath}} \simeq 0.83$, $f_{\mathrm{cont}} \simeq 0.9$, hence $\mathcal{R}_{\mathrm{rel}} \simeq 1.6$; but evaluating the source weight required to reproduce the published CLASH relation [4] across its data-supported acceleration window gives $\mathcal{R} \simeq 3.7$ at $g_{\mathrm{bar}} = 10^{-10}\,\mathrm{m\,s^{-2}}$ rising to $\mathcal{R} \simeq 7.8$ at $10^{-11}\,\mathrm{m\,s^{-2}}$. The linear lift $(1-\epsilon)B_{\mathrm{bath}}f_{\mathrm{cont}}$ is short of the observed peak residual by a factor of roughly 2.5–4, and no escape through missing baryons (a multiple of the X-ray gas mass would be required), hydrostatic bias (the masses are lensing-based), or sample heterogeneity is available at that magnitude.

The linear candidate is excluded on amplitude. Because $B_{\mathrm{bath}} \le 1$ and $f_{\mathrm{cont}} \le 1$, the linear candidate bounds the relaxed residual by $\mathcal{R}_{\mathrm{rel}} < 1 + (1-\epsilon) \simeq 1.81$. The measured peak residual of relaxed clusters is 3–5 [4, 16], well above this bound, so the linear lift is excluded. The exclusion is specific to the lift function: the channel-split ontology itself makes a structural prediction about the residual's shape that the data support, taken up next.

The hook morphology. Independently of the lift amplitude, the channel split predicts the radial shape of the cluster residual. The decoupled BCG stellar component dominates cluster centers and carries weight 1, so the local residual starts near unity. Farther out, the virialized gas continuum dominates and raises the residual toward the bath-weighted value. At the lowest accelerations, the deep branch compresses a bounded source weight W toward $\sqrt{W}$ in acceleration terms and lowers the residual again. The resulting profile is a *hook*: near unity in the stellar-dominated center, maximal where the bath dominates at intermediate acceleration, and closer to the galaxy relation in the deep outskirts. Current measurements report this morphology [16, 17]; neither a total-baryon modified-gravity law with no relaxed-cluster excess nor a constant offset gives the same shape.

The channel split gives the observed radial shape, but the linear lift underestimates its amplitude: the predicted peak is $\simeq 1.5$ for CLASH-like parameters against the observed 3–5. The lift function must grow faster with bath development, reach 3–5 in developed clusters, and remain small enough for X-ray-faint groups to stay near the galaxy relation.

At the saturation weight $W_{\rm bath} = 1/\epsilon$ the predicted mass residual for developed-bath parameters is $\mathcal{R}_{\rm sat} = f_{\rm dec} + f_{\rm cont}/\epsilon \simeq 4.7$–$4.9$, inside the required window and at the upper edge of the measured peak band, and the radius-resolved profile then peaks at $\simeq 3.9$ in acceleration terms for CLASH-like parameters against the observed 3–5. The same weight applied uniformly at the group scale overshoots by a factor of ~ 2, so saturation must depend on coherence development: massive relaxed clusters sit at or near the bound, while X-ray-faint groups remain far below it, with the group-scale data requiring $W_{\rm bath} \lesssim 1.4$ there.

One caveat accompanies the saturated profile: the predicted central residual depends on the stellar/gas decomposition and on excluding multiphase cool-core gas from the coherent bath. The deep-outskirt question—power-law continuation [4] versus convergence toward the galaxy relation [17]—is adjudicated directly below.

The ceiling against cluster data. Inverting the X-COP hydrostatic measurements [67] gives twenty-four source-weight tests. For each point, S solves

$$\frac{S g_{\rm bar}}{1 - \exp[-\sqrt{S g_{\rm bar}/a_0}]} = g_{\rm obs}.$$

Every point respects $S \leq 1/\epsilon$; the maximum is $S = 3.96$, and the relaxed systems span $S \simeq 1.8$–2.7 at R_{500} and 1.4–2.2 at R_{200}.

The same inversion adjudicates the deep end: the power-law continuation requires $S \simeq 6$–8 at $g_{\rm bar} \simeq 1$–$2 \times 10^{-11}\,\mathrm{m\,s^{-2}}$, precisely the accelerations of the R_{500}–R_{200} points, which sit at $S \simeq 1.5$–2.7; within this sample the deep end converges rather than continuing, and the strongest published challenge to the bound is not borne out.

The methodological caveat is that the continuation was fit to lensing-based masses of higher-redshift systems while the inversion here uses local hydrostatic masses; breaching the bound at R_{500} would require the non-thermal-corrected masses to be low by a factor of $\simeq 2.3$, well beyond any claimed hydrostatic bias. The measured radial run of the source weight—$S \simeq 3.7$–4.9 in the hook-peak window, where the saturation band is reached, declining to $\simeq 2.2$ at R_{500} and $\simeq 1.5$ at R_{200}—is the quantitative target the coherence-growth profile must reproduce.

Direct and fluctuation-based turbulence measurements find low non-thermal support in relaxed systems at all probed radii [68, 69], so the decoherence agent gating the lift cannot be the cluster-to-cluster turbulence level: it must grow with radius even in fully relaxed atmospheres. The coherence-growth profile between the fixed endpoints, so constrained, is the object the channel-selection theorem must deliver, with the group end requiring $W_{\rm bath} \lesssim 1.4$.

The decoupled-fraction form is excluded by the mass trend. A second candidate class assigns the residual to the decoupled stellar fraction, $\mathcal{R} = 1 + 4.32\, B_{\rm bath} f_{\rm dec}$ with $f_{\rm dec} = f_\star/(f_\star + f_{\rm gas})$. Its amplitude can cross the observed band, but its mass trend is wrong. Because $f_{\rm dec}$ *falls* with mass while $B_{\rm bath}$ rises, their product peaks at the group or poor-cluster scale and declines toward massive clusters. The observed residual rises from groups to massive clusters. The trend therefore assigns the residual to the continuum, although its lift amplitude remains underived. The relaxed-cluster amplitude and Bullet morphology are distinct observables and require separate tests.

Bullet-type mergers. The relaxed residual and Bullet morphology use the same branch coefficient ϵ with different source expressions because the gas occupies different states. A relaxed atmosphere enters through the bath-lift term; shocked displaced gas carries the suppressed weight while collisionless cores remain decoupled. The two regimes share one conditional coefficient, not one universal scalar law.

In the merger, then, the decoupled galaxies and subcluster cores retain the unsuppressed projection and the shocked diffuse gas couples at ϵ. With a gas/galaxy baryon ratio near 5.7, the gas contributes $\epsilon \times 5.7 \simeq 1.07$ in the entropic channel against the galaxy contribution of 1.0: the projection brings the two components to near-parity, removing the factor ~ 5.7 by which the gas would otherwise dominate, but it does not by itself invert them. The inversion is completed by projected compactness. For two roughly symmetric outgoing components the ratio of one edge peak to the central gas contribution scales as

$$\frac{\Sigma_{\rm edge}}{\Sigma_{\rm gas}} \sim \frac{f_{\rm dec}}{2\epsilon f_{\rm gas}} \frac{A_{\rm gas}}{A_{\rm edge}}, \qquad \frac{f_{\rm dec}}{2\epsilon f_{\rm gas}} \simeq 0.47,$$

so an edge peak dominates the projected map once the shocked gas is spread over more than about twice the projected area of a compact outgoing core—a condition the observed morphology satisfies by a wide margin. The projection rule and this geometry therefore produce the observed gas/lensing inversion—by projection and geometry together, not by projection alone and not by transport lag—as a spatial surface-density prediction rather than an integrated-mass argument. This is consistency, not yet a test of the coefficient. In standard flexible lens reconstructions the gas weight is degenerate with free halo and substructure components, so a model that fits comparably well with or without the fixed X-ray gas map constrains ϵ only weakly; Bullet-type mergers are thus consistent with the projection rule but do not yet measure ϵ. Peak location alone is in any case insensitive to the coefficient, since sufficiently broad shocked gas yields clump-centered peaks across a wide range of gas weights; the coefficient is tested only by the resolved amplitude fit below.

Relation to the transport sector. The telegrapher sector of Section 17 is not the cluster mechanism; it governs how the field propagates and relaxes once the source weights are set. The source-projection rule supplies the static weights $\chi_{\rm ent}$; the transport sector then evolves them. This division avoids the failure mode of a transport-only account, in which a field sourced equally by all baryons cannot hold a lensing peak on the outgoing collisionless component. The full merger observable is obtained by evolving $\chi_{\rm ent}(x,t) = \rho_{\rm dec}(x,t) + [\cdots]$ through the causal equation with the observed geometry as input.

Falsifiers and open status. The rule makes quantitative predictions beyond the relaxed normalization. *(i)* The linear candidate's ceiling $\mathcal{R}_{\rm rel} < 1.81$ lies below the measured peak residual of relaxed clusters [4, 16], which excludes that form of the lift. The channel split predicts a hook: a residual near unity in BCG-dominated centers, one peak where the virialized bath dominates at intermediate acceleration, and convergence toward the galaxy relation in the deep outskirts. A profile monotonic in acceleration, or one that peaks in the stellar-dominated center, would falsify the channel split. The capacity bound supplies a ceiling, $\mathcal{R}_{\rm rel} \le f_{\rm dec} + f_{\rm cont}/\epsilon \simeq 5$, for developed-bath parameters. All twenty-four X-COP source-weight inversions respect it. A confirmed relaxed-cluster residual well above this bound would falsify the capacity bound.

(ii) At fixed mass, X-ray-bright bath-developed systems (larger $B_{\rm bath}$, larger $f_{\rm cont}$) should deviate more from the galaxy RAR than X-ray-faint systems; the residual turns on with the developed diffuse atmosphere, not with mass alone, so two systems of equal mass but different bath development should separate.

(iii) Resolved lensing maps test the three source components directly. Because the relaxed and merger regimes use different source expressions, the convergence is a *three-component* channel-weighted map—a relaxed virialized-bath component, a shocked non-equilibrium continuum at the suppressed weight, and a decoupled collisionless component,

$$\boxed{\kappa_{\rm obs}(x,y) = A\Big[W_{\rm bath}\,\Sigma_{\rm bath}(x,y) + \epsilon\,\Sigma_{\rm shock}(x,y) + \Sigma_{\rm dec}(x,y)\Big] + b}$$

with the branch value $\epsilon = g_{\rm share,eff}/4\pi^2 \simeq 0.188$ or with ϵ floated as a test, and $W_{\rm bath}$ fit within $[1, 1/\epsilon]$. Recovering $\epsilon \simeq 0.19$ across relaxed and merging systems would support both the cluster source rule and the inherited transverse normalization. A best fit near 1 or 0 would falsify the cluster branch. The fit must use the channel-weighted baryonic maps without free dark haloes, which would otherwise absorb the gas weight.

Three open items remain at the theory level. First, the relaxed residual and Bullet morphology use one inherited coefficient with two regime-specific source expressions. The boundary between the virialized and shocked regimes is not derived.

Second, suppression of a phase-averaged continuum and collective lift of a virialized bath are independent premises. Neither follows from the current microscopic source map. Their derivation must also reproduce the measured radial decline and the proposed branch endpoints.

Third, identifying the decoupled component with stellar or galaxy mass and the continuum with gas is a coarse split; intracluster light and tidally stripped stars blur it at a level the resolved-map fit would expose. Abell 520, whose reported gas-coincident dark core is disputed, is a phase-state stress case rather than a direct test: a re-cohering or quasi-bound central component would raise its effective $B_{\rm bath}$ and return lensing toward the gas. A confirmed *young* merger with a statistically secure gas-centered, galaxy-free lensing peak would leave no time for that re-coherence and would challenge the model.

This sector is a structured, falsifiable proposal. The branch value of ϵ is inherited rather than re-fitted, $B_{\rm bath}$ is measured from the hot-atmosphere fraction, and current data support the trend and hook morphology while excluding the linear lift candidate. The microscopic transverse normalization, coherence-growth profile, resolved-map test, and channel-selection theorem remain open.

19. Cosmology and the Hubble-Tension Sector

The scalar capacity field has two cosmological roles, and the sector works only if they stay separate. Its homogeneous mode $\overline{S}(t)$ affects the background expansion and the sound horizon; its inhomogeneous fluctuations $s(x,t)$ still govern local weak-field gravity. The cosmological sector is the homogeneous continuation of the same medium, not an unrelated dark-energy component appended to the weak-field theory: what changes is the kinematic regime, not the ontology, as the background mode becomes dynamically relevant on horizon scales while the local branch stays encoded in the fluctuations.

The cosmological sector uses the same field split,

$$S(x,t) = \overline{S}(t) + s(x,t),$$

where $\overline{S}(t)$ is the homogeneous mode and $s(x,t)$ the inhomogeneous sector responsible for local weak-field dynamics. The vacuum baseline is fixed by apparent-horizon capacity,

$$S_\infty(t) = \pi\frac{R_A(t)^2}{L_*^2}.$$

This is the horizon-normalized representation of the same entropy field used locally. It is compatible with the cell-normalized source theorem because local observables depend on $\delta S/S_\infty$ and $\kappa/(\gamma S_\infty)$ rather than on an absolute entropy unit.

Two independent results clarify what this baseline means. First, the type II_1 static-patch construction makes empty de Sitter the maximum-entropy gravitational state and places excited semiclassical states below it [23]. Second, Bianconi obtains the same de Sitter area scaling from a bulk rather than a horizon entropy. In the $c = 1$ convention of her low-curvature Friedmann approximation, the local geometric-relative-entropy density and the causal-diamond four-volume scale as

$$\frac{\delta s}{\delta v} \simeq \overline{\omega}_{[1]} H^2, \qquad V_{\mathrm{dS}} \sim H^{-4},$$

so their product gives

$$S_{\mathrm{GfE}} \simeq \frac{\overline{\omega}_{[1]}}{\ell_P^4 H^2} \propto H^{-2} \propto A_{\mathrm{dS}}.$$

Integrating Bianconi's local volumetric information density over the observer's causal diamond gives an area-sized finite capacity [24]. Her result does not fix the coefficient used here, replace ℓ_P by the independently calibrated L_*, or derive the homogeneous evolution. Her intrinsic geometric k-temperatures scale as H^2, whereas the Gibbons–Hawking temperature used in Section 15 scales as H.

Because the entanglement field couples to the trace of the stress-energy tensor, the homogeneous mode is largely dormant during radiation domination but can become active near matter–radiation equality. The conditional proposal would reduce the sound horizon and shift the CMB-inferred Hubble constant upward: the mechanism has the required sign and turns on at the required epoch. Its joint evolution with the committed component has not been computed, and a full Boltzmann calculation and likelihood analysis remain open.

The local weak-field predictions are protected by the separation between $\overline{S}(t)$ and $s(x,t)$. This is the role of the shear-lock logic: changing the homogeneous background mode does not rewrite the local static Poisson branch that governs galactic dynamics and lensing. The two regimes therefore use one scalar medium without changing the galactic coefficients, and a full perturbation calculation must still test that separation dynamically.

20. The Saturated Phase and the Cosmic Microwave Background

The conditional transverse normalization of Section 15 is now applied to the early universe. Its galactic response remains below the capacity ceiling, but the estimated recombination-era demand exceeds it. The resulting saturated phase contains a conserved count of committed channels that redshifts as a^{-3}. The dust theorem below is conditional on the pinned realization of that phase, and the numerical abundance inherits the transverse coefficient ϵ.

Saturation requirement at recombination. Use $y \equiv g_{\mathrm{pert}}/a_0(z)$ for the acceleration ratio and $x \equiv \sqrt{y}$ for the thermal argument of Section 15. The linear-domain estimate $y \lesssim 2 \times 10^{-3}$ today and $y \propto \sqrt{a}$ gives $y \lesssim 6 \times 10^{-5}$ at $z \simeq 1100$, hence $x \lesssim 8 \times 10^{-3}$ and $\nu(x) \gtrsim 10^2$. This is well above the proposed ceiling $1/\epsilon = 5.32$, so the adopted response branch requires saturation if the same estimate applies to the relevant perturbation modes. The relation $a_0(z) = \epsilon c H(z)$ keeps the background ratio $cH/a_0 = 1/\epsilon$ fixed at every redshift.

Absolute capacity, source-normalized recruitment, and coherence. Three quantities that answer different questions must be kept separate. The availability field q measures the absolute local capacity remaining for ordinary transactions; in the static normalization it obeys

$q = 1+2\Phi/c^2+O(c^{-4})$. It is not a universal identity with $-g_{00}$ in arbitrary FLRW coordinates. Recruitment saturation is instead defined relative to each source's cap. For a Lagrangian source element A of baryonic mass M_A, let $M_{c,A}$ be the mass-equivalent committed allocation assigned to that source. Per-defect bookkeeping gives the cap and its normalized occupancy,

$$M_{c,A}^{\max} = \frac{M_A}{\epsilon}, \qquad \boxed{\sigma_A = \frac{M_{c,A}}{M_{c,A}^{\max}} = \epsilon\frac{M_{c,A}}{M_A} \in [0,1].}$$

The label A is physical bookkeeping: allocations belonging to distinct defects may overlap spatially, but they cannot be counted twice. In the continuum A becomes a Lagrangian source label, so σ_A is source-supported rather than a local projector attached independently to every cell.

Coherence is likewise a property of the committed flow, not a microscopic Boolean register. Write $\mathcal{C}=1$ when the source-labelled currents combine into one hypersurface-orthogonal, single-stream flow with a single-valued renewal phase; locally this requires vanishing curl of the normalized flow, and globally it requires vanishing phase holonomy. At first shell crossing the Lagrangian map loses invertibility and several velocities occupy one Eulerian point, so $\mathcal{C}$ falls to zero. The physical pinned branch is therefore

$$\boxed{\sigma_A = 1 \text{ for every recruited source}, \qquad \mathcal{C}=1,}$$

generally with the absolute availability still close to its vacuum value. Terminal strong-field saturation is the different condition $q_{\rm geo} = 0$. Cosmological pinning saturates a small active subsector; it does not exhaust the vacuum budget.

The pinned reading and the dust theorem. The *pinned reading* is the statement that the saturated bath holds exactly at the source-normalized ceiling and stays there. The transfer law quantizes channel transport at one bit per tick as an upper bound; saturation is the attainment of that bound, so on the pinned reading every committed channel advances by exactly one unit per tick, and while the phase remains pinned no channel can decommit, since there is no slack for it to relax into. The committed count is then conserved and dilutes only with the expanding volume.

The unit-timelike normalization is not a further independent condition once saturation and coherence are granted. Let θ be the coherent renewal phase, increasing by one on each electron-calibrated interval τ_*. The dimensionful clock $\phi = \tau_*\theta$ therefore satisfies $d\phi/d\tau = 1$ along a committed worldline. Coherence identifies the flow covector as $u_\mu = -\nabla_\mu\phi$. Hence

$$1 = \frac{d\phi}{d\tau} = u^\mu\nabla_\mu\phi = -u^\mu u_\mu, \qquad \boxed{X = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi = \frac{1}{2}.}$$

Thus the spectral tick fixes the normalization of the coherent clock; no cosmological coefficient is inserted. This proves the constraint within the pinned coherent branch. It does not derive the transition that makes $\sigma_A \to 1$ and $\mathcal{C} \to 1$.

The coarse-grained field ϕ consequently carries a constraint rather than a generic kinetic term. The leading action consistent with it is

$$S_{\rm comm} = \int d^4x\,\sqrt{-g}\,\lambda\left(X - \tfrac{1}{2}\right),$$

whose variation gives $\nabla_\mu(\lambda\,\partial^\mu\phi) = 0$ and, on the constraint surface,

$$T_{\mu\nu} = \rho\, u_\mu u_\nu, \qquad \rho = E_0\, n, \qquad p = 0, \qquad c_s^2 = 0, \qquad \rho \propto a^{-3},$$

with $u_\mu = -\partial_\mu \phi$ and n the conserved spatial density of committed cells. This belongs to the constrained-scalar class [70]. Conditional on the pinned reading, it is pressureless dust. The construction inherits caustic formation and the need for a small-scale completion. The manuscript does not derive the transition from the unconstrained weak-field branch onto $X = \frac{1}{2}$ or the commit/decommit energy accounting. Related relativistic Milgromian dynamics provides a useful comparison [30].

Uniqueness of the carrier. The constraint coupling is the unique survivor of the dynamics classes examined (Appendix M). Relaxational response kernels are excluded twice over: a growth clock calibrated on the cluster radial decline misses cosmological development by a factor of order thirty, and the precision of the measured acoustic peaks bounds any oscillatory leakage of a lagged response below one part in five hundred, a rejection no causal filter achieves over the few oscillation periods available before recombination. Bound-type rail readings sit at the sound-speed pole and carry no perturbation; plateau approaches have $c_s^2 = -1/(2n{+}1) < 0$ and are gradient-unstable; the released branches have $c_s^2 = \frac{1}{2}$ and 1 and free-stream. A general no-go for relaxation-plus-cap carriers is recorded in Appendix M; the conserved committed density is precisely the additional integrating variable that the no-go requires and the constraint reading supplies.

Abundance. At full saturation the committed weight is the ceiling, giving

$$\frac{\Omega_c}{\Omega_b} = \frac{1}{\epsilon} = \frac{4\pi^2}{g_{\mathrm{share,eff}}} = 5.321 \qquad \text{against the measured } 5.364 \pm 0.065,$$

a 0.7σ agreement with the abundance derived from the commitment count [46]; equivalently $\Omega_m/\Omega_b = 1 + 1/\epsilon = 6.32$. A standard Einstein–Boltzmann computation [71] with the cold component tied to this value and the acoustic scale θ_* held fixed reproduces the quoted Planck best-fit temperature spectrum with a root-mean-square residual of 0.15% over $\ell = 2$–2500 and 0.08% in the third-peak region. At the likelihood level, the tied model carries one fewer free parameter than ΛCDM and sits at $\Delta\chi^2 = +2.15$ against the six-parameter best fit on the compressed Planck TT,TE,EE likelihood [37]. This fixed-θ_* comparison tests the inherited dust abundance only. It does not demonstrate the separate Section 19 claim that the homogeneous mode reduces the sound horizon. A joint Boltzmann calculation must evolve the homogeneous mode, commitment transition, and tied dust component together; that calculation remains open.

The acceleration and abundance relations contain a parameter-free structural test that does not depend on the numerical value of the phase-cell coefficient. Since $a_0 = \epsilon c H_0$ and $\Omega_c/\Omega_b = 1/\epsilon$,

$$\boxed{a_0 \frac{\Omega_c}{\Omega_b} = cH_0}.$$

Using the RAR scale $(1.20 \pm 0.02) \times 10^{-10}\,\mathrm{m\,s^{-2}}$, the Planck abundance ratio 5.364 ± 0.065, and $H_0 = 67.4 \pm 0.5\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ gives

$$\frac{a_0(\Omega_c/\Omega_b)}{cH_0} = 0.983 \pm 0.022$$

[1, 46]. This two-percent agreement tests the reciprocal structure of the two sectors; the quoted band uses the random error on the RAR scale, and its mass-to-light systematic widens it accordingly. It does not derive either conditional premise: the transverse loading controls a_0, while the pinned recruitment reading controls the abundance.

That the committed density attains the ceiling exactly, rather than a development-dependent fraction of it, follows only within the recruitment assumptions of this branch. In the notation above, the assumption is $\sigma_A \to 1$ for every recruited Lagrangian source element. Commitment is source-driven: cells receiving no demand commit nothing, and each source's allocation is capped at $1/\epsilon$ per unit source mass. In the recombination estimate above the relevant modes demand $\nu \gtrsim 10^2$, far above the cap, while the homogeneous background remains below it. With abundant supply and irreversible commitment in the pinned regime, each source recruits to its cap and allocations are assumed to add. The committed density is then

$$\rho_c = \frac{1}{\epsilon}\,\rho_b$$

pointwise at a commitment epoch. The manuscript has not derived that epoch or shown that commitment completes before the modes used in the Boltzmann calculation begin their relevant acoustic evolution. The initial condition $\delta_c = \delta_b$ is therefore an explicit assumption of the current numerical check, not an output of the recruitment argument. The abundance remains conditional on the pinned reading and per-defect bookkeeping; the transition history and energy accounting are open.

Conditional conservation: release at the caustic. The committed component must behave as conserved dust while the linear bath remains pinned, yet the galactic branch (Section 15) requires collapsed systems to follow the baryonic response law with no surviving collisionless halo, so the completion is a conditional-conservation law: $\nabla_\mu J^\mu_{\text{commit}} = 0$ in the pinned regime, with decommitment upon release. The constrained phase fixes the release mechanism itself. Because the coherent committed flow has one single-valued phase, it is irrotational and single-stream. If $x^i(t, \mathbf{a})$ is its Lagrangian map, first shell crossing occurs at

$$\mathcal{J} = \det\left(\frac{\partial x^i}{\partial a^j}\right) = 0.$$

Beyond that event the Eulerian velocity is multivalued, so no single phase can represent every stream and $\mathcal{C} : 1 \to 0$. The caustic — the known breakdown locus of the constrained-scalar class — is here the decommitment event, converting committed capacity into the local response branch. Under the synchrony definition of commitment, one counter per cell ticking with the local phase, decommitment at first stream-crossing is forced rather than chosen: a single fixed cell cannot remain synchronized with two distinct phase branches, so conversion is event-like at the first caustic, a result holding at the same conditional grade as the pinned reading itself.

Re-entry is forbidden on entropic grounds. Recommitment of a virialized region would require shedding vorticity and re-synchronizing with the advanced global clock — a spontaneous recoherence — so decommitment is irreversible, with the door's orientation inherited from the thermodynamic arrow, which remains an open conditional extension (Section 22, Appendix G.7). The resulting partition is the one the data require: the linear cosmological field never shell-crosses and remains committed dust, while collapsed systems have shell-crossed and retain none. The alternative, commitment tracking the instantaneous local acceleration, is excluded directly by rotation-curve data: below g_c the binned radial-acceleration residuals lie within 0.05 dex of the response law [1], against the +0.40 dex excess that locally regrown dust at the cosmic ratio would produce. One completion remains open and is recorded — the energy bookkeeping of conversion at the caustic — together with the consequence that late nonlinear structure growth proceeds on the response branch.

The sharp geometric prediction is registered as a falsifier. Infalling material between turnaround and first shell-crossing is single-stream and still committed, so clusters should carry a surviving committed component on their infall streams, terminating at the splashback surface [72], with an excess gravitating rim in that shell, a sharp edge at the outermost caustic, and none inside it. The recruitment count fixes the rim's amplitude as well as its geometry: along a stream of which a fraction $f_{\rm rel}$ has already shell-crossed, the surviving committed density is $(1 - f_{\rm rel})\,\rho_b/\epsilon$, so the prediction specifies location, edge, and magnitude together.

Domain structure. Writing $y = g/a_0$ and $x_{\rm th} = \sqrt{y}$, the release threshold obeys $\nu(x_{{\rm th},c}) = 1/\epsilon$. Thus

$$x_{{\rm th},c} = -\ln(1-\epsilon) = 0.2082, \qquad y_c = x_{{\rm th},c}^2 = 0.0433,$$

and $g_c = y_c a_0 \simeq 5.3 \times 10^{-12}\,{\rm m\,s^{-2}}$ today. The earlier notation used x_c for y_c; the separate symbols here remove that ambiguity. Whether linear cosmological modes stay on one side of this threshold throughout their evolution must be established in the joint Boltzmann calculation.

Distinction from horizon saturation. The committed cosmological phase — channels advancing at capacity, a running clock — is distinct from the terminal saturation of the strong-field sector (Section 21), where capacity is exhausted and transactions cease. Whether the two termini are stages of one process is open and carries a stated consistency burden: the gravitating energy of committed capacity must coincide with the mass already accounted at infinity, with no double counting. This is recorded as an open question shared by the strong-field and cosmological sectors.

Part VI. Strong Fields, Many-Pasts, and Microstructure

The strong-field branch treats the zero-capacity boundary, and Many-Pasts supplies the record-conditioned history ontology in which the substrate's renewal process operates. The separate refresh theorem selects the memoryless dressing used in scale setting, while Section 23 asks for its microscopic dynamics. The same grading applies here: the exact spherical reduction and the operational quantum measure are controlled results, while the boundary microphysics, the arrow of time, and the condensate embedding carry their stated conditional or open grades.

21. Strong-Field Action: Spherical Closure and Its Boundary

The bounded variable remains the natural strong-field order parameter,

$$q = \frac{S_{\rm ent}}{S_\infty} \in [0,1].$$

Serial composition and weak-field matching select $N^2 = q$ in a static exterior. That relation is meaningful after the asymptotic Killing time has fixed the normalization of the lapse; it is not, by itself, a covariant field equation in a general foliation.

Why the former multiplier action is not retained. An exploratory ADM term $\sqrt{h}\,\lambda(N^2 - q)$ makes the problem look variational, but if q has no independent bulk dynamics or matter coupling its variation gives $\lambda = 0$. The remaining constraint merely renames the lapse, which is gauge dependent, and supplies neither the capacity Poisson equation nor a new covariant relation. Giving q its own kinetic term would instead add a scalar degree of freedom and reopen the fifth-force, stability, and double-counting problems. The multiplier construction is therefore a diagnostic control case, not the parent action.

Exact spherical reduction. Spherical symmetry supplies the invariant completion that the generic lapse constraint lacks. Write

$$ds^2 = h_{ab}(x)dx^a dx^b + R^2(x)d\Omega^2, \qquad a, b \in \{0, 1\}, \qquad x^0 = ct.$$

After integrating the Einstein–Hilbert plus GHY action over the two-sphere and removing a two-dimensional total derivative, the bulk action is

$$I_{\rm sph} = \frac{c^3}{4G}\int d^2x\,\sqrt{-h}\,\Big[R^2\,{}^{(2)}\!R + 2h^{ab}\partial_a R\partial_b R + 2\Big] + I^{(2)}_{\rm matter} + I^{(2)}_{\partial}.$$

Its vacuum equations imply conservation of the Misner–Sharp mass

$$M_{\rm MS} = \frac{c^2 R}{2G}\left(1 - h^{ab}\partial_a R\partial_b R\right), \qquad \nabla_a M_{\rm MS} = 0.$$

The capacity-adapted invariant is therefore

$$\boxed{q_{\rm geo} \equiv h^{ab}\partial_a R\partial_b R = 1 - \frac{2GM_{\rm MS}}{c^2 R}.}$$

Thus in the preferred metric-only construction, spherical q is a composite geometric scalar and its vacuum profile is a first integral of the metric equations. It is not an auxiliary field and it is not an additional propagating mode.

For a static asymptotically flat vacuum, $M_{\rm MS} = M$ and Birkhoff's theorem gives

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)c^2dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2,$$

so

$$q_{\rm geo} = N^2 = 1 - \frac{2GM}{c^2 r}.$$

This is the nonperturbative action-level realization of the bounded capacity relation in the strongest sector where an invariant local definition is currently available.

The $q = 0$ surface. The equation $q_{\rm geo} = 0$ locates a marginal sphere. A well-posed exterior variational problem is obtained first on a stretched timelike boundary $q = \epsilon > 0$ with the usual GHY term and fixed induced data, and then by taking the null limit with the corresponding null and joint terms. The Einstein action determines that universal gravitational boundary bookkeeping. It does not determine a new substrate boundary Hamiltonian, microscopic reflectivity, or the rule that excises $q < 0$.

Accordingly, the restriction

$$\mathcal{M}_q = \{q_{\rm geo} > 0\}$$

is a physical postulate about the domain of the capacity EFT, not a consequence of varying the Einstein action. The exterior solution and horizon location are closed within spherical metric reduction; the claim that the classical interior is absent, and the dynamics experienced at the saturation surface, remain conditional on a microscopic boundary theory. Appendix F and Appendix N state this boundary between result and interpretation explicitly.

Status of the construction. The Schwarzschild exterior, its standard Hawking temperature, and ordinary exterior perturbation equations follow from the metric parent. The channel identity yielding a coefficient 1/4 remains a normalization identity until the channel-to-area map is derived. Rotating and charged GR exteriors remain valid solutions of the baseline parent action, but a covariant capacity scalar that identifies their saturation surface has not yet been constructed. The universal strong-field result is therefore spherical and exterior; boundary microphysics, dynamical saturation, nonspherical capacity geometry, and any controlled departure from GR remain open.

22. Many-Pasts: The History-Space Ontology

Many-Pasts makes a conservative operational claim and a distinct ontological claim. Laboratory records obey standard quantum mechanics. The ontology assigns a conditional measure to the decoherent past histories compatible with the one realized present.

The operational weight. For a decoherent family, the joint history weight and its record-conditioned form are

$$p(h, P) = \mathcal{D}(h, h), \qquad p(h \mid P) = \frac{\mathcal{D}(h, h)}{\sum_{h' \in \mathcal{H}_P} \mathcal{D}(h', h')}.$$

Equivalently, one may define $D(h, P) = -\ln p(h, P)$ on the support of the decoherent joint measure, so that $p(h \mid P) \propto e^{-D(h,P)}$. The exponential is a reparameterization of a normalized quantum probability, not an additional classical measure placed on interfering paths. Projective measurements and general quantum instruments therefore retain their Born probabilities. Appendix G gives the construction and the no-signaling proof.

Branch realization without many worlds. The weight also answers what it means for one outcome to be realized. There is no forward branching into co-real worlds and no collapse event. A definite present is a present with definite macroscopic records, and the histories the weight supports are exactly those compatible with those records; alternative outcomes correspond to alternative present records, not to coexisting branches. Probability is the measure this weighting assigns over the admissible pasts of the one realized present.

Record retention and the resolution of the past. The physical degrees of freedom that distinguish histories are present records. A durable record can be a detector state, an environmental correlation, or the exported history register of the renewal dilation. Such a record keeps distinct those past alternatives that remain distinguishable in the present record algebra. If orthogonal fine records R_i are later merged into a coarser record $\bar{R} = \sum_{i \in I} R_i$, decoherence and additivity give

$$p(h, \bar{R}) = \sum_{i \in I} p(h, R_i), \qquad p(h \mid \bar{R}) = \frac{\sum_{i \in I} p(h, R_i)}{\sum_{h'} \sum_{i \in I} p(h', R_i)}.$$

The history measure therefore loses resolution by ordinary quantum coarse-graining when the present loses a record. The ontology contains no separate archival copy of a distinction that has disappeared from every physical register. This does not erase consequences already carried into the current state. It identifies earlier alternatives with identical surviving records as the same present physical state.

No future pull. The class operators and unitary maps determine the joint measure before conditioning. The factor $p(h \mid P)$ updates the description of earlier alternatives after P is recorded; it does not enter the Hamiltonian, the channel, or the class operator that produced P. A record that has not yet formed supplies no conditioning event in the realized present. Many-Pasts therefore adds no retrocausal force, final-boundary dynamics, or Born-rule bias. It gives a physical reading to retrodictive conditioning already present in standard quantum mechanics.

Familiar quantum examples. In a double-slit experiment, the alternatives through the two slits remain inside one coarse class operator until a durable which-path record decoheres them. Their cross terms are therefore retained before the record and suppressed after it. In an EPR or Bell experiment, the present is a joint record of the pair and the detectors. The joint measure gives the usual nonclassical correlations, while the local marginals remain independent of the remote setting. Measurement creates a stable record and thereby identifies the decoherent family on which conditional probabilities can be used. These are standard quantum calculations with a record-conditioned history-space reading.

The arrow of time. The same framework proposes to orient time through conditional typicality. The maximum-caliber replacement process does not solve this problem. At stationarity it obeys detailed balance identically, $p(b)K_*(b,b') = p(b')K_*(b',b) = p(b)p(b')$, and is time-reversal symmetric as a stochastic process. The reversible dilation also has an inverse. Histories with a low-entropy past and increasing future entropy dominate only after a past-boundary condition and a substrate mixing or large-deviation theorem suppress Boltzmann-fluctuation histories. This is not an added law of laboratory probability; it is the open typicality result stated precisely in Appendix G.7.

Bianconi supplies a useful compatibility result, but not that missing theorem. In her low-curvature matter- and radiation-dominated Friedmann approximations, the local geometric-relative-entropy density decreases as the universe dilutes while the integrated entropy grows with the expanding volume and the integrated energy approaches a constant [24]. This demonstrates in a concrete information-geometric action that local ordering and a global entropy increase need not conflict. It does not select a low-entropy boundary, establish substrate mixing, or show that the Many-Pasts conditional measure favors ordinary histories over Boltzmann fluctuations. The process-level arrow therefore remains open exactly where Appendix G.7 places it.

No external cycle ledger. If a proposed cosmological evolution reaches a state whose present record algebra contains no witness of an earlier macroscopic era, the descriptions "first beginning" and "return to the same beginning" are not distinct states within this ontology. Distinguishing them would require a register that survives the record-free interval or an external time parameter that counts passages. Many-Pasts supplies neither. This conditional observation does not derive a cyclic universe, a unique beginning, or a nucleation rate. It removes an otherwise hidden meta-history from the ontology.

Where the weight is used. Many-Pasts supplies the history-space realization in which the electron dressing operates, while faithful full-support resolution proves the independence of successive passes. Appendix H shows that this condition is equivalent to maximum path entropy and uniquely selects the replacement process; the lightest-defect criterion selects the same temporal independence together with independent channel layers. The record-conditioned viewpoint also enters the proposed macroscopic arrow of time and the conditional cosmological extensions. It changes no laboratory law.

Local renewal and reversible history export. Complete local renewal has a precise quantum form. If the new cell must contain no information about the old cell even when the old cell is entangled with a reference, the one-tick channel is uniquely

$$\mathcal{E}_*(\rho) = \rho_* \operatorname{Tr} \rho.$$

This local channel need not destroy information globally. A reversible dilation transfers the old cell into an environment register while a fresh admissible register becomes the new present. Many-Pasts interprets the exported correlations as history degrees of freedom. The identification is structural: Stinespring dilation guarantees an environment, while Postulate III supplies its history-space reading. It does not derive the decoherence functional or show that an indefinitely long history can be stored in finite microscopic resources.

Coherence under renewal. The replacement channel acts only on the renewed closure register. Its Heisenberg dual is

$$\mathcal{E}_*^\dagger(O) = \mathrm{Tr}(\rho_* O)I,$$

so no later observable of that register can recover an input off-diagonal. The global dilation can nevertheless retain coherent information in its complement. If two spatial branches export discarded states $|e_x\rangle$ and $|e_y\rangle$, their reduced off-diagonal is multiplied by $\langle e_y|e_x\rangle$. A cell-addressed mark record makes those states orthogonal and would destroy position coherence in one update. The allowed free vertex instead transports the marked fiber as part of the coherent defect system and leaves the discarded renewal record branch-independent. Ordinary environmental interactions may then reduce the overlap in the usual way. Appendix H.11 gives the proof and distinguishes this condition from the weaker statement that a record merely "moves with" a worldline.

The remaining quantum-dynamical condition. Renewal and the record-overlap theorem identify the persistent marked sector in which coherent probability and phase data must live; they do not derive its dynamics. Caticha's Hamilton–Killing reconstruction gives a sufficient target: if coarse-graining this sector yields coordinates (P, Φ) with the canonical symplectic form and Fisher-compatible metric stated in Appendix H.11, then $\Psi = \sqrt{P}\, e^{i\Phi/\hbar}$ and linear unitary Schrödinger evolution follow, with a corresponding nonrelativistic local-$U(1)$ extension [25, 26]. This imports the mathematical theorem, not Caticha's epistemic interpretation of probability, and supplies no evidence that the present vertex satisfies its hypotheses. It reduces the quantum-emergence problem to a specific microscopic question: does the decorated reversible dilation generate the required probability–phase geometry?

Status. The operational measure is mathematically complete because it imports the standard decoherence functional and quantum instruments; within those kinematics the Born form is additionally the unique consistent record measure (Section 3.3, Appendix G.4). A substrate derivation of that functional is still missing. The foundational faithful full-support condition closes the classical replacement kernel on history space, reference decoupling closes the form of the quantum replacement channel, and interferometric consistency fixes the no-which-path condition on its marked dilation. The decorated tetrahedral transfer vertex prepares the diagonal fresh state, exports the previous closure register, and realizes the finite charged marked event. Deriving the slow probability–phase dynamics from that action, long-time history storage, relativistic quantum fields, and the thermodynamic arrow retain their open or conditional grades.

23. Microstructure Hamiltonian and Underlying Dynamics

The UV closure chain now has an explicit finite action on the scale-setting side and a candidate condensate realization on the geometry side. Appendix H derives the replacement process, its quantum channel, the state-weighted determinant transfer, and a decorated native-cell vertex. The vertex prepares the diagonal fresh closure state, exports the previous replaceable closure register, transports the persistent hard-core charged fiber without writing a free which-path record, and fixes the graph that dresses the electron and heavier shells. Its geometric GFT embedding and condensate stability remain separate tasks.

On the geometry side, the realization is a GFT/condensate picture [55, 56] in which spacetime emerges from a condensate of discrete tetrahedral building blocks and fermionic defects of the same substrate appear macroscopically as matter. In Madelung form,

$$\sigma(x) = \sqrt{n(x)}e^{i\theta(x)},$$

the condensate hydrodynamics generate a positive scalar stiffness for the logarithmic-density variable. This supplies a possible condensate origin for the EFT kinetic term. Appendix C remains the source of its explicit coefficient.

On the defect side, the closure ensemble fixes the stationary marginal p_{η_*} and maximum path entropy selects

$$K_*(b, b') = p_{\eta_*}(b').$$

This fixes the dimensionless history process but cannot produce seconds. The electron, already the theory's single dimensionful anchor and its lightest charged one-bit defect, supplies the clock through the positive transfer spectrum below.

Requiring the output cell to decouple from every reference system forces the replacement channel $\mathcal{E}_*(\rho) = \rho_* \operatorname{Tr} \rho$. The decorated vertex chooses the diagonal maximum-entropy completion

$$\rho_* = \sum_b p_*(b)|b\rangle\langle b|, \qquad p_*(b) = Z^{-1} e^{-\eta_* K^2(b)},$$

and prepares it from the amplitude $A_*(b) = \sqrt{p_*(b)}$. A reversible register permutation moves the old cell into history and the fresh amplitude into the present. For the complete 1680-state tetrahedral register this is one native circuit layer. A direct circuit made only from disjoint face comparisons still requires the three perfect matchings of the four-face graph; the decorated action uses the complete tetrahedron as its local gate.

On the factorized seven-channel history space let

$$|v^{(7)}\rangle = \bigotimes_{m=-3}^{3} |\sqrt{p_{\eta_*}}\rangle_m, \qquad P^{(7)} = |v^{(7)}\rangle\langle v^{(7)}|.$$

On the commutative cell algebra, likelihood multiplication by $p_*(b)$ has the state-weighted determinant

$$\Delta_{\tau_p}(\mathsf{R}) = \exp\left(\sum_b p_b \ln p_b\right) = e^{-g_{\text{share,eff}}}.$$

The seven-channel determinant is $r = e^{-7g_{\text{share,eff}}}$. It is the almost-sure geometric transfer rate of the record-conditioned renewal history; the annealed equality probability would instead involve the collision entropy. Compressing the one-bit charged loop to its determinant line gives the positive scalar survival transfer

$$T_{\text{surv}} = 1 - r.$$

The Euclidean transfer Hamiltonian

$$H_{\text{surv}} = -\frac{\hbar}{\tau_*} \ln T_{\text{surv}}$$

therefore has the exact raw gap

$$E_{\text{raw}} = -\frac{\hbar}{\tau_*} \ln(1 - r).$$

The closure amplitude has an exact Gaussian linearization. Projecting the same amplitude through the two directed scalar returns gives $u = 8\eta_*/49$, and its canonical unitary dilation supplies the factor $(1 - u)^{21/2}$. The nine present/history closure polarizations give the marked weight ζ_* of Section 13.4. The electron graph contributes $Z_e = (1 + \zeta_*)(1 + 7\zeta_*^2)$, so the dressed transverse identification is

$$E_e = \frac{3}{2} Z_e E_{\text{raw}}.$$

Identifying this lowest charged transfer gap with the electron rest energy fixes the update time,

$$\boxed{\tau_* = -\frac{3}{2} Z_e \frac{\hbar}{m_e c^2} \ln(1-r)}.$$

Analytic continuation supplies the phase frequency $E_e/\hbar$. The identification of the same spectral tick with the causal cell scale gives $L_* = c\tau_*$.

The causal length of that spectral update is

$$\boxed{L_* = c\tau_* = -\frac{3}{2} Z_e \lambda_e \ln\left(1 - e^{-7g_{\mathrm{share,eff}}}\right)}.$$

The baseline rare-event correction $-\ln(1-r) = r[1 + O(r)]$ differs from r only at order 10^{-23}. The finite marked response changes the scale by $Z_e - 1 = 0.00530828$, which is the physically relevant correction resolved by the microscopic vertex.

Retained temporal information raises the recurrence mass by e^{I_t}, so electron lightness independently selects $I_t = 0$. Fermionic exclusion caps the occupied channel count at seven and subadditivity gives $\Delta_k \geq 0$, hence the lightest resolved one-bit defect has $k = 7$ and $\Delta_7 = 0$ at fixed per-channel marginal. The unit mixing gap, charged survival gap, and coherent GFT Hessian remain distinct objects. The decorated transfer action fixes the first two and the marked edge Hessian. A geometric GFT calculation must still determine the condensate spectrum and prove that no additional light source-coupled mode appears.

The finite scale-setting dynamics is therefore specified end to end: renewal fixes the fresh marginal, the determinant fixes the baseline recurrence, the decorated vertex fixes the marked event and routing, positivity fixes the transfer gap, and the electron fixes the cadence. The complete edge-Hessian and graph-enumeration audits are finite and reproducible. Long-time history capacity, stable condensate embedding, and the first-principles inhomogeneous metric influence functional remain open.

The cell-order calculations show that the closure weight, refresh, history tilts, and conserved field do not supply vacuum curvature stiffness. They do couple geometry to persistent closure failure, and Section 24 measures that local response. The condensate branch still owes a microscopic kinetic term, while the simulations import their extended vacuum geometry from the CDT host. Postulate III conditions histories within that specified state and does not select it.

Part VII. The Substrate on a Dynamical Lattice

This part tests the cell ensemble numerically on a dynamical simplicial geometry, and then applies the resulting joint measure to the equilibrium vacuum and the cosmological term.

24. Lattice Tests: Compatibility, Defect Response, and Transport

The single-cell coefficient chain of Part II can be evaluated in closed form. Its many-cell consequences require numerical tests: survival of the vacuum ensemble on fluctuating geometry, local strain around a persistent closure failure, and transport toward the $1/r$ deficit of Section 12. The calculation places the unchanged cell ensemble on a dynamical simplicial geometry. Appendix J gives the ensemble definitions, controls, and measured tables.

24.1 The host geometry and the cell identification

The host is a causal-dynamical-triangulations ensemble: four-dimensional triangulations of $S^1 \times S^3$ weighted by the Regge action [74]. CDT is used because it independently sustains an extended four-dimensional de Sitter-like phase [75, 76]. That phase is an external host datum for the substrate test, not a state selected by Many-Pasts.

The identification is immediate, and it respects a separation the theory requires. Each spatial tetrahedron of a slice hosts one cell's boundary data: its four triangles carry the labels $m = -3, \ldots, 3$ of the $j_{\text{eff}} = 3$ sector of Section 5, each slice triangle is shared by exactly two cells, and the slice gluing supplies adjacency. Nothing else of the lattice enters. The theory's cell is its label configuration, a dimensionless unit of capacity; no size is ever assigned to it, and the granularity stays in the capacity, as Section 1.2 requires. The lattice simplices are regulator scaffolding, as they are throughout lattice gravity. The identification is nevertheless geometric in the sense that matters: the joint admissibility statistics of the labels depend on how the slice is glued, so the closure weighting can in principle tell one geometry from another, and that channel is the entire coupling between the theory and the host.

Two boundaries of the design are stated at the outset. First, the host supplies a dynamical, curvature-carrying simplicial geometry with the theory's cell structure, and nothing more is asked of it. The calculation does not assume the CDT ensemble is the theory's own vacuum, and it withholds any claim that the substrate itself derives four-dimensional emergence. At the tuned host couplings used below, increasing N_{41} from about 40,000 to 100,000 raises both the Hausdorff estimator and the concentration of volume in an extended region, as expected when moving farther into that finite-volume phase; the order of the visible phase boundary and the joint continuum limit remain open (Appendix J.1). Second, the weighting coupled to the host is the paper's own: the admissibility energy of Sections 5–6 at the closed point, with $\eta = \eta_*$ and injectivity enforced. It is not a spin-foam vertex amplitude. The distinction drawn in Section 29.8 was kept operational: a spin-foam amplitude was coupled first as a neighboring-theory control, and the failures exposed by that exercise supplied the control methodology used below (Appendix J.3).

24.2 The coupled ensemble and its controls

The coupled system is the joint Gibbs measure

$$\pi(g, m) \;\propto\; \exp\Big(- S_{\text{Regge}}(g) - \varepsilon\,(N_{41} - \bar{N})^2 - \beta \sum_{\text{cells } c} \big[E_c(m) - \mu\big]\Big),$$

$$E_c(m) = \eta_*\, K^2(m_c) + \lambda\, n_{\text{coll}}(m_c),$$

where the sum runs over the slice cells, K^2 is the closure invariant of Appendix B, n_{coll} counts label collisions (injectivity is imposed as a penalty whose hard limit is the constraint, with the residual collision fraction reported so the softness stays visible), ε pins the slice volume, and μ is the per-cell label free energy, computed by thermodynamic integration so that the closure term cannot masquerade as a shift of the bare cosmological coupling. The labels carry a uniform base measure, so at $\beta = 0$ the label entropy cancels exactly and the bare host is recovered identically. The physical weighting fixed by Part II is $\beta = 1$ at η_*: once that chain is accepted there is no coupling dial left free, and the intermediate β values serve only as a diagnostic interpolation.

Every run opens by recomputing the single-cell chain on its own tables and refuses to proceed unless $g_{\text{share,eff}} = 7.4198$ and $\langle K^2\rangle_{\eta_*} = 3/(2\eta_*) = 50.223$ are reproduced, so the object coupled to the lattice is verifiably the object counted in Part II. Every quoted triangulation is connected, simplicial, and correctly foliated, and comparisons use matched total volume at the same regulator. The principal control shuffles the 210 label-orbit energies while preserving their values and permutation symmetry. This destroys the closure structure without changing the energy histogram or sampling machinery, so only a difference from that shuffled control is attributed to closure. These validity conditions and the interpretation of each possible outcome were recorded before the corresponding data were examined (Appendix J.3).

24.3 What the closure sector cannot supply: vacuum stiffness

Three cell-order calculations bound the closure sector's contribution to vacuum stiffness. The static closure weight induces only about one percent of the bare geometric coupling. The selected memoryless kernel, modeled as cell-by-cell redraws, has a closed stationary state with vacuum admissibility 0.536 and no detectable curvature action. A separate closure-class history tilt, solved by a Doob transform on a ring, also remains short-ranged and well below the required coupling. These are calculations of specified effective models; they do not derive the refresh kernel from Postulate III.

Two further calculations extend the exclusion beyond the label sector, to the conserved capacity field that Section 24.7 introduces as the carrier of the long-range sector. Even a field the labels cannot see might rank geometries on its own, because integrating out a conserved Gaussian field induces a purely geometric action: $\frac{1}{2}\ln\det' L(g)$ per channel, the spanning-tree entropy of the slice graph by the matrix-tree theorem. Computed on degree-matched proxies for smooth extended and for crumpled geometry, and on real engine slices, that entropy is nearly universal at fixed coordination: the per-cell differential is a few times 10^{-4}, and with all seven channels the phase-tipping force is of order 10^{-3} against the bare $k_0 \simeq 2.2$. The theory's own non-Gaussianity closes the loophole tighter still. The finite budget of Postulate I gives the field a saturation mass $m^2 = 2/g_{\mathrm{share,eff}} \simeq 0.27$, and the massive determinant suppresses precisely the soft modes that carried the residual sensitivity, shrinking the differential by a further factor of three at the budget mass and toward zero beyond. What the field retains is a local renormalization of the host's couplings, of order 0.08 per cell in the seven-channel count: enough to relocate the host's phase boundaries, and unable to create the phase.

Within the tested models, static weight, refresh, history tilts, and conserved fields do not supply vacuum stiffness. The refresh does couple geometry to the density of closure failure, which the defect experiment measures. The same pass verifies the locality fracture: injectivity-preserving unit shifts split the 1680 states into 48 components of 35 states. This excludes that local realization of the full-entropy kernel; it does not derive the nonlocal physical operator.

24.4 The externally hosted vacuum and the role of conditioning

The exclusions show that the specified substrate does not dynamically select its vacuum geometry. Two questions called emergence of spacetime must therefore be separated. The first is kinematic encoding: the closure invariant K^2 measures the failure of the four oriented faces to close, adjacency carries proximity, and the marked-transfer dictionary of Section 13 converts entanglement increments into meters. The cells are dimensionless capacity units rather than sites of a preferred spatial grid. The second question is dynamical selection among crumpled, branched, and smooth extended geometries. Section 24.3 finds that none of the tested substrate mechanisms performs that ranking.

Many-Pasts cannot fill that dynamical gap. Appendix G normalizes the probabilities of alternative present records before conditioning, so $p(h \mid P)$ describes histories compatible with an already specified present and does not select which P occurs. In the simulations the smooth extended vacuum is supplied by the chosen Regge/CDT host phase as an external background datum. Many-Pasts may condition the compatible decoherent histories within that datum, but it neither ranks candidate geometries nor explains why the extended present is realized. The coefficients of Part II are computed on that host state, and the medium dresses it rather than generating it. A substrate derivation of the prior state or of a probability measure over alternative vacuum geometries remains open.

Regge/CDT is a useful external host because it supplies tetrahedral cells, dynamical curvature, no rigid preferred spatial grid, and a demonstrated extended four-dimensional phase [75, 76].

This choice is operational, but the completed fixed-regulator runs are not a vacuum derivation by the substrate. The host supplies the geometry on which the label theory is tested; the substrate then produces quantitative dressing of its couplings. Integrating out the labels gives $S_{\text{eff}} = S_{\text{host}} - \log Z_{\text{label}}$. Appendix J.6 reports the predicted extensive volume shift and its measured scaling family, including the independently predicted shuffled-control line. The corresponding curvature-sector shifts remain to be tested statistically. The hierarchy is explicit: cell-level non-closure reweights the glued tetrahedra, and integrating out the labels converts that reweighting into corrections to the host's volume- and curvature-sector couplings. This is presently a quantitative interface with an externally supplied vacuum geometry. Section 24.9 states the additional critical-surface test that would promote the interface to a continuum embedding.

24.5 Compatibility: the weighting on dynamical geometry

At $N_{41} \simeq 100{,}000$, three eighty-slice ensembles were evolved from the same tuned host with respectively zero closure coupling, the physical weighting $\beta = 1$, and $\beta = 1$ after shuffling the closure energies among label orbits (Appendix J.5). Their total-simplex counts agree within 0.6%, and every final triangulation remains connected, simplicial, and correctly foliated. The label sector orders exactly as the closure weight demands: the collision fraction falls from 0.654 at $\beta = 0$ (against the uniform-measure prediction $1 - 840/2401 = 0.650$) to 0.077 under the physical weighting, while the shuffled control stays high at 0.736. The Hausdorff values remain 3.55–3.59 and the volume profiles remain extended across the matched ensembles. Repeating the physical weighting with forty rather than eighty slices gives collision fraction 0.078 and $d_H = 3.74$. Thus the microscopic closure structure is strongly active without destabilizing the host geometry, and the shuffled comparison shows that the ordering follows the closure structure rather than the energy histogram or sampler.

24.6 The defect experiment: geometry responds to the theory's mass

The theory's matter is persistent closure failure: committed capacity, maintained against the refresh (Sections 3.2 and 23). The experiment inserts it by hand and repeats the comparison with two independent random seeds. In each repetition, one ensemble holds one hundred well-separated cells in maximal closure failure (all four faces at $m = 0$: six collisions, $K^2 = 48$), while a control ensemble holds the same number in the best-closed injective configuration ($m = \{0, 1, 2, 3\}$, the minimum $K^2 = 40.67$). Anchoring, protection from geometry moves, and every other update rule are identical. Every final triangulation remains connected and correctly foliated, and all one hundred marked cells survive in each ensemble. Subtracting the closed-cell control therefore isolates the physical effect of failure content. The final analysis pools the two seeds and uses across-seed scatter where replicated and integrated-autocorrelation-time-corrected standard errors otherwise.

The capacity response is measured with high statistical significance. At the first shell the mean closure energy is 1.6802 ± 0.0026 around failure pins and 1.7260 ± 0.0008 around closed pins, a difference -0.0458 at 16.8σ. The collision fraction changes from 0.0783 ± 0.0002 to 0.0698 ± 0.0009, a 9.2σ separation. These label observables largely return by the second shell, as the short closure correlation length requires. The geometric channel appears in a different observable and farther out: at the third shell the mean coordination is lower around failure pins by 0.0398, a 4.0σ separation from the closed-cell control after autocorrelation and seed scatter. Shell-cell counts do not separate significantly at this precision, and the first- and second-shell coordination differences are consistent with zero. The data establish a local geometric response, although several preview signals disappear under the completed error treatment.

Persistent closure failure measurably strains the surrounding capacity state and changes the local coordination field. The closed-cell control carries the same anchoring without the failure, isolating the failure content as the source. This finite-lattice result supplies the microscopic source-to-strain seed used by the weak-field sector. The defect-only experiment reaches three lattice steps and establishes neither continuum curvature nor a value of G. The separately conserved instrument of Section 24.7 tests transport beyond that short closure range.

24.7 Reaching Newtonian range: the conservation requirement

The strain of Section 24.6 dies within a lattice step, as the sub-cell correlation length requires. Macroscopic transport needs a different operator. If free capacity is only a per-cell budget, local maximum-entropy re-equilibration gives $(2z - L)\delta\mu = -m/\chi$ on the slice adjacency ($z = 4$ faces per cell, L the slice Laplacian). That operator has no small-momentum pole; on a real slice the response falls eight orders of magnitude within thirteen steps. A locally rebalanced budget therefore screens at the substrate scale.

Committed stock and maintenance throughput. Particle formation and persistence are different operations. Let $J^{\mu}_{c,A}$ carry the committed stock belonging to source A. Its balance law is

$$\nabla_{\mu} J^{\mu}_{c,A} = \Gamma_{\mathrm{form},A} - \Gamma_{\mathrm{rel},A}.$$

For an already-existing stable defect both rates vanish, so its mass-equivalent committed stock remains constant. The defect nevertheless draws a maintenance throughput

$$\mathcal{P}_{\mathrm{maint}} = \alpha_{\mathrm{maint}} \rho_{\mathrm{def}}.$$

This is the rate at which the surrounding renewal bandwidth services persistence; it is not continuing conversion into more matter. Spatial free-bandwidth flux enters the defect event, while the reversible update exports the previous replaceable closure state along an internal history edge. The history register is an output of the renewal circuit, not an ordinary spacetime fluid, so no four-current J^{μ}_{hist} is assumed. In free propagation that output carries no cell address of the persistent mark. This stock–throughput distinction removes the apparent secular growth of a static particle.

Augmented-graph Ward identity. The following result holds for the refresh-bandwidth completion. Form the directed renewal event graph whose edges are spatial bandwidth transport, ordinary temporal continuation, and the history output of a maintenance event. Let B be its incidence matrix and j_e the oriented edge throughput. In the absence of formation, release, or terminal absorption, each local reversible gate has one incoming active slot for every outgoing active or history slot. Its exact Kirchhoff identity is

$$\Delta_t n_v + \sum_e B_{ve} j_e = 0.$$

At a stable defect the history-edge current is $j^H_v = \mathcal{P}_{\mathrm{maint},v}$; it is an outflow from the present register, not an increase of committed stock. Summing over any region cancels every internal edge and leaves only boundary transport and the explicitly counted history outputs.

On a fixed spatial slice, a local linear constitutive law has the quadratic operator

$$L_f = B_s W B_s^T,$$

where B_s is the spatial incidence matrix and W contains positive face conductances. Since $B_s^T \mathbf{1} = 0$,

$$\boxed{L_f \mathbf{1} = 0.}$$

For a connected slice the constant mode is the only exact zero mode. Locality and spatial isotropy then give

$$\lambda_f(k) = D_q k^2 + O(k^4)$$

near $k = 0$; a nonzero constant term would violate the incidence identity and represents leakage. The steady equation on a compact slice is

$$L_f \mu_q = \alpha_{\text{maint}} \left(\rho_{\text{def}} - \overline{\rho}_{\text{def}}\right),$$

where the subtraction removes the constant mode. In an extended three-dimensional limit its pseudoinverse has

$$G_f(r) \sim \frac{1}{4\pi D_q r},$$

so a persistent maintenance demand produces the unscreened $1/r$ profile without tuning a scalar mass. Conservation proves the gapless transport pole within this completion. It does not prove that refresh bandwidth is the physical carrier, that defects couple only to it, or that the resulting scalar is the metric capacity coordinate.

The continuum junction condition. Let the renormalized static response be $\delta q = \chi_q \mu_q$, $J_q^i = -D_q \nabla^i \mu_q$, and $\mathcal{P}_{\text{maint}} = \alpha_{\text{maint}} \rho_{\text{def}}$. Away from the compact zero-mode subtraction,

$$\nabla^2 q = \frac{\chi_q \alpha_{\text{maint}}}{D_q} \rho_{\text{def}}.$$

The static bridge $q = 1 + 2\Phi/c^2$ and $\nabla^2 \Phi = 4\pi G_* \rho$ therefore require

$$\boxed{\frac{\chi_q \alpha_{\text{maint}}}{D_q} \longrightarrow \frac{8\pi G_*}{c^2}}$$

after renormalized-operator and lattice-spacing matching. The three quantities on the left are lattice outputs. The electron-derived G_* enters only after their dimensionless regulator dependence has been removed, so the continuum comparison is target-blind rather than a fit.

This transport experiment is performed on a separate forty-slice realization at $N_{41} \simeq 100{,}000$ (Appendix J.8). The field begins at the vacuum anchor 7.4198, moves only through antisymmetric face fluxes, and is recycled through the compact zero mode; its mean remains 7.4198 through all 4000 sweeps. Absorption reads the persistent excess failure above the measured vacuum dressing, so the source strength is produced by the label dynamics rather than assigned by the transport law. Five fixed source values, $m = 0, 1, 2, 3, 6$, are represented by thirty-two well-separated defects each. The $m = 0$ value measures commitment-independent boundary dressing; the positive values test the source and response laws independently. Within-pin series use Sokal integrated-autocorrelation corrections, and final errors include across-pin and across-level scatter.

Four distinct measurements give a consistent finite-lattice source-to-field chain. First, conservation is exact. Second, after subtracting the $m = 0$ dressing, the emergent maintenance charge is additive,

$$Q - Q(0) = (0.1440 \pm 0.0025)m, \qquad Q(0) = 0.0075.$$

Third, the shell-one deficit divided by that independently measured charge is constant across the positive source values,

$$\left.\frac{\Delta f_1 - \Delta f_1(0)}{Q - Q(0)}\right|_{m=1,2,3,6} = 4.9965 \pm 0.5244,$$

where the uncertainty is the larger of propagated error and across-level scatter. Sources of different strength therefore couple through one finite-lattice junction ratio. The dimensionful continuum coefficient $8\pi G_*/c^2$ still requires a continuum scaling and operator-matching calculation. Fourth, the dressing-subtracted profile remains positive through eight shells and fits

$$\Delta f(d) \propto d^{-0.62\pm0.11}.$$

The screening diagnostic gives $d\ln(\Delta f\, d)/dd = +0.1062 \pm 0.0551$, statistically consistent with zero and implying the finite-range bound $\xi > 4.6$ lattice steps. The exact Ward identity and the absence of detected screening therefore agree: the tested carrier is conservation-protected and long-ranged over the usable graph radius.

Two qualifications are visible in the same data. The proportional source fit has a maximum relative residual of 0.242, driven mainly by the $m = 6$ source falling below the extrapolation from weak sources; this is finite-capacity saturation, not precision linearity at arbitrary multiplicity. The exponent differs from the ideal three-dimensional massless Green-function value -1; the closed-slice zero mode and finite slice radius flatten the measured tail, but a larger-volume scaling study must determine whether the exponent tends to -1. These measurements establish the tested finite-lattice behavior while leaving the continuum scaling and absolute junction normalization open.

24.8 What the finite-lattice results establish

The fixed-lattice results establish four points. The admissibility sector orders on a dynamical host without destabilizing its matched geometry. Persistent closure failure produces a replicated local capacity response and an autocorrelation-corrected local coordination response. Conditional on refresh bandwidth as the carrier, the exact augmented-graph identity protects the long-range pole. The separate transport calculation then verifies conservation, approximately additive source strength, one lattice junction ratio across the tested source values, a power-law tail, and no detected screening. None of these mechanisms selects the vacuum geometry; CDT supplies that host state externally, while the substrate measurably dresses its N_{41} coupling direction. The exact Regge volume projection of that direction is given in Section 25. The remaining CDT questions are qualitatively different: whether the joint system has a continuous critical limit, how its lattice spacing and operators match to continuum observables, and whether the absolute junction coefficient tends to $8\pi G_*/c^2$. Section 26 records those distinct grades.

24.9 The capacity-decorated continuum target

The present runs compare coupled and control ensembles at fixed regulator. A continuum completion requires a stronger object: a critical trajectory of the joint geometry–capacity measure. For a causal triangulation T, let $Z_{\text{cap}}[T; \mathbf{g}_{\text{cap}}]$ be the trace over its closure labels, defect marks, renewal gates, source-labelled committed allocations, and history outputs. The joint partition function is

$$Z(\mathbf{g}) = \sum_{T\in\mathfrak{T}_{\text{CDT}}} \frac{1}{C_T} e^{-S_{\text{CDT}}[T;\kappa_0,\Delta,\kappa_4]} Z_{\text{cap}}[T; \mathbf{g}_{\text{cap}}],$$

and integrating out the capacity sector gives

$$S_{\text{eff}}[T] = S_{\text{CDT}}[T] - \ln Z_{\text{cap}}[T].$$

The theory fixes $\mathbf{g}_{\text{cap}}$ at its microscopic values. The regulator couplings κ_4, κ_0, and Δ are then used to locate infinite volume and a continuous geometric transition. CDT supplies evidence that continuous transition lines can occur and can support a conventional continuum limit [77, 78]. The pre-registered volume pair now confirms that the tuned point moves in the coherent extended direction as volume grows; it does not establish the order of the visible boundary or an intersection with a continuous critical line.

The additive vacuum term and the finite continuum volume coupling are distinct coordinates of this approach, but the volume normalization is not an adjustable unknown. Section 25 derives the exact Regge operator

$$\mathcal{V}_4 = a^4(v_{41}N_{41} + v_{32}N_{32})$$

and its corresponding source direction in the (κ_4, Δ) plane. On a fixed-ratio trajectory the scalar matching is $t_4 = \overline{v}_4 a^4 \rho_{\mathrm{vol,R}} + o(a^4)$, with $\overline{v}_4 = (v_{41} + \xi v_{32})/(1+\xi)$. The continuum calculation must therefore measure the limiting α, ξ, lattice scale, and mixed critical eigenoperator, rather than fit a free Z_V. Vacuum normalization removes the complete capacity-sector bulk coefficient at every fixed regulator. Consequently a finite source-independent capacity density cannot hide in t_4/a^4: it would already be an extensive term before the limit and would have been included in μ_{cap}. If the common-parent critical limit exists, Postulate I then excludes an additional independent host volume action and gives $\Lambda_{\mathrm{R}}^{\text{empty equilibrium}} = 0$. The currently external CDT host does not satisfy that antecedent by assumption, so its own cosmological coupling remains a regulator input rather than a result of these runs.

Let $\mathfrak{C}_*$ denote the critical manifold on which the geometric correlation length diverges in lattice units after the cosmological coupling is tuned to infinite volume. The distinct symbol keeps this geometric locus separate from the coherence observable $\mathcal{C}$. The physical capacity completion exists only if the fixed capacity slice intersects that manifold,

$$\boxed{\mathcal{P}_{\mathrm{cap}} \cap \mathfrak{C}_* \neq \varnothing, \qquad \frac{\xi_{\mathrm{geom}}}{a} \longrightarrow \infty.}$$

This requirement introduces no capacity fit: failure of the fixed slice to intersect $\mathfrak{C}_*$ would reject this CDT embedding. Geometric criticality removes the ultraviolet regulator. The augmented-current Ward identity has a different job: it protects the infrared transport pole. The two limits need not be the same phase transition.

Appendix J.9 states the finite-size-scaling, operator-mixing, current, and junction tests. A successful capacity-decorated CDT transfer matrix would provide one complete geometric embedding of the finite marked vertex. Until those tests are passed, CDT remains the external host used by the simulations, while the stable GFT condensate remains an alternative microscopic completion.

25. Equilibrium Vacuum and Cosmological Term

The normalization $q = 1$ defines the unstrained vacuum reference. The regulated joint ensemble makes the corresponding subtraction exact, rather than leaving it as an unspecified cancellation. At fixed triangulation let

$$Z_{\mathrm{cap}}[T] = \mathbb{E}_{m\sim\mathrm{unif}} \exp\left[-\beta \sum_c E_c(m)\right]$$

and define its homogeneous bulk free energy per cell by

$$\mu_{\mathrm{cap}}(a,\beta) = -\lim_{N_{\mathrm{cell}}\to\infty} \frac{1}{\beta N_{\mathrm{cell}}} \ln Z_{\mathrm{cap}}^{\mathrm{vac}}(a, N_{\mathrm{cell}}).$$

Its dependence on the homogeneous regulator couplings is suppressed in the notation; the definition is applied along the trajectory whose continuum limit is being tested. The vacuum-normalized capacity factor is

$$\widehat{Z}_{\mathrm{cap}}[T] := e^{\beta\mu_{\mathrm{cap}}(a,\beta)N_{\mathrm{cell}}(T)} Z_{\mathrm{cap}}[T].$$

By construction,

$$\boxed{-\lim_{N_{\rm cell}\to\infty}\frac{1}{\beta N_{\rm cell}}\ln\widehat{Z}^{\rm vac}_{\rm cap}=0.}$$

This is the lattice grand-potential statement. In the thermodynamic limit $F_{\rm cap}=\mu_{\rm cap}(a,\beta)N_{\rm cell}+o(N_{\rm cell})$, so the centered bulk potential

$$\Omega_{\rm cap}=F_{\rm cap}-\mu_{\rm cap}(a,\beta)N_{\rm cell}$$

has zero density in the homogeneous reference state. The cancellation is invariant under an arbitrary change of microscopic energy origin. If

$$E_c\mapsto E_c+C,\qquad \mu_{\rm cap}\mapsto\mu_{\rm cap}+C,$$

then

$$\boxed{\widehat{Z}_{\rm cap}[T]\mapsto\widehat{Z}_{\rm cap}[T]}$$

exactly. When the bulk cell count admits the usual thermodynamic reading, Gibbs–Duhem writes the same relation as $\Omega_{\rm cap}/V=-P$; an isolated equilibrium reference has $P=0$ [73]. The partition identity above is stronger for the regulated model because it does not require the conditional refresh current to serve as that thermodynamic charge. A large homogeneous source-independent zero-point term therefore cannot reappear as a capacity-sector observable; it cancels against the uniquely shifted bulk free energy, while defect and geometry-dependent differences remain.

Integrating out the normalized capacity sector gives

$$S_{\rm eff}[T]=S_{\rm CDT}[T]-\ln\widehat{Z}_{\rm cap}[T].$$

The homogeneous extensive capacity term is absent from this action. Only the geometry- and defect-dependent free-energy difference remains to source strain, as required by the information–geometry and mass–entropy postulates. This result does not use the conditional refresh-current Ward identity. Transport conservation protects the infrared pole; vacuum normalization follows from the equilibrium partition measure. They are different conserved structures.

Appendix J has already tested the finite-regulator algebra. Omitting the centering restores the term $+\beta\mu_{\rm cap}N_{\rm cell}=+(\beta\mu_{\rm cap}/2)N_{41}$ in the geometric action. Against the quadratic volume pin it predicts

$$\Delta N_{41}=-\frac{\beta\mu_{\rm cap}}{4\varepsilon},$$

and the measured uncentered-minus-centered displacement is -62.8 against -62.2. The run does not prove a continuum cosmology, but it verifies the coefficient and sign of the subtracted microscopic bulk term. One bookkeeping refinement matters here. In the standard CDT action basis,

$$S_{\rm CDT}=-(\kappa_0+6\Delta)N_0+\kappa_4(N_{41}+N_{32})+\Delta(2N_{41}+N_{32}),$$

the restored N_{41} term is the coupling displacement $\delta\Delta=+\beta\mu_{\rm cap}/2$, $\delta\kappa_4=-\beta\mu_{\rm cap}/2$, not a pure shift of κ_4. At fixed simplex ratio it contains the expected volume chemical potential; its orthogonal component renormalizes the regulator asymmetry. The measured displacement therefore verifies the predicted N_{41} direction, which is the exact statement licensed by the control.

The same conclusion is independent of the centering convention. The leading exponential growth of the decorated canonical sum defines a critical surface in the full (κ_4,Δ) plane. An extensive change of microscopic energy origin translates the bare coupling vector and that critical surface by the same displacement displayed above. Their normal difference is invariant, and it vanishes at the infinite-volume surface. On a trajectory that holds Δ fixed after the other relevant directions have been projected out, this normal coordinate is the familiar

$$t_4 := \kappa_4 - \kappa_4^c \longrightarrow 0.$$

Thus critical tuning absorbs the additive bulk term without turning an energy-origin choice into an observable. The remaining question is the normalization of the physical volume direction, to which we now turn.

The continuum volume matching can now be made explicit instead of being hidden in an unspecified Z_V. After Wick rotation, the exact Regge four-volumes of the two simplex types are [76]

$$V_{41} = a^4 v_{41}(\alpha), \qquad v_{41}(\alpha) = \frac{\sqrt{8\alpha - 3}}{96},$$
$$V_{32} = a^4 v_{32}(\alpha), \qquad v_{32}(\alpha) = \frac{\sqrt{12\alpha - 7}}{96},$$

with $\alpha > 7/12$ so both Euclidean simplex types are nondegenerate. Here a is the spatial regulator edge length; it is not the substrate length L_*. The integrated volume operator is therefore the kinematic identity

$$\boxed{\mathcal{V}_4[T] = a^4 \big[v_{41}(\alpha) N_{41} + v_{32}(\alpha) N_{32}\big].}$$

Let $\rho_{\mathrm{vol,R}}$ denote the coefficient of $\mathcal{V}_4$ in the dimensionless Euclidean effective action. If G_{R} is the continuum Newton coefficient and $L_G^2 := \hbar G_{\mathrm{R}}/c^3$, then

$$\rho_{\mathrm{vol,R}} = \frac{c^3 \Lambda_{\mathrm{R}}}{8\pi\hbar G_{\mathrm{R}}} = \frac{\Lambda_{\mathrm{R}}}{8\pi L_G^2}.$$

On the matched weak-field branch $G_{\mathrm{R}} = G_*$ and therefore $L_G = L_*$. Keeping L_G visible until that junction passes prevents the volume matching from assuming the gravitational result it is meant to join. The two action-conjugate volume couplings are therefore

$$\boxed{t_{41} = a^4 v_{41}(\alpha)\rho_{\mathrm{vol,R}}, \qquad t_{32} = a^4 v_{32}(\alpha)\rho_{\mathrm{vol,R}}.}$$

Equivalently, in the (κ_4, Δ) basis a pure volume-source displacement obeys

$$\boxed{\begin{pmatrix} \delta\kappa_4 \\ \delta\Delta \end{pmatrix} = a^4 \rho_{\mathrm{vol,R}} \begin{pmatrix} 2v_{32} - v_{41} \\ v_{41} - v_{32} \end{pmatrix}.}$$

Substitution returns $\delta(\kappa_4 + 2\Delta) = a^4 v_{41}\rho_{\mathrm{vol,R}}$ and $\delta(\kappa_4 + \Delta) = a^4 v_{32}\rho_{\mathrm{vol,R}}$, so this is an operator identity, not a fitted matching. At the isotropic point $\alpha = 1$, $v_{41} = v_{32} = \sqrt{5}/96$ and the Δ component vanishes, providing a direct check.

The earlier one-parameter formula is the fixed-ratio compression of this two-operator statement. If $\xi = N_{32}/N_{41}$ and $N_4 = N_{41} + N_{32}$, then

$$\overline{v}_4(\alpha, \xi) = \frac{v_{41}(\alpha) + \xi v_{32}(\alpha)}{1 + \xi}, \qquad t_4 = \overline{v}_4(\alpha, \xi) a^4 \rho_{\mathrm{vol,R}} + o(a^4).$$

Thus the geometric part of the previously unnamed factor is fixed:

$$\boxed{Z_V^{\mathrm{geom}}(a) = \overline{v}_4\big(\alpha(a), \xi(a)\big).}$$

The critical calculation still has to show that $\alpha(a)$ and $\xi(a)$ approach finite limits, set a by a target-blind observable, and project out any mixed relevant eigenoperator. Those are continuum-existence and scale-setting tests; they are no longer freedom to choose the volume normalization.

This matching never identifies a CDT regulator simplex with the physical capacity cell. In the $x^0 = ct$ convention one native cell has four-volume L_*^4. The average number of regulator simplices in such a block is

$$n_*(a) = \frac{L_*^4}{a^4 \overline{v}_4},$$

and its volume coupling is

$$\boxed{t_* := n_*(a)t_4 = \rho_{\mathrm{vol,R}} L_*^4 = \frac{\Lambda_{\mathrm{R}} L_*^4}{8\pi L_G^2} \xrightarrow{G_{\mathrm{R}}=G_*} \frac{\Lambda_{\mathrm{R}} L_*^2}{8\pi}.}$$

The cutoff and the simplex factor cancel. CDT can therefore supply arbitrarily fine scaffolding while L_* remains the finite physical capacity scale, as required by Section 24.1.

There is then no surviving capacity-sector 0/0 ambiguity. The definition of $\mu_{\mathrm{cap}}(a, \beta)$ removes the complete thermodynamic bulk coefficient at every fixed regulator, not merely the part that diverges as $a \to 0$. If a nonzero finite $\rho_{\mathrm{cap,R}}$ remained, then at any fixed a the normalized vacuum action would contain

$$\rho_{\mathrm{cap,R}} a^4 \big[v_{41} N_{41} + v_{32} N_{32}\big],$$

which is still linear in the number of cells and therefore contradicts the defining zero bulk density of $\widehat{Z}^{\mathrm{vac}}_{\mathrm{cap}}$. Hence, on every regulator and on every continuum subsequence for which the volume operator has a limit,

$$\boxed{\rho^{(\mathrm{eq})}_{\mathrm{cap,R}} = 0, \qquad \Lambda^{(\mathrm{eq})}_{\mathrm{cap,R}} = 0.}$$

The complete equilibrium bulk term must be normalized before the regulator is removed. This order avoids inferring a continuum density from the ratio of two quantities that separately vanish.

What about an independent host cosmological term? In the present simulations it remains possible because CDT is deliberately an external scaffold: $\widehat{Z}_{\mathrm{cap}}[T]$ normalizes the capacity sector at fixed T, not the externally supplied host measure. In a completed theory, however, Postulate I says that geometry and capacity are the same substrate, so the vacuum normalization must apply to the bulk coefficient of the *joint* geometry–capacity measure. That joint coefficient is the leading exponential growth removed when the cosmological coupling is placed on its infinite-volume critical surface. Retaining another source-independent host volume action after this common-parent normalization would split geometry back off from capacity and undo the already-used foundational identification. Writing $\mathfrak{C}_*$ for the common-parent critical manifold defined in Section 24.9, the continuum statement is consequently the conditional theorem

$$\boxed{\big(\mathcal{P}_{\mathrm{cap}} \cap \mathfrak{C}_* \neq \varnothing\big) \text{ and Postulate I} \quad \Longrightarrow \quad \Lambda_{\mathrm{R}}^{\text{empty equilibrium}} = 0.}$$

This adds no new premise. It applies Postulate I to the exact vacuum normalization once a common-parent continuum limit exists. The existence of that decorated critical limit remains to be demonstrated empirically and mathematically, so the currently externally hosted lattice calculation does not yet license an unconditional claim about the observed Universe. Nor does equilibrium cancellation imply a matter-tracking residual, a relaxation law, evolving dark energy, or $w \neq -1$. Any observed homogeneous acceleration must come from a state-dependent, boundary, or nonequilibrium sector with covariantly conserved stress energy, or else falsify the common-substrate equilibrium completion; its equation of state must be derived separately.

A controlled nonequilibrium target. The external results also identify the correct next object more tightly than the phrase "state-dependent energy" alone. Let $C_0 > 0$ be the covariance of a regulated Gaussian capacity block in the centered equilibrium state and $C > 0$ the covariance of another state on the same support. The dimensionless relative covariance is

$$\mathcal{G} = C_0^{-1/2} C C_0^{-1/2}.$$

The Kullback–Leibler divergence between the corresponding centered Gaussian measures is the basis-independent identity

$$\boxed{\Gamma_{\mathrm{rel}}(C\|C_0) = D_{\mathrm{KL}}(\mathcal{N}(0,C)\|\mathcal{N}(0,C_0)) = \frac{1}{2}\operatorname{Tr}\left[\mathcal{G} - I - \ln \mathcal{G}\right].}$$

If $\lambda_i > 0$ are the eigenvalues of $\mathcal{G}$, each contribution $\lambda_i - 1 - \ln \lambda_i$ is nonnegative and vanishes only at $\lambda_i = 1$. Thus

$$\Gamma_{\mathrm{rel}} \geq 0, \qquad \Gamma_{\mathrm{rel}} = 0 \Longleftrightarrow C = C_0,$$

and for $\mathcal{G} = I + X$,

$$\Gamma_{\mathrm{rel}} = \frac{1}{4}\operatorname{Tr} X^2 + O(X^3).$$

Appendix H.10 derives this functional rather than merely recognizing it: at finite regulator it is the quadratic-source Legendre transform of the equilibrium-normalized determinant of any positive dressed/reference Gaussian Hessian pair on a common physical support. The trace-log and positive mismatch forms are therefore closed as functional identities. The substrate must still derive the physical operator, its support and trace multiplicity, its dynamics and dimensions, and the conserved covariant response used in cosmology. This introduces no new premise or fitted potential: it is the exact dimensionless relative-information functional of any positive Gaussian fluctuation sector. Multiplication by the appropriate thermodynamic scale turns it into a relative free energy. Bianconi independently promotes the same operator-convex form to a covariant geometric-relative-entropy theory,

$$\Lambda_G = \frac{1}{2\beta}\operatorname{Tr}\left[G - I - \ln G\right],$$

and obtains a low-energy Einstein limit and a local thermodynamic first law [24]. The result therefore supplies a well-motivated target class for the homogeneous influence functional: equilibrium has zero relative energy, while a distinguishable state can carry a positive mismatch energy without restoring a source-independent vacuum constant.

The missing cosmological action must still derive the relevant covariance or response operator, its normalization and dimensions, its time evolution, and a conserved metric stress tensor. Those results must determine $w(z)$. Bianconi's G-field has its own derivative terms in the modified field equations, so it cannot be identified with q, varied as a new scalar, or imported wholesale without contradicting the metric-only decision of Section 11. Relative information nevertheless supplies a consistent local action, energy, and thermodynamic language for departures from the maximum-information vacuum. The microscopic theory here must still determine which departure the Universe occupies.

Part VIII. Closure Status, Falsifiability, and Comparisons

26. Closure-Status Table

The closure bookkeeping is concentrated here in one place; the rest of the text states results and refers here for their status.

The leading word in each status uses a fixed vocabulary. *Closed* means derived within the stated postulates and ensemble. *Fixed* means no phenomenological freedom remains once the named branch is adopted. *Conditional* means the result follows if a named reading or completion holds. *Frontier* or *open* means the piece is structured but incomplete. *Empirical support* denotes comparison with data rather than derivation, and *audit task* marks an independent check still required. A conditional premise does not demote every theorem downstream of it. Rows therefore separate exact identities and within-model lemmas from the microscopic or empirical status of their premises.

Quantity / Claim	**Sector**	**Status**	**Type of Support**	**Where Established**
Ω_{tet}, $g_{\text{share,max}}$	UV counting	Closed	exact combinatorics	Part II, App. B
η_*	admissibility closure	Closed	stationary normalized closure evidence $\ln Z + \frac{3}{2}\ln\eta$ maximized on the exact K^2 spectrum; the 3/2 is the determinant weight of the three closure-defect components	Part II, App. B
$g_{\text{share,eff}}$	UV entropy	Closed	exact weighted evaluation	Part II, App. B
Memoryless refresh kernel	information / scale theorem	Closed under the foundational faithful full-support condition	applying that same condition to histories gives maximum path entropy, not a new premise: $H(B_{t+1} \mid B_t) = g_{\text{share,eff}} - I(B_t; B_{t+1})$ is maximal iff $I = 0$, uniquely giving $K(b, b') = p_{\eta_*}(b')$	Part I, App. D, G–H
Renewal / charged-position coherence	quantum-channel interface	Exact channel and record-overlap theorems; microscopic phase dynamics open	$\mathcal{E}_*^\dagger(O) = \text{Tr}(\rho_* O)I$ makes the renewed output coherence-blind, while a discarded record multiplies a path off-diagonal by $\gamma_{xy} = \langle e_y \vert e_x\rangle$; cell-addressed marked records are excluded, and the free vertex must retain the marked fiber coherently with branch-independent renewal output; this is a consequence of the imported quantum sector and interferometry, not a new postulate	§3.3, §22, App. H.11
Decorated charged marked-transfer vertex	UV dynamics	Closed as a finite transfer action; stable geometric embedding open	Gaussian closure amplitude fixes $\sqrt{\eta_*}$; two directed singlet returns give $u = 8\eta_*/49$; canonical dilation gives $\sqrt{1-u}$; present/history response strands give nine orthonormal marked states; complete Hessian and 56,800-state graph audits pass; the measured G selects the routing integer uniquely over $\vert n\vert \leq 200$ (§13.5)	§13, §23, §24.9, App. H, J.9, K
Gaussian GFT–geometric-relative-entropy bridge	UV interface	Exact functional theorem; physical embedding open	for positive dressed and equilibrium GFT Hessians on a common physical support, the normalized determinant gives $2\ln \widehat{Z}_{\text{G}} = -\,\text{Tr}'\ln\Theta$ and its quadratic-source Legendre transform gives $\frac{1}{2}\,\text{Tr}'(\mathcal{G} - I - \ln\mathcal{G})$; the marked record sector preserves bosonic support on its established branch and its internal fiber is $V_0 \oplus V_1 \oplus V_2$; one specified geometric gluing tensor must still pass the condensate/spectrum, covariant assembly/normalization, and transfer-positivity audits	§3, §25, App. H.10, N.6–N.8
Substrate length L_* (marked-transfer map)	scale setting	Fixed within the decorated transfer action and electron anchor	state-weighted determinant gives $r = e^{-7g_{\text{share,eff}}}$; the marked vertex gives $Z_e = (1+\zeta_*)(1+7\zeta_*^2)$; positivity and the electron anchor give $L_* = -(3/2)Z_e\lambda_e \ln(1-r)$	Part I, App. D, H
J_{bare}, $J_{\text{eff}}^{\text{tree}}$	UV edge kernel	Closed	tetrahedral isotropy identity	Part II, App. C
$\Sigma_{\text{ret}} = 65/9$	finite-loop UV	Conditional minimal return-sector completion	seven diagonal return channels plus one projected singlet are specified; an explicit microscopic return operator must derive their relative weights and exclude further motifs	Part II, App. C

Quantity / Claim	Sector	Status	Type of Support	Where Established
$J_{\mathrm{eff}}^{(\mathrm{ren})}$	finite-loop UV	Conditional on the minimal return operator	algebraic Dyson resummation once Σ_{ret} is specified	Part II, App. C
γ	continuum stiffness	Conditional loop-dressed value	Euclidean normalization is exact for the one-sublattice edge convention; the numerical value inherits the minimal return operator	Part II, App. C
Green-matched source projection	UV source map	Closed in the canonical weak-field branch	exact defect counting + tetrahedral on-site Green function	App. C
κ/γ	source-to-stiffness ratio	Closed in the canonical weak-field branch	$\sigma_{\mathrm{def}} = \rho/\kappa_m(L_*)$ plus $G_{\mathrm{tet}}(0)$ and tetrahedral 4/3 projection	Part III, App. C–D
Weak-field action / bridge law	EFT / gravity	Closed for the ordinary longitudinal branch	Einstein–Hilbert + GHY reduced in Newtonian gauge gives I_{Newton}; $\delta S = -2S_\infty \Phi/c^2$ gives $I_{\mathrm{cap}}^{\mathrm{static}} = Z_S I_{\mathrm{Newton}}$ with no extra scalar	Part III, App. D, N
G_*	gravitational scale	Parameter-free output of the decorated scale branch; historical postdiction	$G_* = (9/4)(\hbar c/m_e^2) Z_e^2 \ln^2(1 - e^{-7g_{\mathrm{share,eff}}}) = 6.6742890772 \times 10^{-11}$, or -0.073σ relative to CODATA; the residual was known before the vertex was constructed	Part I, App. D, H, L
Matched G	weak-field gravity	Algebraic identity in the chosen normalization; not an independent determination	substituting the closed source map and S_∞^{cell} into $G = c^2\kappa/(8\pi\gamma S_\infty)$ gives identically $G = c^3 L_*^2/\hbar = G_*$	Part III, App. C–D
Electron anchor	mass / length sector	Fixed empirical elementary anchor	one-bit fermionic defect supplies λ_e for length setting and $m_e/\ln 2$ for mass–entropy map	Part III, App. D
Seven-sector additivity and lightest branch	UV scale theorem	Closed entropy theorem; exact conditional mass minimum	subadditivity gives $H_k = kg_{\mathrm{share,eff}} - \Delta_k$ with $\Delta_k \geq 0$; at fixed admissibility marginals, the adopted recurrence mass $m_k \propto e^{\Delta_k - kg_{\mathrm{share,eff}}}$ is minimized uniquely at the fermionic ceiling $k = 7$ and $\Delta_7 = 0$	App. D, H
a_0	galactic EFT	Exact conditional consequence; loading and horizon coupling open	the canonical transverse doublet gives two action–angle pairs and hence angular Haar volume $(2\pi)^2$ for one quantum action cell; $a_0 = [g_{\mathrm{share,eff}}/(2\pi)^2]cH_0$ follows after loading one sharing entropy into that cell and coupling it reversibly to the horizon bath	Part IV, App. N
Rank-one transverse response	galactic EFT	Closed within the one-invariant leading EFT	$C_\times^2 = A_L A_T$, $\delta R_*/\delta X = -C_\times/A_T$, and $C_\times/A_T = \omega_L/\omega_T$ follow exactly; tetrahedral symmetry gives $Z_L = Z_T$ at leading derivative order	Part IV, App. N
RAR law	galactic EFT	Exact consequence of a conditional leading completion	thermal matching reduces to $\hbar\omega_T = k_B T_H$ in the clamped-oscillator reading; A_T must remain environment-independent, and the influence functional must use the clamped response and yield $n_B(x)$ without an extra spectral factor	Part IV, App. N
Baseline no slip / lensing	weak-field metric	Closed for the ordinary longitudinal branch	varying the unreduced Einstein scalar constraint gives $\nabla^2(\Phi - \Psi) = 0$ and hence $\Phi = \Psi$ under asymptotic flatness	Part III, App. D, N
Transverse-sector lensing	galactic metric response	Open — principal action task	RAR fixes the radial temporal response only; $\Gamma_\perp[g_+, g_-]$ must derive $\Delta\Psi_\perp$, the lensing kernel, and the covariant Ward identity	Part IV, App. N

Quantity / Claim	Sector	Status	Type of Support	Where Established
PPN leading values	weak-field metric	Closed for baseline; transverse decoupling open	Einstein parent gives $\gamma_{\rm PPN} = \beta_{\rm PPN} = 1$ and no extra longitudinal mode; the transverse influence kernel must be shown negligible and free of preferred-frame effects at high acceleration	Part III, App. F, N
Telegrapher relation $D/\tau_0 = c^2$	transport	Closed in canonical transport branch	causal closure	Part V, App. E
Canonical $\tau_0^{-1} = H_0$ branch	transport	Fixed in the minimal transport closure	no-new-IR-scale choice	Part V, App. E
Hubble-tension mechanism	cosmology	Structurally supported extension	homogeneous trace-coupled mode	Part V, App. E
Equilibrium capacity-vacuum subtraction	cosmological constant / lattice dynamics	Exact regulator identity; conditional continuum theorem	the vacuum-normalized factor $\widehat{Z}_{\rm cap} = e^{\beta\mu_{\rm cap}N_{\rm cell}} Z_{\rm cap}$ removes the complete capacity-sector bulk coefficient at every regulator and is energy-origin invariant; the uncentered control verifies the predicted N_{41} coupling displacement; the exact two-simplex Regge volume fixes $Z_V^{\rm geom} = \overline{v}_4$ and, on the matched $G_{\rm R} = G_*$ branch, gives $t_* = \Lambda_{\rm R} L_*^2/(8\pi)$ without setting $a = L_*$; if the decorated common-parent critical limit exists, Postulate I excludes an independent host volume term and implies $\Lambda_{\rm R}^{\text{empty equilibrium}} = 0$; continuum existence and dark-energy dynamics remain open	§25, §24.9, App. J.2, J.6, J.9
Saturated-phase committed dust	cosmology	Conditional transition; normalization and dust theorem closed within the pinned coherent branch	source-labelled saturation is $\sigma_A = 1$ while absolute availability remains nonzero; one coherent phase advancing per electron-calibrated tick gives $X = 1/2$ rather than adding it separately; the constraint action then gives $p = 0$, $c_s^2 = 0$, and $\rho \propto a^{-3}$; recruitment and coherence onset remain open	Part V, §20, App. M
Committed-capacity abundance $\Omega_c/\Omega_b = 1/\epsilon$	cosmology	Conditional on the transverse normalization, pinned reading, and per-defect bookkeeping	$\sigma_A = \epsilon M_{c,A}/M_A$ is defined on each Lagrangian source label; over-demanded sources are assumed to reach $\sigma_A = 1$ and allocations add, giving $\rho_c = \rho_b/\epsilon$; the recruitment dynamics, epoch, and joint Boltzmann evolution remain open	Part V, §20
$a_0(\Omega_c/\Omega_b) = cH_0$	cross-sector structure	Exact conditional identity independent of the cell value	multiplication cancels ϵ; current central values give $a_0(\Omega_c/\Omega_b)/(cH_0) = 0.983 \pm 0.022$; the acceleration loading and committed-abundance premises remain separate	Part IV, Part V, §20
Post-fixation tests of ϵ	empirical support	Supported across heterogeneous tests; no combined significance assigned	a_0 is $+2.6\%$ from the local RAR scale; Ω_c/Ω_b agrees at 0.7σ; all twenty-four X-COP inversions obey the capacity ceiling; two redshift samples report evolution of a_0 in the predicted direction; these comparisons share theory premises and are not statistically independent	§15, §18, §20, §27.3
Caustic release / conditional conservation	cosmology	Conditional under the synchrony reading	committed dust is proposed to decommit at first shell-crossing of the irrotational clock flow; splashback-rim falsifier; conversion bookkeeping open	Part V, §20
Diffuse source projection $\epsilon = g_{\rm share,eff}/4\pi^2$	cluster sector	Conditional, with no additional coefficient	inherits the compact two-phase transverse normalization used for a_0/a_H; the cluster extension adds no further dial	Part V, §18

Quantity / Claim	Sector	Status	Type of Support	Where Established
Hot-atmosphere factor $B_{\rm bath}$	cluster sector	Operationally closed; microscopic origin open	$M_{\rm hot,vir}/(f_{b,\cos}M_{500})$ from X-ray/SZ; no per-system fitted knob	Part V, §18
Cluster residual $\mathcal{R}_{\rm rel} = 1 + (W_{\rm bath} - 1)f_{\rm cont}$	cluster sector	Hook morphology and trend direction supported; linear lift candidate excluded; lift function open	linear candidate's ceiling 1.81 lies below measured peak residual 3–5 [4, 16, 17]; capacity bound fixes saturation at $1/\epsilon$, giving $\mathcal{R}_{\rm sat} \simeq 4.7$–$4.9$ and a radius-resolved peak $\simeq 3.9$ inside the measured band; the bound holds against all twenty-four X-COP source-weight inversions, which also exclude the deep power-law continuation within that sample; coherence-growth profile open	Part V, §18
Relaxed vs. merger source expressions	cluster sector	Conditional — two regimes, one coefficient	residual rides on virialized bath; Bullet on shocked gas + decoupled clumps; regime boundary not yet derived	Part V, §18
Channel-selection rule (suppression + collective lift)	cluster sector	Conditional — suppression open; linear lift excluded	transverse-suppression premise (participation $\Rightarrow$ weight ϵ) not yet derived from the source map; linear form of the lift excluded by the measured peak residual; saturation endpoint fixed at $1/\epsilon$ by the channel capacity bound; coherence-growth profile between the endpoints not yet derived	Part V, §18
Bullet gas/lensing inversion	cluster sector	Structurally supported; consistent but untested (ϵ not yet measured)	ϵ brings gas/galaxy to near-parity, compactness completes the inversion; existing flexible reconstructions non-discriminating because gas weight is halo-degenerate	Part V, §18
Resolved cluster lensing-map test	cluster sector	Open — principal empirical task	three-component $\kappa \propto (1+(1-\epsilon)B_{\rm bath})\Sigma_{\rm bath}+\epsilon\Sigma_{\rm shock}+\Sigma_{\rm dec}$; predicted best-fit $\epsilon \simeq 0.19$, floated on baryonic maps with free dark haloes disallowed; not yet performed	Part V, §18
Bounded capacity q, $N^2 = q$	strong field	Fixed static constitutive rule; covariant generalization open	multiplicative composition and weak-field matching fix the static lapse map; a generic lapse is foliation dependent	Part VI, App. F, N
Former ADM multiplier action	strong field	Excluded as a parent completion	with no independent q dynamics, variation gives $\lambda = 0$ and only renames the foliation-dependent lapse; adding dynamics introduces an extra mode	Part VI, App. F, N
Spherical parent reduction	strong field	Closed in the metric-only branch	exact two-dimensional reduction gives $q_{\rm geo} = (\nabla R)^2 = 1 - 2GM_{\rm MS}/(c^2R)$ as a composite first integral and returns Schwarzschild in static vacuum	Part VI, App. F, N
Capacity-exhaustion horizon $q = 0$	strong field	Geometric zero closed in spherical symmetry; domain termination conditional	$q_{\rm geo} = 0$ is the marginal sphere; identifying it with substrate saturation and excluding $q_{\rm geo} < 0$ requires the bounded-domain postulate and boundary theory	Part VI, App. F, N
Hawking temperature and exterior ringdown	strong field	GR-matching branch closed; capacity boundary condition open	Schwarzschild exterior fixes the wave operator and Euclidean temperature; $\mathcal{R} = 0$ follows if future-horizon regularity is retained, while $\Gamma_{\partial q}$ must determine the physical substrate reflectivity	App. F
Bekenstein–Hawking area-law bridge	strong field	Exact normalization identity within the cell-normalized horizon convention; channel-to-area count open	per-channel cut entropy $\ln 2$ from fermionic face exclusion + channel density from $G_\perp = (2/3)G_{\rm tet}(0)$; product $n_{\rm hor}S_\infty^{\rm cell} = 1/4$ as exact identity	App. C, F

Quantity / Claim	Sector	Status	Type of Support	Where Established
Horizon formation and boundary microphysics	strong field	Geometric marginal-surface formation closed; saturation dynamics open	GR collapse can form $q_{\mathrm{geo}} = 0$; a bounded causal q_{cap} evolution, its equality to q_{geo}, the boundary action, and relaxation spectrum remain to be derived	Part VI, App. F
Rotating / charged stationary exteriors	strong field	Baseline metric solutions closed; capacity map open	Einstein / Einstein–Maxwell gives Kerr / Reissner–Nordström / Kerr–Newman, but no nonspherical covariant capacity scalar or domain rule has yet been derived	App. F
Many-Pasts Born compatibility	quantum foundations	Operationally complete; measure unique under stated quantum-record hypotheses; kinematics imported	Gleason fixes every normalized noncontextual additive measure on a sufficiently rich record-projector lattice in dimension > 2; generalized records transfer that measure to medium-decoherent pure-state histories; Hilbert-space kinematics, state, record completeness, and the substrate origin of the decoherence functional remain premises	Part VI, App. G
No-signaling in operational branch	quantum foundations	Exact within the operational branch	summing a remote quantum instrument gives a trace-preserving map, leaving the local marginal independent of the remote setting	Part VI, App. G
Record-retention reading of Many-Pasts	quantum foundations	Closed as an ontological consequence of Postulate III and the imported record measure	the realized present includes its physical records; erasing fine record distinctions pushes the joint measure forward by ordinary coarse-graining; $p(h \mid P)$ is formed after the joint dynamics and supplies no future-to-past force	§3.3, §22, App. G.6
Arrow-of-time account	quantum foundations	Open conditional extension	requires a Substrate Past Hypothesis, elapsed time below relaxation and recurrence, and a mixing/large-deviation theorem showing ordinary entropy-increasing histories dominate the conditioned ensemble	Part VI, App. G
Microstructure Hamiltonian	UV realization	Finite marked-transfer action closed; stable geometric embedding open	the decorated vertex prepares the diagonal fresh amplitude, exports history, produces the 21-block antisymmetric edge Hessian, and fixes the three routing traces; a geometric completion must realize the oriented decoration and full source-coupled spectrum, either through a stable GFT condensate or a capacity-decorated CDT transfer matrix on a continuum critical trajectory	Part VI, §24.9, App. H, J.9
Charged-lepton spectrum	particle-sector extension	Fixed within the decorated marked-transfer and shell action; no fitted lepton coefficient	closure-spectrum collapse gives three shells; the baseline shell algebra is dressed by $Z_\mu = 1 + \zeta_*$ and $Z_{\tau,2} = 1 + (2/7)\zeta_*$, giving $m_\mu/m_e = 206.768280237$ and $m_\tau/m_e = 3477.343310$, both within 1σ	§13, App. H–I
Abelian / weak gauge hosting	gauge sector	Coherent extension; Standard Model identification external	baseline redundancy supplies the Abelian gauge template; primitive spin data carry an $SU(2)$ action, while weak chirality, doublets, and hypercharge assignments are not derived	App. I.2
Persistent open-route color algebra	gauge sector	Conditional derivation from one open-route premise	a reference-preserving open three-route record is a reversible qutrit channel; quotienting the channel phase gives $PU(3)$ with $\mathfrak{su}(3)$ Lie algebra and eight traceless endpoint operators; phase-invariant channel-return fidelity gives the adjoint Wilson form	App. I.2
Primitive color transfer $t_8 = 13/14$	gauge sector	Fixed within the minimal identity-kernel completion	primitive $j_0 = 3/2$ recoupling gives $G = (9/10)P_{\mathbf{1}} + (21/20)P_{\mathbf{2}}$; maximum faithful throughput and route-frame isotropy give $\mathcal{E} = 1_{\mathbf{1}} \oplus (13/14)I_{\mathbf{8}}$	App. H.9, I.2

Quantity / Claim	Sector	Status	Type of Support	Where Established
Diamond causal color regulator	gauge sector	Conditional on the minimal product causal stack and equal primitive heat times	four-link mixed loops, six-link spatial loops, the embedded diamond volume, and electric–magnetic isotropy give $a_t/d = 4\sqrt{2}/3$ and $g^2_{\mathrm{graph,HK}} = 2\sqrt{6}\, g^2_{\mathrm{HK,step}} = 0.24203562353$; graph-to-continuum matching remains open	App. I.2
Physical QCD matter sector	gauge / particle sector	Open and outside the present premises	the global $SU(3)$ endpoint lift and central $U(1)$ identification, triality-carrying quark defects, two weak species per generation, hypercharge, a lattice fermion action, finite scheme matching, confinement, and hadron observables are not derived by the projective pure-gauge route construction	App. I.2
Vacuum stiffness from the substrate	lattice dynamics	Excluded, five branches	induced curvature coupling $c_0 = +0.019$ against bare $k_0 \simeq 2.2$; refresh stationary state geometry-blind to exponential accuracy; history-tilt ceiling twenty times below requirement; free conserved field induces near-universal spanning-tree entropy; budget saturation mass suppresses the residual soft modes	§24.3, App. J.4
Vacuum geometry in the lattice realization	lattice dynamics	Open; externally hosted	tested substrate mechanisms do not select an extended phase; the Regge/CDT host supplies the vacuum geometry, while Many-Pasts conditions histories only after a present record is specified	§24.4, App. G, J.4
Capacity-decorated CDT continuum limit	geometric embedding	Open, with explicit tests; finite-volume host behavior measured	at the same tuned couplings, increasing N_{41} from about 42,000 to 102,000 raises d_H from 3.31 to 3.56 and the blob score from 1.46 to 1.57, consistent with motion farther into the extended finite-volume region; the fixed microscopic capacity slice must still intersect a continuous geometric critical manifold with $\xi_{\mathrm{geom}}/a \to \infty$, and the boundary order, operator projection, and continuum scaling remain open	§24.9, App. J.1, J.9
Ensemble fracture under local dynamics	lattice dynamics	Verified	explicit construction: 48 components $(4! \times 2)$ of 35 states each under injectivity-preserving unit shifts, conserved (ordering, parity) charge	§24.3, App. D.4, H.6, J.4
Substrate contribution to the host volume term	lattice dynamics	Measured as a scaling family, demonstration volume	closure free energy per cell, computed from η_* alone, displaces the equilibrium volume by the predicted amount: single point -62.2 predicted, -62.8 measured; eight coupling–volume-penalty combinations span -10 to -158 at ratios 0.96–1.07, with the concave β-curve and $1/\varepsilon$ scaling confirmed; the shuffled control lies on its separately predicted line at ratio 1.00; curvature-sector shifts remain to be measured with comparable statistics	§24.4, App. J.6
Admissibility weighting on dynamical geometry	lattice dynamics	Finite-lattice compatibility measured	at $N_{41} \simeq 100{,}000$, three matched eighty-slice ensembles remain connected and correctly foliated with 0.6% total-volume spread; the collision fraction changes from 0.654 without the weighting to 0.077 with it, while the shuffled-energy control remains 0.736; the corresponding d_H values remain 3.55–3.59, and a forty-slice repetition gives collision fraction 0.078	§24.5, App. J.5

Quantity / Claim	Sector	Status	Type of Support	Where Established
Local response of geometry to persistent closure failure	lattice dynamics	Demonstrated in two independent matched comparisons	for each of two random seeds at $N_{41} \simeq 100{,}000$, one hundred failure defects are compared with one hundred identically anchored closed cells, with every marked cell surviving: first-shell closure-energy and collision separations are 16.8σ and 9.2σ; third-shell coordination separates at 4.0σ; shell-cell count and first-/second-shell coordination do not separate significantly after autocorrelation and seed corrections	§24.6, App. J.7
Long-range propagation of the strain field	lattice dynamics	Conditional Ward theorem; finite-lattice source-to-field behavior measured; continuum junction open	on a separate forty-slice realization at $N_{41} \simeq 100{,}000$, conservation is exact; $Q - Q(0) = (0.1440 \pm 0.0025)m$; the dressing-subtracted shell-one deficit per charge is 4.9965 ± 0.5244 across $m = 1, 2, 3, 6$; the profile is $d^{-0.62\pm0.11}$ through eight shells and the screening fit is consistent with zero, giving $\xi > 4.6$; the 24.2% maximum source-linearity residual and nonideal exponent are explicit finite-size and saturation diagnostics, while carrier identification, critical scaling, and $\chi_q \alpha_{\mathrm{maint}}/D_q \to 8\pi G_*/c^2$ remain open	§24.7–24.9, App. J.8–J.9
Numerical consistency checks	validation layer	Supportive audit layer	cross-sector consistency tests and executable reproduction block	App. K, O
EFT consistency checklist	field-theory audit	Supportive audit layer	ordinary longitudinal branch has no extra scalar degree of freedom; static source sign and quadratic form checked; telegrapher stability belongs to the conditional transport completion	App. D

This table is the epistemic map used for the rest of the discussion.

Cosmological row. The saturated phase enters the ledger as follows. The per-source variable σ_A separates saturation of recruitment from exhaustion of absolute capacity. Conditional on $\sigma_A = 1$ and one coherent tick-normalized phase, $X = 1/2$ follows and the constrained-scalar dust form is a theorem. The abundance $1/\epsilon = 5.321$ stands against the measured 5.364 ± 0.065 and inherits the transverse normalization together with the recruitment and per-defect assumptions. The dynamics that drive $\sigma_A \to 1$, establish coherence, and account for conversion energy remain open.

27. Falsifiability and Observational Tests

27.1 Static weak-field falsifiers

The static weak-field sector stands or falls on a small number of concrete checks. The most direct are the shape and tightness of the galaxy RAR transition [1, 41], the baryonic Tully–Fisher scaling, and the weak-lensing response, where stacked galaxy–galaxy lensing probes the relation two decades below the rotation-curve regime [42]. That lensing measurement already carries two results for this framework. The measured relation between g_{obs} and g_{bar} agrees with the extrapolation of the same interpolation function used here, under the stated assumption that photons respond to the modified potential exactly as in general relativity — so the no-slip target of Section 16 is observationally viable across the extended range. The same analysis, however, reports a difference of at least 6σ between the lensing relations of early- and late-type galaxies at fixed stellar mass, and notes that a gravity modification depending on the baryonic acceleration alone cannot produce such a split. The transverse response of Section 15 depends only on g_{bar}, so this observation is a present tension rather than a future test. The resolution proposed by the same authors is baryonic bookkeeping rather than new response physics: if early types carry circumgalactic gas with $M_{\mathrm{gas}} \approx M_\star$, their g_{bar} is underestimated and the split closes [42]. Establishing that gas budget is a requirement this framework inherits. Baseline no slip and GR PPN values now follow from the Einstein parent reduction. Persistent gravitational slip associated with the low-acceleration excess would instead falsify the proposed transverse single-mode completion. Solar-System and laboratory bounds [3, 50, 52] require the transverse kernel to decouple at high acceleration and exclude any independently coupled static scalar force.

Wide binaries supply an independent solar-neighborhood discriminator for the proposed local transverse rule. If the gap is controlled by the total local baryonic field, a binary embedded in the Galactic field $g_{\mathrm{ext}} \simeq 1.4\text{–}1.9 \times 10^{-10}\,\mathrm{m\,s^{-2}}$ approaches the boost $1 + n_B(\sqrt{g_{\mathrm{ext}}/a_0}) \simeq 1.4\text{–}1.5$ once its internal field falls below the external term. This is a conditional prediction of the local influence-kernel completion, not a consequence of the ordinary transport equation. Current Gaia analyses divide between a low-acceleration boost and Newtonian consistency [34, 35]; a settled Newtonian result would falsify this local transverse branch.

27.2 Dynamical falsifiers

The dynamical extension has two independent observational tests. *Source-projection* tests ask whether relaxed clusters follow the predicted hook profile—near unity in BCG-dominated centers, maximal where the virialized bath dominates, and lower in the deep outskirts [4, 16, 17]—and whether resolved merger maps prefer $\epsilon \simeq 0.19$. The linear lift candidate already fails on amplitude (Section 18). *Transport* tests ask whether $D/\tau_0 = c^2$ evolves those source weights correctly through a merger. Systems such as the Bullet Cluster [2] probe both. A failure of the propagation law would reject the causal completion even if the static branch survived.

27.3 Cosmological falsifiers

Cosmology presents a different kind of test. The question there is whether a full Boltzmann treatment allows the trace-coupled homogeneous mode to reduce the sound horizon without spoiling the CMB or structure-growth observables. If it cannot, the cosmological extension fails on its own terms. The empirical target is set by the current measurement spread: early-universe inferences near 67.4 [46], distance-ladder determinations ranging from $\simeq 70$ [48] to 73 [47], and a tension whose proposed resolutions are reviewed in Di Valentino et al. [49].

The relation $a_0(z) = cH(z)g_{\mathrm{share,eff}}/(4\pi^2)$ predicts measurable redshift evolution. For a Planck-like background, $H(2.3)/H_0 \simeq 3.47$. A disk with $g_{\mathrm{bar}} \simeq 2 \times 10^{-10}\,\mathrm{m\,s^{-2}}$ then has $g_{\mathrm{obs}}/g_{\mathrm{bar}} \simeq 2.02$, compared with 1.39 for an epoch-independent scale. Observed outer rotation curves at these redshifts indicate strong baryon dominance [36, 40], although pressure-support and stacking systematics leave the comparison unsettled. Controlled observations showing no stronger boost than matched $z = 0$ systems would falsify $a_0 \propto H(z)$.

The equilibrium-vacuum route has separate continuum requirements, now with no free volume normalization. The capacity-decorated trajectory must approach a common-parent critical limit, show finite limiting α and simplex ratio ξ, set a by a target-blind observable, and recover the exact two-simplex volume source after mixed-eigenoperator projection. A nonzero source-independent *capacity* bulk coefficient after vacuum normalization would directly falsify the regulator identity. A continuum completion that still requires an independent host cosmological action would instead falsify the Postulate I common-substrate identification. Only if both conditions hold does the conditional theorem $\Lambda_{\mathrm{R}}^{\text{empty equilibrium}} = 0$ apply. The value or constancy of w tests whatever state-dependent or nonequilibrium cosmological dynamics is eventually supplied; it does not test the exact equilibrium partition identity by itself.

A MUSE sample of 79 star-forming galaxies at $0.33 < z < 1.44$ finds a radial-acceleration scale that rises with redshift [38]; a resolved low-redshift H I sample tentatively reports the same direction [39]. An epoch-independent a_0 predicts no such evolution. The comparison is already quantitative. The reported scale at $z \sim 1$, $a_0 = 2.38^{+0.12}_{-0.10} \times 10^{-10}\,\mathrm{m\,s^{-2}}$, is an enhancement of 1.98 over the local value, which $a_0 \propto E(z)$ produces at $z = 1.18$; the prediction sits at -2.3σ if the effective redshift of the high-z data is 1.0, -1.0σ at 1.1, and $+0.3\sigma$ at 1.2, so the verdict turns on the sample's effective redshift. The reported linear slope $a_1 = (1.59 \pm 0.10) \times 10^{-10}$ likewise exceeds the $E(z)$-anchored effective slope of about 1.19 over the sampled range, but $E(z)$ is convex and a linear fit absorbs curvature, so the slope comparison settles nothing by itself. The clean form of the test is the per-bin $a_0(z)$ values compared directly with $E(z)$: parameter-free, computable from published data, and able to fail. Current mass-to-light and pressure-support uncertainties remain comparable to the effect and enter that comparison as stated by the authors.

Saturated-phase falsifiers. Three tests bind the saturated phase: a full Einstein–Boltzmann implementation must jointly evolve the homogeneous mode, commitment transition, and constrained fluid with abundance fixed at $1/\epsilon$; the recruitment derivation must determine its epoch and survive scrutiny of the pinned reading and per-defect bookkeeping; and the threshold $g_c \simeq 5.3 \times 10^{-12}\,\mathrm{m\,s^{-2}}$ must produce a domain assignment compatible with linear observables. The release law adds a fourth test: a surviving committed component on cluster infall streams terminating at the splashback surface; its confirmed absence, or a halo-like component persisting inside collapsed systems, would falsify the caustic-release mechanism.

27.4 Correlated-constant falsifiers

The inferred L_*, induced G_*, and matched weak-field G are not independent legs: the matched route carries the same electron length calibration. Their agreement is a normalization audit rather than a separate falsifier. The marked action adds two independent tests because the same ζ_* enters the muon and tau through different graph polynomials. A future independent determination of L_* would provide a fourth test of the shared vertex.

27.5 Many-Pasts status

Many-Pasts imports ordinary quantum instruments, so it predicts no laboratory departure from the Born rule or no-signaling. Its discriminating burdens are theoretical and now separable. The substrate must reproduce the decoherence functional rather than assume it; the retained (P, Φ) sector must derive the symplectic and Fisher-compatible metric structures needed for the Caticha Hamilton–Killing bridge; and a conditional-typicality calculation must suppress high-entropy-past and Boltzmann-fluctuation histories. Failure of the first two would remove the proposed microscopic quantum completion, while failure of the third would remove the proposed arrow; none would constitute a new experimental violation of standard quantum mechanics.

28. What the Theory Would Have to Get Wrong to Fail

The failure modes are not all equally severe, and they are ordered here by how much of the theory each would remove.

Kills the core ontology. A demonstrated failure of mass–entropy equivalence, an internal incoherence in the Many-Pasts weighting, or evidence that geometry cannot be read as the long-wavelength form of an entanglement-capacity substrate would remove the foundations on which everything else rests.

Kills the static capacity-response closure or its transverse completion. A weak-field UV coefficient chain that cannot be reconciled with an independently validated microscopic derivation would break the ordinary static closure. An RAR transition shape that departs from the proposed bosonic law, persistent low-acceleration slip, unacceptable residual PPN effects, or violation of the metric Ward identity would falsify the transverse completion while leaving the ordinary Einstein/capacity equivalence intact.

Kills an extension only. If the cosmological trace-coupled homogeneous mode cannot survive a full Boltzmann likelihood confrontation, the cosmology sector fails while the static weak-field branch stands. If the saturated phase fails any of its commitments — the dark-to-baryonic abundance departing from the capacity ceiling, linear-regime observables departing from the inherited form below the release threshold, or no committed component surviving on cluster infall streams out to the splashback surface — the committed-dust reading fails in the same contained way. If $q_{\rm cap}$ does not track $q_{\rm geo}$ during collapse, or if the derived boundary response conflicts with black-hole observations, the bounded-domain interpretation fails while the spherical Einstein reduction remains. None of these touches the weak-field core.

Requires modification, not death. A fuller graph calculation may revise the separate conditional stiffness self-energy, and strong-field boundary spectroscopy may change without altering the Einstein exterior. A failure of the decorated charged vertex would be more serious because the same ζ_* enters the electron scale and both heavier-lepton ratios. The positive-spectrum clock conversion survives only if another microscopic charged action replaces it.

One scale-setting commitment cuts across these tiers. The high-precision G_* rests on sevenfold additivity, the record-conditioned determinant transfer, and the decorated marked vertex. A microscopic Hessian with a collective edge mode, persistent endpoint polarization, or off-diagonal charged propagation would fail the finite action and displace the correlated G and lepton results. If the electron identification or mass–entropy ontology failed, the scale-setting derivation of Appendix H would not go through.

29. Comparison with Other Approaches

The galactic excess and cosmological abundance are assigned to two phases of one medium. The comparisons below distinguish that proposal from nearby alternatives while retaining the conditional grades of Sections 15 and 20.

29.1 Relative to ΛCDM

The contrast with ΛCDM begins at the level of ontology. Here visible matter is interpreted as localized defects in a vacuum-capacity medium. The ordinary longitudinal response is a reduced representation of Einstein gravity; the proposed extra galactic response is carried by a transverse substrate sector. The UV entropy enters both branches, but only the ordinary branch closes without the transverse matching conditions. Lensing and cosmology remain extensions.

29.2 Relative to MOND-like interpolation programs

MOND-like programs [18, 54, 20] usually begin from an acceleration law or interpolation function. Here the same law is reconstructed from a rank-one capacity EFT and equilibrium bosonic free energy. Its horizon and GFT matching conditions are explicit, so the comparison turns on whether those conditions can be derived and whether their metric kernel passes lensing and precision tests.

29.3 Relative to Verlinde-style emergent gravity

Verlinde-style emergent-gravity programs share the broad intuition that gravity may be entropic [21, 53, 28, 29], but they are usually formulated through thermodynamic reasoning or horizon-inspired force laws. The present framework specifies tetrahedral counting, admissibility closure, edge coupling, return dressing, and Euclidean normalization before reaching the continuum EFT. Its correctness is an empirical question.

29.4 Relative to TeVeS and other multi-field modified gravities

Multi-field relativistic MOND completions such as TeVeS [19] introduce additional scalar and vector fields alongside the metric to obtain relativistic lensing and cosmology. Here the ordinary longitudinal response adds no field beyond the Einstein metric. The transverse thermal response is nevertheless an additional effective sector, and its retarded metric kernel must derive its own slip and lensing behavior.

29.5 Relative to AeST

The closest modern comparator is the AeST theory of Skordis and Złośnik [30], which combines the metric with a dynamical timelike vector and a scalar field and can reproduce MOND-scale galaxy phenomenology while fitting the CMB power spectrum. The present construction differs in its finite counting input and is less developed cosmologically: the joint Boltzmann treatment remains open here.

29.6 Relative to scalar-tensor gravity

The reduced capacity functional of Section 10 can be mistaken for the scalar part of a Brans–Dicke-type theory [51]. The action reconstruction shows otherwise: in the ordinary static branch it is the Einstein constraint action after a field redefinition and carries no independent scalar. A scalar–tensor or disformal theory is retained only as a control case for sectors that might genuinely require an additional mode; if adopted, ordinary fifth-force, slip, and PPN constraints would apply.

29.7 Relative to quantum-mechanical interpretations

Because Many-Pasts occupies the role of an interpretation of quantum mechanics (Sections 3.3, 22), it should be placed against the standard options. It posits one realized present, adds no collapse term, and introduces no hidden sharp values. Its operational probability theory is the decoherent-histories formalism: unresolved alternatives retain amplitudes, decoherent record histories receive diagonal probabilities, and conditioning on the present occurs only after the alternative records are normalized. The new content is the ontology assigned to that conditional measure and its proposed substrate realization. Born statistics and no-signaling are inherited from the quantum instruments; a substrate derivation of the decoherence functional remains open.

29.8 Relative to CDT, spin foams, and group field theory

The construction uses discrete tetrahedral boundary data, with a fermionic base label $j_0 = 3/2$ paired into the seven-state effective boundary representation $j_{\text{eff}} = 3$; this places it near the quantum-tetrahedron vocabulary of simplicial spin networks [57, 58, 59]. Its selection principle differs from spin-foam and GFT programs: it solves a finite boundary-counting problem and routes admissibility closure to capacity entropy rather than using a vertex amplitude as the primary history weight. Section 24 supplies a separate working interface: the label ensemble is coupled to a CDT host [75, 76]. In the completed runs the host supplies the dynamical vacuum geometry and the paper tests how its own admissibility weighting dresses it. Section 24.9 and Appendix J.9 define the stronger alternative in which the finite marked transfer becomes part of a capacity-decorated CDT transfer matrix and the fixed capacity couplings must land on a continuum critical surface. Until that scaling program succeeds, the fixed foliation and other debated features of CDT belong to the external scaffold, while the substrate's proposed granularity remains in the capacity variables.

29.9 Relative to algebraic and information-geometric gravity

Three continuum results address separate claims made here. The observer-dressed de Sitter algebra establishes a maximum-entropy empty gravitational state. The horizon modular calculation equates the relative information of a coherent excitation with its Killing energy flux and, after the area input, with Einstein curvature. The geometric-relative-entropy action gives a local bulk information functional with an Einstein limit, a positive Legendre energy, cosmological thermodynamics, and de Sitter area scaling [23, 22, 24]. They support the continuum ontology but do not solve the ultraviolet problem. This paper must still derive the local bound, matter commitment, dimensional scale, and response coefficients from one finite microstructure. Bianconi's independent dynamical G-field belongs to a modified-gravity theory and is not imported into the metric-only longitudinal branch.

The coefficient comparison is explicit. On the matched branch $L_*^2 = \hbar G_*/c^3$, the horizon normalization gives

$$S_\infty = \frac{A_{\rm dS}}{4L_*^2} = \frac{c^3 A_{\rm dS}}{4\hbar G_*}.$$

The observer-dressed de Sitter construction uses this area coefficient and gives $S_{\max} - S_{\rm gen}(E) = \beta_{\rm dS} E + O(E^2)$ for a small central excitation. The horizon modular calculation uses $S_{\rm rel} = c^3 \delta A/(4\hbar G_*)$ and obtains

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G_*}{c^4}\langle T_{\mu\nu}\rangle.$$

The bulk route derived here gives

$$\nabla^2 \delta S = -\frac{\kappa}{\gamma}\rho, \qquad \frac{\Phi}{c^2} = -\frac{\delta S}{2S_\infty}, \qquad G_* = \frac{c^2\kappa}{8\pi\gamma S_\infty},$$

and hence $\nabla^2\Phi = 4\pi G_*\rho$. The two continuum routes therefore use the same coupling after the channel-to-area normalization is granted. A microscopic area operator must still derive that normalization.

At Gaussian order, the relationship to Bianconi is sharper. Appendix H.10 proves that the normalized determinant of any positive dressed GFT Hessian relative to its equilibrium Hessian on the same physical support has exactly the trace-log form used by geometric relative entropy, while its quadratic-source Legendre transform is exactly the positive covariance mismatch $\frac{1}{2}\operatorname{Tr}'(\mathcal{G} - I - \ln\mathcal{G})$. This closes the functional bridge, not the microscopic operator identification. The latter requires the decorated gluing tensor to pass the condensate/spectrum, covariant assembly/normalization, and transfer-positivity audits stated there.

The cosmological scaling also agrees at the level of form. With $R_A = c/H$,

$$S_\infty = \frac{\pi c^2}{H^2 L_*^2}, \qquad S_{\rm GfE} \simeq \frac{\overline{\omega}_{[1]} c^2}{\ell_P^4 H^2}, \qquad \frac{S_{\rm GfE}}{S_\infty} \simeq \frac{\overline{\omega}_{[1]} L_*^2}{\pi \ell_P^4}.$$

Bianconi therefore obtains the same H^{-2} dependence, but her couplings do not determine the finite-cell coefficient used here.

30. Conclusion

One physical picture runs through the paper. The vacuum is a finite medium of entanglement capacity; matter is that capacity tied up in stable, localized defects; a particle's mass measures the entanglement its defect commits; and gravity is the capacity strain the surrounding medium carries once that commitment is made. The theory derives general relativity as the medium's low-energy geometry. On galactic scales, the same medium produces the additional long-range response usually attributed to dark matter.

The central result is a tightly specified static weak-field construction with its remaining assumptions exposed. The tetrahedral ensemble fixes the sharing entropy; faithful renewal and the decorated marked-transfer vertex fix the electron-anchored substrate length; the edge kernel and Green-matched source map fix the ordinary response; and the separate minimal loop operator supplies the stated conditional stiffness correction. The weak-field bridge then rewrites the Einstein constraint sector in the capacity variable.

Newtonian gravity is the point-source limit of the Einstein/capacity action equivalence, and baseline lensing and PPN values follow from that parent. Within the one-invariant transverse EFT, the rank-one identity, static response coefficient, clamped-frequency ratio, and two-oscillator angular measure are exact. The one-entropy loading, thermal anchoring of the clamped response, influence kernel, and lensing response remain open. The marked-transfer action gives G_* at -0.073σ and the muon and tau ratios at -0.535σ and $+0.481\sigma$. These are historical postdictions from one zero-continuous-fit action, not three blind predictions.

The same medium extends into sectors the paper holds more tentatively. Transport gives the field a finite propagation speed and the lag that clusters and mergers require. The cluster sector reads lensing anomalies as a phase-dependent projection of an already-fixed coefficient. The cosmological mode shifts the sound horizon in the direction the Hubble tension requires. In the saturated early universe the medium becomes a conserved committed density that gravitates as pressureless dust. In strong fields the exact spherical reduction identifies the composite q_{geo} and recovers Schwarzschild, while domain termination, boundary microphysics, and the nonspherical capacity map remain open. These are extensions and frontier completions, not closed results, and the closure table marks the distinction.

The microscopic construction has also been tested on a dynamical simplicial lattice. At matched $N_{41} \simeq 100{,}000$, the admissibility weight strongly orders the microscopic labels while the host geometry remains extended and nearly unchanged. Two independent comparisons between identically anchored failure defects and closed cells give local capacity separations of 16.8σ and 9.2σ and a third-shell coordination separation of 4.0σ after autocorrelation and seed corrections. A separate conserved-field calculation finds exact conservation, approximately additive emergent source charge, one junction ratio across the tested source strengths, a power-law tail through eight shells, and no detected screening. Increasing the host volume from about 42,000 to 102,000 raises both the Hausdorff estimator and the concentration of volume in the extended region. A continuum critical point and the absolute G junction remain to be measured. Section 26 records the present grades.

The joint measure also closes a sharper equilibrium-vacuum statement. Normalizing by the complete homogeneous capacity free energy at each regulator makes the theory invariant under arbitrary extensive shifts of microscopic energy origin, and the uncentered control verifies the predicted N_{41} coupling displacement. The exact CDT simplex volumes fix the volume-operator normalization and, on the matched $G_{\text{R}} = G_*$ branch, block to $t_* = \Lambda_{\text{R}} L_*^2/(8\pi)$ without identifying the regulator spacing with L_*. A finite source-independent capacity density therefore cannot survive the normalization by hiding behind a^4. Conditional on the still-open decorated common-parent continuum limit, Postulate I excludes a separate host volume action and gives zero cosmological term for the empty equilibrium branch. This is not yet a theory of the observed homogeneous acceleration and yields no equation-of-state prediction without additional state-dependent or nonequilibrium dynamics.

Three independent continuum constructions support different parts of the proposed ontology. The type II_1 de Sitter algebra realizes an entropy-maximizing empty gravitational state. The horizon modular calculation relates an excitation's relative information to energy flux and, after the area input, to Einstein curvature. The geometric-relative-entropy action gives a local bulk information dynamics with an Einstein limit, a positive mismatch energy, a first law, and H^{-2} de Sitter entropy [23, 22, 24]. Section 25 derives the same positive mismatch functional from a Gaussian covariance identity. These results support the continuum ontology but do not derive the finite cell, coefficients, cosmological state, or ultraviolet completion used here.

For any successful Gaussian geometric-condensate realization, equilibrium normalization produces a trace-log relative-Hessian action, and its quadratic-source Legendre transform produces the positive covariance mismatch $\frac{1}{2}\,\mathrm{Tr}'(\mathcal{G} - I - \ln \mathcal{G})$. The marked transfer already supplies support preservation on its established branch and the $V_0 \oplus V_1 \oplus V_2$ internal representation. Three properties of the geometric gluing tensor remain to be checked: its stable physical spectrum and source overlaps, its covariant assembly and normalization, and its positive Euclidean transfer and Lorentzian continuation.

Many-Pasts assigns a record-conditioned ontology to the decoherent histories compatible with the present. Its operational branch changes no laboratory prediction: no-signaling follows from standard quantum instruments, and the Born form is the unique normalized noncontextual additive measure on a sufficiently rich record-projector lattice. The Hilbert-space kinematics and decoherence functional remain imported. The past is represented at the resolution retained by present physical records; erasing a record coarsens the same joint measure, and conditioning on a record after it forms supplies no future-to-past dynamics. Faithful full-support resolution selects the memoryless dressing kernel. The decorated vertex prepares its diagonal fresh closure state, reversible dilation exports the old replaceable register into history, and the marked spectrum plus electron anchor fixes the clock and phase frequency. The complementary-channel calculation shows how renewal can coexist with interference: the marked position fiber remains in the coherent system, and the discarded free-renewal record carries no path label. Caticha's Hamilton–Killing reconstruction identifies the conditions required for the persistent sector to yield coherent quantum dynamics. Whether the decorated dilation enforces them remains a specific open derivation problem. Durable history capacity, relativistic quantum fields, and the Past-Hypothesis/mixing package also remain open. None is a new founding premise.

The particle appendix also contains a conditional projective color sector: a persistent open tetrahedral route, transported reversibly, gives the $PU(3)$ qutrit-channel group, its $\mathfrak{su}(3)$ algebra, the adjoint Wilson form, and the fixed primitive transfer $t_8 = 13/14$, and the same primitive fusion reproduces the nine-state representation required by the marked present/history fiber (Appendix I.2). The global $SU(3)$ endpoint lift, chiral quark matter, continuum-scheme matching, confinement, and hadron observables remain outside the completed derivation.

The remaining work is specific. A geometric completion must realize the oriented present/history decoration, provide durable history capacity, and prove the required relative-mode gap and source projection. The GFT route must select a stable condensate; the CDT route must show that the fixed capacity slice intersects a continuous critical surface and satisfies the renormalized junction condition. The transverse influence functional must reproduce the RAR without an extra form factor and determine lensing, slip, and Ward identities. The separate finite-loop stiffness operator still needs its graph audit; cosmology needs a full Boltzmann likelihood; the cluster lift needs its coherence profile and lensing-map test; and the strong-field boundary needs its channel-to-area map and spectroscopy. The color branch needs a derived chiral matter action and graph-to-continuum matching before it can be compared with physical QCD. The marked event, its edge Hessian, finite routing graph, and primitive marked-fiber representation are no longer on the open list.

One ultraviolet count fixes the coefficients of the closed sector in advance. A measurement inconsistent with any of those linked coefficients would falsify that sector. The calculations above specify those comparisons.

Appendix A: Symbol Dictionary and Canonical Conventions

Appendix A gathers the conventions used throughout the technical material that follows, fixing the units, field definitions, and couplings in one place before the denser calculations begin.

Plain-language terms. Several physical words recur throughout the paper and are collected here in plain form before the symbols:

- *Capacity* — the entanglement support locally available in the vacuum medium.
- *Defect* — a stable, localized commitment of that capacity; coarse-grained, a particle.
- *Deficit* — capacity no longer freely available near a defect, $\delta S = S_\infty - S_{\mathrm{ent}}$.
- *Capacity strain* — the extended deficit profile whose fractional value gives the weak-field potential and whose gradient gives the gravitational field.
- *Committed capacity* — capacity locked into transferring on behalf of a defect; the conserved carrier of the saturated phase (Section 20).
- *Dressing* — the cloud an elementary defect builds by resolving the boundary sectors; its determinant transfer and marked response set the length in the decorated scale branch.
- *Saturation* — the regime in which the capacity bath has no slack left, with the available transfer channel at its ceiling.
- *Pinned reading* — the statement that the saturated phase holds exactly at that ceiling and stays there.
- *Admissibility* — the weighting that favors boundary configurations close to a regular, isotropic local cell.
- *Closure* — the condition that the four oriented face data sum to zero, so the cell closes into a regular volume; K^2 measures the failure of closure.
- *Many-Pasts* — the postulate that one recorded present is supported by its compatible decoherent pasts. Their operational probabilities are the diagonal decoherence-functional weights conditioned on that present; $e^{-D(h,P)}$ is only shorthand for those normalized quantum probabilities.

A.1 Units, signature, and entropy normalization

All dimensional quantities are expressed in SI units unless noted otherwise. The metric signature is $(-,+,+,+)$. Covariant spacetime integrals use $x^0 = ct$, so the Einstein–Hilbert coefficient is $c^3/(16\pi G)$; after writing $dx^0 = c\,dt$, the ADM coefficient is $c^4/(16\pi G)$. Entropies are measured in nats, so Boltzmann's constant is absorbed into the entropy normalization. The canonical UV cell has spatial scale L_* and volume $V_* = L_*^3$; it is the bipartite primitive cell containing the two paired tetrahedral sites, so the site density is $2/L_*^3$ and the four-cell $\Delta V_4 = L_*^4/c$ is assigned one per primitive cell — the convention under which $\gamma_Q = 4\hbar c J/(3L_*^2)$ and $\kappa/\gamma = 3L_*/(4G_{\mathrm{tet}}(0)\kappa_m(L_*))$ are simultaneously exact (Appendix C.4–C.5). In the decorated electron-anchored marked-transfer branch,

$$L_* = -\frac{3}{2}\, Z_e \lambda_e \ln\!\left(1 - e^{-7 g_{\mathrm{share,eff}}}\right), \qquad \lambda_e = \frac{\hbar}{m_e c}, \qquad Z_e = (1+\zeta_*)(1+7\zeta_*^2).$$

The conventional Planck length $L_P = \sqrt{\hbar G/c^3}$ is used only as a comparison scale or in standard gravitational thermodynamic expressions after the gravitational scale has been identified.

These conventions matter because the argument repeatedly moves between a dimensionless ultraviolet counting problem and a dimensionful continuum EFT. The units and signature make those descriptions directly comparable.

A.2 Core scalar variables

The canonical continuum variable is the vacuum-relative coarse-grained entanglement field

$$S_{\mathrm{ent}}(x),$$

with vacuum baseline S_∞ and deficit

$$\delta S(x) = S_\infty - S_{\mathrm{ent}}(x).$$

For nonlinear work the bounded occupancy fraction is

$$q(x) = \frac{S_{\mathrm{ent}}(x)}{S_\infty} = 1 - \frac{\delta S}{S_\infty} \in [0,1].$$

The absolute entropy unit is fixed only after choosing a cell or horizon normalization. Under a constant rescaling of S_{ent}, the quantities S_∞ and κ/γ rescale together, leaving $\delta S/S_\infty$ and $\kappa/(\gamma S_\infty)$ invariant. The source channel is

$$\chi(x) = -\frac{T^\mu{}_\mu}{c^2},$$

which is the continuum trace channel of the localized defect sector and reduces to the ordinary mass density ρ in the nonrelativistic static limit.

A.3 Couplings and derived observables

The main-text conventions are

$$\begin{aligned}
\gamma &: \ \text{entanglement-field stiffness}, && (12)\\
\kappa &: \ \text{continuum defect–entropy coupling}, && (13)\\
\kappa_m(\ell) &: \ \text{mass-per-entropy map at scale } \ell, && (14)\\
\zeta_* &\equiv 9e^{-g_{\mathrm{share,eff}}}\left(1 - \frac{8\eta_*}{49}\right)^{21/2}, && (15)\\
Z_e &\equiv (1+\zeta_*)(1+7\zeta_*^2), && (16)\\
L_* &\equiv -\frac{3}{2}\,Z_e\lambda_e \ln\!\left(1 - e^{-7g_{\mathrm{share,eff}}}\right) \quad \text{in the decorated marked-transfer branch}, && (17)\\
G_* &= \frac{c^3L_*^2}{\hbar} = \frac{9}{4}\,\frac{\hbar c}{m_e^2} Z_e^2 \ln^2\!\left(1 - e^{-7g_{\mathrm{share,eff}}}\right), && (18)\\
G_{\mathrm{tet}}(0) &: \ \text{tetrahedral on-site Green constant}, && (19)\\
g_{\mathrm{share,max}} &= \ln(1680), && (20)\\
g_{\mathrm{share,eff}} &: \ \text{admissibility-weighted sharing entropy}, && (21)\\
J_{\mathrm{bare}},\ J_{\mathrm{eff}}^{\mathrm{tree}},\ J_{\mathrm{eff}}^{(\mathrm{ren})} &: \ \text{UV edge couplings}, && (22)\\
a_0 &= \frac{cH_0 g_{\mathrm{share,eff}}}{4\pi^2}. && (23)
\end{aligned}$$

The canonical weak-field bridge and Newton closure are

$$\frac{\Phi}{c^2} = -\frac{\delta S}{2S_\infty}, \qquad \frac{\kappa}{\gamma} = \frac{3L_*}{4G_{\text{tet}}(0)\kappa_m(L_*)},$$
$$G = \frac{c^2\kappa}{8\pi\gamma S_\infty}.$$

Collected in one place, these formulas also make clear which quantities are downstream of the closure chain. The UV data determine the stiffness and source-to-stiffness ratio first; the observable weak-field constants appear after the bridge and fixed S_∞ normalization are applied.

A.4 Notation map

One notation set is used throughout. The effective sharing entropy is denoted $g_{\text{share,eff}}$, the scalar variable is always the vacuum-relative field S_{ent} or its deficit δS, and the weak-field bridge is used in the single form stated above. The principal extension-sector symbols are:

$$\begin{aligned}
\epsilon &\equiv \frac{g_{\text{share,eff}}}{4\pi^2}, & \nu(x) &\equiv \frac{1}{1-e^{-x}},\\
x &\equiv \frac{E_\perp}{k_B T_H}, & &\\
W_{\text{bath}} &: \text{ cluster bath source weight}, & &\\
B_{\text{bath}} &: \text{ measured bath-development fraction}, & &\\
D &: \text{ capacity diffusivity}, & \tau_0 &: \text{ transport relaxation time},\\
D/\tau_0 &= c^2, & &\\
\sigma_* &\equiv \pi/g_{\text{share,eff}}, & &\\
a_{\text{UV}} &\equiv 1/\operatorname{Var}_{\eta_*}(K^2). & &
\end{aligned}$$

In the transverse EFT, $\Delta C = \alpha_C + g_C X + \beta_C R^2$ is the single capacity mismatch, $u_C = F''(0)$ its inverse susceptibility, A_L, A_T, and $C_\times$ are the entries of its (X, R) Hessian, and $W(g)$ is the conservative response potential satisfying $W'(g) = \nu(\sqrt{g/a_0})$; Z_0 is the common transverse kinetic coefficient and ω_L, ω_T the clamped channel frequencies with $\omega_{L,T}^2 = A_{L,T}/Z_0$ in the units of Section 15. These symbols describe a conditional effective completion rather than additional closed UV coefficients.

Appendix B: UV Boundary Ensemble and Admissibility Closure

Appendix B gives the finite ultraviolet counting problem explicitly. The seven-state tetrahedral ensemble and admissibility weighting determine a unique admissibility-closed entropy from a discrete boundary ensemble. Sections 5–6 use that result.

B.1 Tetrahedral package and logical status

The canonical ultraviolet cell is a tetrahedron, the minimal volumetric simplex in three spatial dimensions. Its four faces are distinguishable relational ports. Each side of a shared face carries a primitive half-integer representation V_{j_0}, and the two sides are compared only after one has been transported and orientation-reversed into the frame of the other. Appendix B.4 shows that the positive oriented-matching operator on

$$V_{j_0} \otimes V_{j_0}$$

transmits the maximal coupled multiplet V_{2j_0}.

The value of j_0 has two possible grades. Conditional on identifying the independently constructed marked present/history fiber with the complete nonmaximal fusion record, the representation content fixes

$$j_0 = \frac{3}{2}, \qquad V_{2j_0} = V_3, \qquad |M| = 7.$$

Without that dynamical incidence identification, each half-integer j_0 defines a discrete zero-parameter branch, and Appendix B.5 compares those branches through the same downstream map.

The counting convention is then fixed by the cell ontology. A letter is the occupation of one channel in a single-copy cell resource, so one letter cannot be routed through two ports. The ports remain distinguishable, and the two global orientations are distinct. The state count and its raw entropy are therefore

$$\Omega_{\text{tet}} = 2P(7,4) = 1680, \qquad g_{\text{share,max}} = \ln(1680) = 7.42654907240.$$

The closure weighting below acts on this finite ensemble. Appendix B.4 derives the primitive pair selection and the conditional $7+9$ complement structure; Appendix B.5 records the neighboring discrete counts and preserves their postdictive status.

B.2 Closure invariant, kernel, and unique fixed point

The canonical scalar closure invariant is

$$K^2(b) = 48 - \frac{1}{3}\big(S^2 - \Sigma^2\big), \qquad S = \sum_{i=1}^{4} m_i, \qquad \Sigma^2 = \sum_{i=1}^{4} m_i^2.$$

The admissibility family is

$$p_\eta(b) = \frac{1}{Z(\eta)} e^{-\eta K^2(b)}, \qquad Z(\eta) = \sum_{b\in B} e^{-\eta K^2(b)}.$$

The admissibility precision η is fixed by stationary normalized closure evidence. The closure constraint is the vanishing of the three-component oriented-face sum $\mathbf{c}(b) = \sum_i \hat{n}_i m_i \in \mathbb{R}^3$. The invariant K^2 is not the squared magnitude of that classical vector, which is $|\mathbf{c}(b)|^2 = (4\Sigma^2 - S^2)/3$; it is the quantum expectation of the squared closure operator. With each face carrying the seven-state $j_{\text{eff}} = 3$ representation $(2j+1 = 7)$ and the product boundary state $|b\rangle = \bigotimes_{i=1}^4 |j, m_i\rangle_{\hat{n}_i}$ built on the tetrahedral normal frame $\hat{n}_i \cdot \hat{n}_j = -\frac{1}{3}$, the closure operator $\widehat{\mathbf{C}} = \sum_i \widehat{\mathbf{A}}_i$ obeys

$$K^2(b) = \langle b|\widehat{\mathbf{C}}^2|b\rangle = \sum_i j(j+1) + 2\sum_{i<k} m_i m_k\, \hat{n}_i \cdot \hat{n}_k = 4j(j+1) - \frac{1}{3}\big(S^2 - \Sigma^2\big),$$

with $4j(j+1) = 48$ for $j = 3$. The decomposition

$$K^2 = |\mathbf{c}(b)|^2 + \sum_i \big[j(j+1) - m_i^2\big]$$

exhibits K^2 as the mean nonclosure plus the irreducible quantum variance of the four face operators. Two remarks fix the status of this formula. First, the expectation is taken in the unprojected product state, without projection onto the gauge-invariant intertwiner subspace; this is the operator content of the framework's soft-closure convention, in which closure enters statistically through the evidence weight $e^{-\eta K^2}$ rather than as an exact constraint, and expectation values in the projected subspace would differ. Second, the seven-label alphabet and the constant 48 are one representation-theoretic choice, not two independent ingredients: given the

$j_{\text{eff}} = 3$ product-state boundary realization and the tetrahedral normal frame, both are fixed together.

The evidence factor follows from the same operator. The closure operator $\widehat{\mathbf{C}}$ has three components, and the normalized isotropic quadratic evidence kernel on that three-dimensional defect space carries the determinant weight $\eta^{3/2}$, so the normalized closure evidence is $\eta^{3/2}Z(\eta)$ and its logarithm is

$$\mathcal{F}(\eta) = \ln Z(\eta) + \tfrac{3}{2}\ln\eta.$$

With $\partial_\eta \ln Z = -\langle K^2\rangle_\eta$, the stationary condition $\mathcal{F}'(\eta) = 0$ is the closure relation

$$\langle K^2\rangle_\eta = \frac{3}{2\eta},$$

so the factor 3/2 is the determinant weight of the three independent closure components, not an equipartition rule imported onto the bounded discrete spectrum; the discreteness, multiplicities, and positive floor of the spectrum remain inside the exact finite sum $Z(\eta)$. The second derivative is $\mathcal{F}''(\eta) = \mathrm{Var}_\eta(K^2) - 3/(2\eta^2)$, and on the exact spectrum $\mathcal{F}'$ has a single interior zero,

$$\eta_* = 0.0298668443935,$$

at which $\mathrm{Var}_{\eta_*}(K^2) = 15.69$ is dwarfed by $3/(2\eta_*^2) = 1681.6$, giving $\mathcal{F}''(\eta_*) = -1665.9 < 0$: η_* is the unique local maximum of the normalized closure evidence. Because the parity-symmetric ensemble is finite, both $Z(\eta)$ and $\langle K^2\rangle_\eta$ are exact finite sums over the spectrum. The distinct closure-defect values and their degeneracies are

K^2	$\frac{122}{3}$	$\frac{134}{3}$	$\frac{142}{3}$	$\frac{146}{3}$	$\frac{152}{3}$	$\frac{154}{3}$
mult	96	96	96	288	192	144

K^2	$\frac{158}{3}$	54	$\frac{164}{3}$	$\frac{166}{3}$	$\frac{170}{3}$
mult	384	192	48	96	48

with total multiplicity 1680 as required. In particular,

$$Z(\eta) = \sum_a n_a e^{-\eta K_a^2}, \qquad \langle K^2\rangle_\eta = \frac{\sum_a n_a K_a^2 e^{-\eta K_a^2}}{\sum_a n_a e^{-\eta K_a^2}},$$

where (K_a^2, n_a) run over the table above. The closed-branch value η_* is therefore the unique interior maximum of an exact finite-spectrum functional, not an unseen numerical fit. The corresponding effective sharing entropy is

$$g_{\text{share,eff}} = -\sum_{b\in B} p_{\eta_*}(b)\ln p_{\eta_*}(b) = 7.41980002357.$$

The closed-branch moments used in the UV stiffness discussion are

$$\begin{aligned}
\langle K^2\rangle_{\eta_*} &= 50.2229154254,\\
\mathrm{Var}_{\eta_*}(K^2) &= 15.6889750078,\\
a_{\text{UV}} \equiv \frac{1}{\mathrm{Var}_{\eta_*}(K^2)} &= 0.0637390269.
\end{aligned}$$

These values quantify the local stiffness of the canonical closure point rather than a tunable phenomenological uncertainty. The closure-saturation product is

$$C_{\text{cl}} := \eta_*\langle K^2\rangle_{\eta_*} = \frac{3}{2}, \qquad C_{\text{cl}}^{-1} = \frac{2}{3}.$$

The value $C_{\mathrm{cl}} = 3/2$ is an identity imposed by the stationarity condition, not an independent numerical cross-check. Its reciprocal is reused as the transverse export factor in the decorated scale-setting branch discussed in Appendix D.4.

The admissibility parameter stops being free here. The kernel introduces η, and the closure condition removes its arbitrariness again by demanding that the fluctuation scale produced by the weighting agree with the weighting itself.

B.3 Rooted reduction and local benchmarks

Rooting on the shared face reduces the exact parity-symmetric ensemble to 140 rooted microstates and 69 rooted closure classes. The rooted classes can be labeled by $\alpha = (m_\bullet, K^2)$, so the same reduced state space supports the local evaluation, the cavity benchmark, and the later shell propagation. Let X denote the root-face label and Y_r the boundary record after r rooted shells. The local information observable

$$\sigma_{\mathrm{ind}}^{(r)} = \frac{H(X \mid Y_r)}{H(X)}$$

has the principal pre-nonlocal benchmarks

$$\sigma_{\mathrm{ind}}^{\mathrm{toy}} = 0.44997, \tag{24}$$

$$\sigma_{\mathrm{ind}}^{\mathrm{loc}} = 0.44708, \tag{25}$$

$$\sigma_{\mathrm{ind}}^{\mathrm{Bethe}}(J=0) = 0.44749. \tag{26}$$

Here the Bethe value is the homogeneous cavity evaluation on the 69×69 rooted-class interaction graph at zero transport coupling,

$$\mu_\alpha \propto w_\alpha \left(\sum_\beta U_{\alpha\beta}(0)\mu_\beta \right)^{z-1}, \qquad \sum_\alpha \mu_\alpha = 1,$$

where $w_\alpha = n_\alpha e^{-\eta_* K_\alpha^2}$ is the rooted-class Gibbs weight, $z = 4$, and $U_{\alpha\beta}(0)$ is the rooted shared-face compatibility matrix before shell transport is turned on. Thus $\sigma_{\mathrm{ind}}^{\mathrm{Bethe}}(J=0)$ is the cavity-theory benchmark of the same explicit rooted ensemble. The horizon target implied by the effective sharing entropy is

$$\sigma_* = \frac{\pi}{g_{\mathrm{share,eff}}} = 0.42340665.$$

The gap between the local benchmarks and σ_* arises from shell and loop structure. The local admissibility closure already satisfies its own condition.

The remaining ultraviolet calculation propagates the fixed local closure ensemble through transport and return structure.

B.4 Channel ontology and derived selection

This section separates four statements that earlier versions ran together. Positive oriented matching selects one transmitted multiplet V_{2j_0}. The choice of that matching rule is part of the primitive ultraviolet architecture and has a spectral falsifier. The marked-fusion identification conditionally fixes $j_0 = \frac{3}{2}$ and the sharp kernel. If that incidence identification is not imposed, $j_0 = \frac{3}{2}$ remains the surviving member of the discrete audit in B.5.

B.4.1 Setup

Two cells share a face. Each contributes a primitive spin-j_0 slot, so the two-sided space is

$$\mathcal{H}_f = V_{j_0} \otimes V_{j_0} = \bigoplus_{J=0}^{2j_0} V_J.$$

The decomposition is multiplicity-free. The face ultimately exports a classical weight alphabet to the admissibility ensemble. The microscopic questions are which coupled sector is transmitted, how the untransmitted information is retained, and why the four port assignments are injective.

B.4.2 Positive oriented matching

Scope. Oriented gluing alone does not select j_0: every half-integer j_0 defines a discrete member with $4j_0 + 1$ transmitted weights. Sections B.4.2e–f give a stronger result conditional on one physical identification: the marked present/history fiber is the faithful record of all nonmaximal primitive-fusion information. Under that identification the independently fixed spin-one closure response uniquely gives $j_0 = \frac{3}{2}$. No smallest-spin rule is used.

B.4.2a Face-sector Hessian

Proposition (exact block decomposition). If the gauge-fixed face Hessian commutes with the diagonal $SU(2)$ action, Schur's lemma gives

$$\boxed{\mathcal{H}^{(2)}(p) = \bigoplus_{J=0}^{2j_0} \left(Z_J p^2 + r_J\right) I_{V_J}.}$$

Thus an $SU(2)$-covariant quadratic kernel cannot mix different total-spin sectors. Its dynamical content is the finite spectrum $\{Z_J, r_J\}$.

For invariant block densities

$$\rho_J = \sum_{M=-J}^{J} |\sigma_{JM}|^2,$$

write the radial part of the homogeneous potential as

$$V_{\text{eff}} = \sum_J r_J \rho_J + \frac{1}{2} \sum_J u_J \rho_J^2 + \sum_{J<K} w_{JK} \rho_J \rho_K.$$

The first instability lies in the block with the smallest r_J. A pure J_* saddle, with $\rho_{J_*}^{(0)} = -r_{J_*}/u_{J_*}$, is locally stable against an unoccupied block J when

$$m^2_{J|J_*} = r_J - \frac{w_{JJ_*}}{u_{J_*}} r_{J_*} > 0.$$

These are the general tests. The minimal matching action below evaluates them explicitly.

B.4.2b Exact coherent-matching kernel

The same geometric face has opposite outward orientations in its two incident cells. Let g_{LR} transport the right frame into the left frame and define the right generator in the common orientation by

$$\widetilde{\mathbf{J}}_R = -\operatorname{Ad}_{g_{LR}} \mathbf{J}_R.$$

Perfect matching compares equal oriented data. If

$$|j,\mathbf{n}\rangle = D^{(j)}(g_{\mathbf{n}})|j,j\rangle$$

is a spin coherent state, the matched pair is

$$|j,\mathbf{n}\rangle_L \otimes |j,\mathbf{n}\rangle_{\widetilde{R}}.$$

The highest-weight identity

$$|j,j\rangle \otimes |j,j\rangle = |2j,2j\rangle$$

and diagonal covariance imply

$$|j,\mathbf{n}\rangle \otimes |j,\mathbf{n}\rangle = F_{2j}^{\dagger}|2j,\mathbf{n}\rangle,$$

where $F_{2j} : V_j \otimes V_j \to V_{2j}$ is the maximal-channel Clebsch–Gordan coisometry.

The direction-independent positive Gram operator of exact matching is

$$\mathcal{G}_{\text{sharp}} = (4j+1)\int_{S^2} \frac{d\Omega_{\mathbf{n}}}{4\pi}\, |j,\mathbf{n};j,\mathbf{n}\rangle\langle j,\mathbf{n};j,\mathbf{n}|.$$

The coherent-state resolution of the identity on V_{2j} gives

$$\boxed{\mathcal{G}_{\text{sharp}} = F_{2j}^{\dagger}F_{2j} = P_{2j}.}$$

Thus exact oriented matching transmits the maximal coupled multiplet. At $j_0 = \frac{3}{2}$,

$$\mathcal{G}_{\text{sharp}} = P_3, \qquad \dim V_3 = 7.$$

The seven-state link is the image of the pair operator on the full sixteen-state space; it is not inserted as a separate alphabet after the tensor product has been formed.

The contraction alternative. Ordinary ε-contraction is a different microscopic operation. It pairs dual legs and leaves an unfused alphabet of $2j_0+1$ states rather than transmitting V_{2j_0}. At $j_0 = \frac{3}{2}$ this gives four letters and the 48-state branch in B.5. The two operations therefore define distinct gluing ontologies with distinct closure spectra. The present theory adopts oriented fusion because the face must continue to carry closure data after gluing.

B.4.2c Finite-width completion and its falsifier

The quantum mismatch of two perfectly aligned spin-j_0 slots has the irreducible floor $2j_0$. Subtracting that floor defines

$$Q_f = \frac{1}{2}\left[\left(\mathbf{J}_L - \widetilde{\mathbf{J}}_R\right)^2 - 2j_0 I\right].$$

On V_J,

$$Q_f\big|_{V_J} = \frac{1}{2}\left[(2j_0)(2j_0+1) - J(J+1)\right] I_{V_J}.$$

Hence $Q_f \geq 0$ and its null space is exactly V_{2j_0}. The positive finite-width transfer

$$T_s = e^{-sQ_f}, \qquad s > 0,$$

forms a semigroup and converges to P_{2j_0} as $s \to \infty$. For $j_0 = \frac{3}{2}$,

$$\boxed{Q_f = 6P_0 + 5P_1 + 3P_2, \qquad T_s = e^{-6s}P_0 + e^{-5s}P_1 + e^{-3s}P_2 + P_3.}$$

A minimal homogeneous action on the complete primitive space is

$$\Gamma_f = \int d^d x \left[Z|\partial\Phi|^2 + r\,\Phi^\dagger\Phi + \kappa\,\Phi^\dagger Q_f \Phi + \frac{u}{2}(\Phi^\dagger\Phi)^2 \right], \qquad Z, \kappa, u > 0.$$

Its potential is

$$V = r\rho + \frac{u}{2}\rho^2 + 6\kappa\rho_0 + 5\kappa\rho_1 + 3\kappa\rho_2, \qquad \rho = \sum_{J=0}^{3} \rho_J.$$

For $r < 0$, the fusion-sector support of the homogeneous minimum is uniquely $J = 3$:

$$\rho_3 = -\frac{r}{u}, \qquad \rho_0 = \rho_1 = \rho_2 = 0.$$

The orthogonal blocks have positive masses

$$\boxed{m_{0|3}^2 = 6\kappa, \qquad m_{1|3}^2 = 5\kappa, \qquad m_{2|3}^2 = 3\kappa.}$$

At fixed total density, every nonmaximal occupation raises the action, so mixed-J support is excluded globally in this minimal model.

The microscopic test is finite. For the reduced physical Hessian define

$$h_J = \frac{1}{2J+1} \operatorname{Tr}_{V_J} \left(P_J \mathcal{H}_f P_J \right).$$

The weak pass condition is

$$h_3 < h_2, \qquad h_3 < h_1, \qquad h_3 < h_0.$$

The minimal-kernel target is

$$\boxed{(h_0 - h_3) : (h_1 - h_3) : (h_2 - h_3) = 6 : 5 : 3.}$$

After condensation the remaining tests are $h_J + w_{J3}\rho_3 > 0$ for $J = 0, 1, 2$. The construction fails if another block is lighter, inequivalent J sectors mix after gauge reduction, a stable mixed-J minimum survives, or the primitive transfer is not positive.

B.4.2d Primitive tensor and effective pushforward

For a shared face (ab), let Q_{ab} denote the transported mismatch operator above. A concrete finite-width primitive tensor is

$$\mathcal{G}_{v,s}^{(0)} = \mathcal{P}_v^{\text{inv}} \left[\bigotimes_{(ab)\in E(v)} e^{-sQ_{ab}} \right] \mathcal{P}_v^{\text{inv}},$$

where $\mathcal{P}_v^{\text{inv}}$ is the cellwise gauge/intertwiner projector. The zero-parameter primitive theory is defined by its sharp form,

$$\boxed{\mathcal{G}_{v,\sharp}^{(0)} = \mathcal{P}_v^{\text{inv}} \left[\bigotimes_{(ab)\in E(v)} P_3^{(ab)} \right] \mathcal{P}_v^{\text{inv}}.}$$

The finite-s family is a positive regulator and a spectral falsifier, not an additional coupling of the sharp theory.

Let

$$F_3 : V_{3/2} \otimes V_{3/2} \longrightarrow V_3$$

obey $F_3^\dagger F_3 = P_3$ and $F_3 F_3^\dagger = I_{V_3}$. On an isolated face,

$$\boxed{F_3 T_s F_3^\dagger = I_{V_3}}$$

for every $s > 0$. For the complete gauge-projected vertex, finite-s independence additionally requires $\mathcal{P}_v^{\mathrm{inv}}$ to preserve the facewise coupled-J decomposition, or equivalently that the spectator/intertwiner dressing be channel-blind. This is a reduced-kernel or commutator audit; it is not assumed silently. The sharp P_3 support is robust when its overlap is nonzero, while finite-width spectral ordering remains the explicit test.

With $\mathcal{F}_v = \bigotimes F_3$, the post-fusion geometric tensor is

$$\mathcal{G}_v^{\mathrm{eff}} = \mathcal{F}_v \mathcal{G}_{v,\sharp}^{(0)} \mathcal{F}_v^\dagger.$$

The marked-transfer vertex of Appendix H decorates this effective object. It is not itself the primitive pair kernel.

B.4.2e Exact marked complement and conditional sharp-kernel selection

Let a marked response be built from two spin-s strands. Then

$$\mathcal{H}_{\mathrm{mark}}(s) = V_s^P \otimes V_s^H = \bigoplus_{J=0}^{2s} V_J.$$

Maximal fusion of two primitive spin-j slots transmits V_{2j} and leaves

$$Q(j) = (V_j \otimes V_j) \ominus V_{2j} = \bigoplus_{J=0}^{2j-1} V_J.$$

Theorem (unique complement match). The representations are isomorphic if and only if

$$\boxed{\mathcal{H}_{\mathrm{mark}}(s) \cong Q(j) \quad \Longleftrightarrow \quad j = s + \frac{1}{2}.}$$

Proof. Both decompositions are multiplicity-free. The marked space contains one copy of each V_J from $J = 0$ through $2s$; the fusion complement contains one copy from $J = 0$ through $2j - 1$. They agree exactly when $2s = 2j - 1$. Equivalently,

$$\dim Q(j) = (2j+1)^2 - (4j+1) = 4j^2, \qquad \dim \mathcal{H}_{\mathrm{mark}}(s) = (2s+1)^2,$$

and the positive solution is $2j = 2s + 1$. □

The marked closure response is independently a three-component vector and therefore has $s = 1$. Conditional on identifying that marked response with the complete fusion record,

$$\boxed{j_0 = \frac{3}{2}.}$$

Explicit complement isometry. On $V_{3/2} \otimes V_{3/2}$ let

$$P_Q = P_0 + P_1 + P_2.$$

The blockwise equivariant isometry

$$\mathcal{I} : Q \longrightarrow V_1^P \otimes V_1^H$$

is defined by

$$\mathcal{I}\,|(j_0 j_0); J, M\rangle = |(1_P 1_H); J, M\rangle, \qquad J = 0, 1, 2.$$

The independent present and history strands have ordered first-excitation basis

$$|a, b\rangle = |a\rangle_P \otimes |b\rangle_H, \qquad \langle a, b|c, d\rangle = \delta_{ac}\delta_{bd}.$$

Let W map this basis to the nine hard-core marked states,

$$W|a, b\rangle = d^\dagger_{ab}|0\rangle.$$

The exact unit Gram matrix gives

$$W^\dagger W = I_9, \qquad WW^\dagger = I_{\mathcal{H}_d}.$$

Define

$$L = W\mathcal{I}P_Q.$$

Then

$$\boxed{L^\dagger L = P_Q, \qquad LL^\dagger = I_{\mathcal{H}_d}.}$$

The complete sharp pair map is

$$U_{\text{sharp}}|\psi\rangle = F_3 P_3|\psi\rangle \otimes |0\rangle_R + |\varnothing\rangle \otimes L|\psi\rangle.$$

The two flags are orthogonal, so

$$\boxed{U^\dagger_{\text{sharp}} U_{\text{sharp}} = P_3 + P_Q = I_{16}.}$$

The seven-state link and nine-state marked record are therefore complementary outputs of an explicit reversible pair map.

Since V_0, V_1, and V_2 occur once, Schur's lemma implies that every other equivariant minimal complement isometry differs by one phase on each block,

$$L' = e^{i\phi_0} L_0 \oplus e^{i\phi_1} L_1 \oplus e^{i\phi_2} L_2.$$

If no microscopic operator compares the blocks coherently, these phases are marked-field conventions. A vertex that does compare them must derive their relative phases.

Why the unit marked weights select the sharp limit. For finite s, the system Kraus amplitude in the canonical dilation is $T_s^{1/2}$ and the complementary amplitude is $\sqrt{I - T_s}$. Its squared norms on the three nonmaximal blocks are

$$\ell_0^2(s) = 1 - e^{-6s}, \qquad \ell_1^2(s) = 1 - e^{-5s}, \qquad \ell_2^2(s) = 1 - e^{-3s}.$$

The H.9 marked vertex instead has one common quadratic normalization,

$$G_{\text{mark}} = I_9 = P_0 + P_1 + P_2.$$

Therefore, within the minimal incidence identification in which this marked vertex is the canonical fusion environment and no compensating J-dependent coupling is added, exact equality of the marked weights requires

$$\boxed{s \to \infty, \qquad T_s \to P_3, \qquad I - T_s \to P_Q.}$$

A finite physical width could survive only with additional block-dependent normalization. The sharp conclusion is conditional on the marked-fusion incidence identification; without it, the equal H.9 Gram matrix and the finite-width fusion transfer describe distinct objects and impose no constraint on one another.

B.4.2f Conditional dimensional endpoint

The closure defect is a spatial vector,

$$C_a, \qquad a = 1, 2, 3.$$

In three spatial dimensions the vector representation of $SO(3)$ lifts to the spin-one irrep of $SU(2)$:

$$d = 3 \quad \Longrightarrow \quad \mathbb{R}^3 \cong V_1 \quad \Longrightarrow \quad s = 1.$$

With the marked-fusion incidence identification, the theorem above gives

$$s = 1 \Longrightarrow j_0 = \frac{3}{2} \Longrightarrow V_{2j_0} = V_3 \Longrightarrow |M| = 7.$$

The minimal three-dimensional simplex has four faces, and the present/history vector record has $3^2 = 9$ states. Single-copy channel capacity makes the four port assignments injective, while the two global orientations give

$$\boxed{\Omega = 2P(7, 4) = 1680.}$$

The complete conditional integer chain is therefore

$$\boxed{d = 3 \Longrightarrow s = 1 \Longrightarrow j_0 = \frac{3}{2} \Longrightarrow \begin{cases} \text{ports} = 4, \\ \text{alphabet} = 7, \\ \text{record} = 9, \\ \Omega = 1680. \end{cases}}$$

For the physical $d = 3$ rotation algebra one may summarize the evaluated identities as

$$\text{ports} = d + 1, \qquad |M| = 2d + 1, \qquad \dim \mathcal{H}_{\text{mark}} = d^2.$$

They are not a claimed continuation to arbitrary dimension. For $d \neq 3$, the vector representation belongs to $\mathrm{Spin}(d)$ and need not be one $SU(2)$ spin. A dimension-general theorem would require repeating the complement analysis in that group.

B.4.3 Injectivity from single-copy channel capacity

The exclusion statement concerns a single cell resource, not Pauli antisymmetry among the four port labels.

Lemma. Assume:

1. the letters index a single copy of the cell-level fermionic channel family, with occupations $n_m \in \{0, 1\}$;
2. a face displaying m is, by definition, that channel occupation routed through the face's port;
3. the four ports are distinguishable relational registers.

Then the admissible boundary states are the injections $f : F_4 \to M$, and their number before orientation doubling is $P(|M|, 4)$.

Proof. If two ports displayed the same letter m, the one cell resource would require $n_m = 2$, contrary to its single-copy ceiling. The assignment is therefore injective. The ports are distinct interfaces to distinct neighbors, so permuting letters between ports changes the routing state; the assignments are ordered. Antisymmetry acts in the occupation algebra of each channel, not by quotienting the relational port index. □

This lemma does not follow from generic fermionic statistics alone. Its physical content is that the four ports draw from one shared channel multiplet rather than four independent copies. That is the same single-copy channel ontology used by the one-bit defect and determinant sectors.

B.4.4 Uniform base measure

Lemma. If the selected face multiplet realizes its full Shannon capacity, its unconditioned base measure is uniform.

Proof. For a probability measure μ on M,

$$H(\mu) \leq \ln |M|,$$

with equality only at $\mu(m) = 1/|M|$. Full use of the fixed alphabet therefore selects the uniform prior. □

This statement concerns the reference measure, not the final closure-weighted ensemble:

$$p_\eta(b) = \frac{1}{Z(\eta)} \qquad \underbrace{\mu_0(b)}_{\text{uniform base measure}} \qquad e^{-\eta K^2(b)}.$$

Covariance alone does not force uniformity, because the face normal permits covariant deformations such as functions of $(\mathbf{J} \cdot \hat{n})^2$. Full-capacity saturation excludes those deformations at the prior level; the closure observable then supplies the physical nonuniformity.

B.4.5 Status ledger

Derived from the primitive matching rule. For any fixed half-integer j_0, exact oriented coherent matching gives the transmitted block V_{2j_0} and an alphabet of $4j_0+1$ states. The single-copy channel ontology gives injectivity; relational ports preserve ordered assignments; and the global face orientation gives the binary factor.

Derived conditional on marked-fusion incidence. The exact relation

$$\mathcal{H}_{\text{mark}}(s) \cong Q(j) \quad \Longleftrightarrow \quad j = s + \frac{1}{2}$$

combines with the three-dimensional vector response $s = 1$ to fix $j_0 = \frac{3}{2}$, seven transmitted states, nine complement states, and $\Omega = 1680$. The exact unit normalization of the marked vertex selects the sharp P_3 kernel within the minimal canonical dilation.

Microscopic premise. The primitive action implements positive oriented matching. The displayed sharp tensor $\mathcal{G}^{(0)}_{v,\sharp}$ is the minimal completion. A more fundamental simplicial calculation must still reproduce its $J = 3$ support or pass the weaker Hessian inequalities of B.4.2c.

Exact but not yet dynamically identified. The representation equivalence, $L^\dagger L = P_Q$, $LL^\dagger = I_9$, and $U^\dagger_{\mathrm{sharp}} U_{\mathrm{sharp}} = I_{16}$ are exact. What remains conjectural is that this complement map is precisely the complete H.9 marked dynamics, including its flags, hard-core mark, phases, and routing.

Data status. Without the incidence identification, $j_0 = \frac{3}{2}$ is selected by the postdictive discrete audit below. With it, the same audit checks an internally selected branch. In neither reading does a continuous parameter choose the alphabet.

B.5 Relaxation and branch audits

The following tables propagate explicit neighboring constructions through one fixed downstream map. They do not form a random sampling distribution and are not assigned "sigma" significances as theoretical alternatives. The marked sector retains the nine response polarizations; a branch with alphabet size n uses $\binom{n}{2}$ pair records, the scalar factor $2/n$, and the second-return coefficient n. Appendix L records that the Newton target influenced the historical construction, so the audit is a conditional and postdictive comparison, not a blind prediction.

B.5.1 Fusion and constituent-spin branches

branch	Ω	$g_{\mathrm{share,eff}}$	$n\, g_{\mathrm{share,eff}}$	G_*/G
$j_0 = \frac{1}{2}$ full reducible space	48	3.8712	15.5	7×10^{31}
$j_0 = \frac{3}{2}$ unfused contraction	48	3.8712	15.5	8×10^{31}
$j_0 = \frac{3}{2}$, $J = 2$	240	5.4785	27.4	2×10^{21}
$j_0 = \frac{3}{2}$, $J = 3$	1680	7.4198	51.9	1.0000
$j_0 = \frac{5}{2}$, $J = 5$	15840	9.6572	106.2	7×10^{-48}
$j_0 = \frac{3}{2}$ full reducible space	87360	11.2616	180.2	4×10^{-112}

The two 48-state rows have the same count but different representation content: the first mixes V_0 and V_1, while the second carries the unfused $V_{3/2}$ alphabet. The family spans about 10^{143} in G_*/G. Holding the rest of the construction fixed, the seven-state branch is the only row near the measured Newton scale.

B.5.2 Counting conventions at seven letters

convention	Ω	$g_{\mathrm{share,eff}}$	G_*/G	m_μ/m_e
$2P(7,4)$	1680	7.4198	1.0000	206.7683
drop orientation doubling	840	6.7267	1.7×10^4	207.82
allow repeated labels	4802	8.4483	5.5×10^{-7}	206.09
quotient port permutations	70	4.2417	3.2×10^{19}	231.01
quotient ports and orientation	35	3.5486	1.1×10^{24}	256.31

These rows change the counting convention while retaining the same seven-letter closure formula and downstream transport. The baseline is the only displayed convention that keeps the Newton and charged-lepton comparisons simultaneously near their measured values.

B.5.3 One continuous prior deformation

To test one nearby nonuniform prior, introduce the parity-even, face-axis-preserving tilt

$$w_\lambda(b) = \exp\left[-\lambda \sum_{i=1}^{4} m_i^2\right]$$

and re-solve the closure condition at each λ. At $\lambda = 0$, exact finite differences of the enumerated ensemble give

$$\frac{dg_{\text{share,eff}}}{d\lambda} = -0.278458, \qquad \frac{d\ln G_*}{d\lambda} = 3.90135, \qquad \frac{d\ln(m_\mu/m_e)}{d\lambda} = 1.37074 \times 10^{-3}.$$

With the present CODATA uncertainty and the baseline residual retained, the one-standard-deviation Newton interval along this one deformation is approximately

$$-5.3 \times 10^{-6} < \lambda < 6.2 \times 10^{-6}.$$

For the muon ratio, $|\Delta\lambda| \simeq 1.6 \times 10^{-5}$ produces one current experimental standard deviation locally. This is a sensitivity scale, not a symmetric bound about zero, because the baseline prediction is not exactly centered. These statements constrain one specified deformation conditional on the frozen downstream map; they do not directly measure the microscopic prior or exhaust all nonuniform measures. Parity-odd tilts have zero linear response by the $m \mapsto -m$ symmetry and require a separate quadratic audit.

Provenance. The entropy near 7.42 was recognized historically by inverting the measured Newton constant, and the decorated vertex was constructed after the remaining Newton and lepton residuals were known. Appendix L retains that chronology. The new result is structural: once the primitive oriented-matching rule and the marked-fusion incidence identification are adopted, the sequence

$$d = 3 \Longrightarrow V_1 \Longrightarrow j_0 = \frac{3}{2} \Longrightarrow V_3 \Longrightarrow 7 \Longrightarrow 1680 \Longrightarrow g_{\text{share,eff}} = 7.419800\ldots$$

contains no adjustable continuous parameter. The branch tables show how sharply nearby stated constructions move the downstream observables; they do not turn the original comparison into a historically blind prediction.

Appendix C: Edge Kernel, Finite Renormalization, and Continuum Matching

Appendix C carries the local boundary ensemble of Appendix B into edge transport, loop dressing, and the continuum stiffness coefficient of the weak-field EFT.

C.1 Channel-averaged isotropy identity and tree coupling

Let $\hat{n}_i$ be the four face normals of a regular tetrahedron. The exact identity

$$\sum_{i=1}^{4} \hat{n}_i \hat{n}_i^{\mathsf{T}} = \frac{4}{3} I_3$$

implies a channel-averaged transverse fraction of $2/3$. The bare edge stiffness is therefore

$$J_{\text{bare}} = \frac{2}{3}\eta_* = 0.0199112296.$$

For a rooted $z = 4$ coarse adjacency graph, the tree-to-lattice map gives

$$J_{\rm eff}^{\rm tree} = \frac{J_{\rm bare}}{z-1} = \frac{2\eta_*}{9} = 0.0066370765.$$

This is the first place where local closure data become a transport law. The tetrahedral identity fixes the isotropic projection, and the rooted branching structure determines how much of the microscopic edge penalty survives as net outward propagation on the coarse graph.

C.2 Horizon target and shell convergence

The horizon-capacity target is

$$\sigma_* = \frac{\pi}{g_{\rm share,eff}} = 0.42340665.$$

At the derived coupling the explicit shell values are

$$\sigma_{\rm ind}^{(2)} = 0.42143, \qquad \sigma_{\rm ind}^{(3)} = 0.42166, \qquad \Delta_{2\to 3} = 0.00023.$$

The residual shift from the target is already small and stable by shell depth $r = 2$, isolating the remaining correction to the loopy local-return sector rather than a broad nonlocal ambiguity.

The shell calculation places the tree branch near the target and assigns the residual discrepancy to local returns. No broad long-range correction is required at this order.

C.3 Finite-loop self-energy closure

The minimal loopy correction is organized as a local Dyson dressing:

$$J_{\rm eff}^{\rm (ren)} = \frac{J_{\rm eff}^{\rm tree}}{1 + J_{\rm eff}^{\rm tree}\Sigma_{\rm ret}}.$$

The specified minimal return operator uses

$$\Sigma_{\rm ret} = 7 + \frac{2}{9} = \frac{65}{9},$$

The two terms have distinct return-channel origins. A short return motif leaves a shared face, explores a local closed loop, and re-enters the same coarse edge before contributing to net long-range transport. In the canonical label basis $m = -3, -2, \ldots, 3$, the minimal operator contains seven label-diagonal returns, one for each face-label channel, contributing

$$\mathrm{Tr}(I_7) = 7.$$

In addition to these label-preserving loops, permutation symmetry allows one collective mode shared across all channels. Writing

$$P_{\rm sing} = |u\rangle\langle u|, \qquad u = \frac{1}{\sqrt{7}}(1, 1, \ldots, 1),$$

this shared return is rank one. Permutation symmetry permits this singlet but does not fix its weight relative to the diagonal returns or exclude longer return motifs. The minimal completion assigns it the same $2/3$ transverse projection used in the tree coupling and the rooted return factor $1/(z-1) = 1/3$ on the $z = 4$ graph. The collective contribution is then

$$\mathrm{Tr}\left(\frac{2}{3}\frac{1}{3}P_{\rm sing}\right) = \frac{2}{9},$$

since $\mathrm{Tr}(P_{\mathrm{sing}}) = 1$. Equivalently,

$$R_{\mathrm{ret}} = I_7 + \frac{2}{9}P_{\mathrm{sing}}, \qquad \Sigma_{\mathrm{ret}} = \mathrm{Tr}(R_{\mathrm{ret}}) = 7 + \frac{2}{9}.$$

Thus 65/9 is fixed inside the stated minimal return operator, not derived from the closure ensemble alone. A microscopic graph-return calculation must derive the relative diagonal and singlet weights, test longer motifs, and determine whether the operator closes on these channels. Within this conditional completion,

$$c_{\mathrm{loop}}^{(\mathrm{ren})} \equiv \frac{J_{\mathrm{eff}}^{(\mathrm{ren})}}{J_{\mathrm{eff}}^{\mathrm{tree}}} = \frac{1}{1 + J_{\mathrm{eff}}^{\mathrm{tree}}\Sigma_{\mathrm{ret}}} \approx 0.95426,$$

and

$$J_{\mathrm{eff}}^{(\mathrm{ren})} \approx 0.00633348.$$

This reproduces the shell-target crossing near $J_{\mathrm{bare,cross}} \sim 0.019$ at the stated level of agreement.

The local Dyson dressing corrects the tree branch for short motifs that recycle amplitude before it contributes to coarse transport. Summing those returns determines the renormalized coupling from the tree coupling.

C.4 Euclidean-action normalization and continuum stiffness

The lattice quadratic form is interpreted canonically as a Euclidean action weight,

$$\frac{I_E}{\hbar} = \frac{J_{\mathrm{eff}}^{(\mathrm{ren})}}{2} \sum_{a,i} (Q_a - Q_{a+L_*\hat{n}_i})^2,$$

where a runs over one sublattice representative of each bipartite primitive cell and i runs over its four outgoing bonds. Every undirected nearest-neighbor edge is counted once. This convention is essential: summing the four bonds from both sublattices would double the variation and the source normalization. The microscopic four-cell is

$$\Delta V_4 = \frac{L_*^4}{c},$$

a coarse-graining convention rather than derived cell geometry (Section 9; the abstract cell complex has no regular-tetrahedron Euclidean embedding). Because every edge occurs once, varying the discrete action gives $J_{\mathrm{eff}}^{(\mathrm{ren})} L_\diamond Q = s$ for a source term $-\sum_a s_a Q_a$, with $L_\diamond = 4I - A$. The diagonal inverse of this same unnormalized Laplacian is the $G_{\mathrm{tet}}(0)$ used in C.5, so a point source obeys $Q(0) = sG_{\mathrm{tet}}(0)/J_{\mathrm{eff}}^{(\mathrm{ren})}$. This removes the factor-of-two ambiguity between the stiffness and source conventions. The same tetrahedral identity then yields

$$\gamma_Q = \frac{4\hbar c}{3L_*^2} J_{\mathrm{eff}}^{(\mathrm{ren})}$$

for the occupancy field Q_{occ}. With the horizon-capacity normalization

$$S = \pi Q_{\mathrm{occ}},$$

the canonical convention $\frac{\gamma}{2}(\partial S)^2$ gives

$$\gamma = \frac{4\hbar c}{3\pi^2 L_*^2} J_{\mathrm{eff}}^{(\mathrm{ren})} = \frac{4\hbar c}{3\pi^2 L_*^2} \frac{2\eta_*/9}{1 + (2\eta_*/9)(65/9)}.$$

Using the substrate-induced scale

$$G_* := \frac{c^3 L_*^2}{\hbar},$$

this is

$$\gamma = \frac{4J_{\text{eff}}^{(\text{ren})}}{3\pi^2}\frac{c^4}{G_*} \approx 8.556 \times 10^{-4}\frac{c^4}{G_*}.$$

This step converts the dimensionless lattice weighting into the dimensionful continuum stiffness used by the weak-field action, after Euclidean normalization and faithful sector-resolution scale setting.

C.5 Local defect insertion and the source-side lattice constant

The stiffness-side matching is not the only UV quantity that can be closed locally. For the canonical rigid defect insertion, excluding one of the seven admissible face labels from one face removes exactly one-seventh of the isotropically averaged local partition weight. Therefore the logarithm of the isotropically averaged partition ratio is exactly

$$\Delta S_{\text{def}} := -\ln\Big\langle \frac{Z_{\text{def}}}{Z_{\text{vac}}} \Big\rangle_{\text{iso}} = \ln\frac{7}{6}.$$

This is the exact isotropic source benchmark in the canonical seven-label ensemble. The isotropically averaged defect free-energy cost differs from it only at $O(10^{-5})$ because the admissibility weighting breaks label symmetry only weakly.

The local source benchmark $\ln(7/6)$ should not be confused with the elementary fermionic one-bit anchor $\ln 2$. The former is the isotropically averaged partition-ratio shift produced by removing one admissible label from the seven-label boundary ensemble. The latter is the intrinsic binary entropy of an elementary occupied/unoccupied fermionic face-exclusion defect. The source theorem uses $\ln(7/6)$ to normalize the local scalar insertion into the lattice response, while the electron anchor uses $\ln 2$ to fix the mass–entropy unit of the elementary fermionic defect.

Exclusion sufficiency lemma. The reduction of the 1680-state boundary ensemble to a single scalar source is, at the classical source level, exact rather than approximate. The canonical exclusion defect is the vacuum ensemble conditioned on the exclusion event A, $P_{\text{def}} = P_{\text{vac}}(\,\cdot\,|A)$, so its likelihood ratio is

$$\frac{dP_{\text{def}}}{dP_{\text{vac}}} = \frac{\mathbf{1}_A}{P_{\text{vac}}(A)},$$

a function of the exclusion indicator $X = \mathbf{1}_A$ alone. The indicator is therefore a sufficient statistic for distinguishing the defect ensemble from the vacuum, and coarse-graining onto it preserves the full classical relative information,

$$D(P_{\text{def}}\|P_{\text{vac}}) = -\ln P_{\text{vac}}(A) = D(P_{\text{def}}^X\|P_{\text{vac}}^X).$$

The scope of the lemma is exactly its statement: one scalar statistic carries all the local classical source information distinguishing the canonical exclusion-defect ensemble from the vacuum. It does not by itself establish that the condensate has a single infrared mode, that orientation or defect-species structure is dynamically irrelevant, or that spatial correlations carry no further information; those are dynamical questions, and the open one is stated in Appendix H.7. For independent exclusions the source is exactly additive, $-\ln P(\bigcap_i A_i) = \sum_i -\ln P(A_i)$, so the leading dilute source is linear in the defect number, with correlation corrections entering at pair order in the density.

To propagate that local defect into the lattice field equation one needs the on-site Green function of the tetrahedral/diamond nearest-neighbor Laplacian. The four-valent diamond graph is adopted as the coarse adjacency realization of the abstract face-sharing tetrahedral boundary complex; the complex enters combinatorially, since regular tetrahedra admit no face-to-face Euclidean tessellation (the dihedral angle $\arccos(1/3) \simeq 70.53°$ does not divide $360°$), so the adopted graph and its Green function are the working objects while cell volumes and face areas remain conventions of the coarse map. Eliminating the two-sublattice structure gives the standard Brillouin-zone representation for the diamond lattice Green function [5]

$$G_{\mathrm{tet}}(0) = \frac{1}{(2\pi)^3} \int_{[-\pi,\pi]^3} \frac{4\, d^3k}{16 - |1 + e^{ik_1} + e^{ik_2} + e^{ik_3}|^2}.$$

This integral is the reproducible source-side lattice constant: it is the self-energy of a unit point insertion for the same scalar mode whose long-wavelength stiffness was matched in Appendix C.4. Joyce's exact evaluation of the diamond-lattice Green function, in this normalization, gives [7, 5]

$$G_{\mathrm{tet}}(0) = \frac{3\Gamma(1/3)^6}{2^{14/3}\pi^4} = 0.4482203943883814\ldots.$$

Thus the source-side graph constant is an exact lattice invariant rather than a fitted numerical coefficient. Direct quadrature with endpoint extrapolation reproduces the same value.

Green-tensor transverse response. The same graph response also fixes the transverse export weight $w_\perp$, the single graph quantity that propagates downstream into the horizon channel count of Appendix F.5 and into the global $\ln(3/2)$ closure-saturation factor of the scale-setting relation. We compute it directly from the four nearest-neighbor bond frame, with no fitting freedom.

Let d_i, $i = 1, \ldots, 4$, be the four diamond nearest-neighbor bond directions,

$$d_1 = \frac{(1,1,1)}{\sqrt{3}}, \quad d_2 = \frac{(1,-1,-1)}{\sqrt{3}}, \quad d_3 = \frac{(-1,1,-1)}{\sqrt{3}}, \quad d_4 = \frac{(-1,-1,1)}{\sqrt{3}}.$$

They obey

$$\sum_{i=1}^{4} d_i^a d_i^b = \frac{4}{3}\delta^{ab}.$$

Distributing the scalar on-site Green response over the tetrahedral bond frame gives

$$\mathcal{G}^{ab}_{\mathrm{loc}} = G_{\mathrm{tet}}(0)\frac{1}{4}\sum_i d_i^a d_i^b = \frac{G_{\mathrm{tet}}(0)}{3}\delta^{ab}.$$

For a local horizon normal $\hat{r}$,

$$P_\perp^{ab} = \delta^{ab} - \hat{r}^a\hat{r}^b,$$

so

$$G_\perp = P_\perp^{ab}\mathcal{G}^{ab}_{\mathrm{loc}} = \frac{2}{3}G_{\mathrm{tet}}(0).$$

Thus the transverse export weight $w_\perp = 2/3$ is the transverse part of the exact local graph response.

Using the field normalization $S = \pi Q_{\mathrm{occ}}$, the rigid local defect shift is

$$\delta Q_{\mathrm{def}} = \frac{\Delta S_{\mathrm{def}}}{\pi} = \frac{\ln(7/6)}{\pi}.$$

The corresponding local source amplitude in lattice units is therefore

$$s_{\rm def} = J_{\rm eff}^{\rm (ren)} \frac{\delta Q_{\rm def}}{G_{\rm tet}(0)},$$

so that

$$\frac{s_{\rm def}}{J_{\rm eff}^{\rm (ren)}} = \frac{\ln(7/6)}{\pi G_{\rm tet}(0)} = 0.109472228\ldots$$

is a pure number fixed by the same UV lattice geometry.

The Green-function constant turns the local insertion into a continuum source theorem. Defining the defect-entropy density by

$$\sigma_{\rm def} = \frac{\rho}{\kappa_m(L_*)},$$

the tetrahedral projection used in the stiffness mapping gives

$$\nabla^2 \delta S = -\frac{3L_*}{4G_{\rm tet}(0)}\,\sigma_{\rm def}.$$

Equating this with the weak-field source equation

$$\nabla^2 \delta S = -\frac{\kappa}{\gamma}\rho$$

closes the canonical source-to-stiffness ratio:

$$\boxed{\frac{\kappa}{\gamma} = \frac{3L_*}{4G_{\rm tet}(0)\kappa_m(L_*)}}.$$

This is the source-side counterpart of the stiffness derivation. The edge-kernel calculation fixes how the scalar capacity mode resists gradients; the Green-matched defect calculation fixes how localized matter defects source that same mode.

The three cancellations, made explicit. The passage from the dimensionless lattice insertion $s_{\rm def}/J_{\rm eff}^{\rm (ren)} = \ln(7/6)/(\pi G_{\rm tet}(0))$ to the source theorem turns on three reductions, none of which leaves a free constant. *(i) The factor π cancels against the field normalization.* The insertion is written for the occupancy field, where one isotropically averaged defect shifts $\delta Q_{\rm def} = \ln(7/6)/\pi$; a fixed defect type (ℓ, f) instead carries $-\ln P(A_{\ell f})$, ranging over $[0.14933, 0.15819]$ with mean 0.15415642 against $\ln(7/6) = 0.15415068$, so the source theorem uses the exact isotropic benchmark under the assumption that the coarse defect population is isotropically averaged over face and label types. Converting to the entropy field through $S = \pi Q_{\rm occ}$ multiplies by π, so the S-field shift is $\delta S_{\rm def} = \pi\,\delta Q_{\rm def} = \ln(7/6)$ and the explicit π does not survive into the source theorem. *(ii) $\ln(7/6)$ is an isotropic source benchmark, not a universal per-defect cost.* For a coarse population averaged over face and label types, the exact mean partition ratio is $6/7$, so the corresponding annealed source normalization is $\ln(7/6)$. A fixed defect type instead carries $-\ln P(A_{\ell f})$. Writing the isotropically averaged source as $\sigma_{\rm def} = \rho/\kappa_m(L_*)$ folds that benchmark and the mass–entropy unit into the density. The geometric coefficient $3L_*/(4G_{\rm tet}(0))$ is fixed by the tetrahedral projection and the cell length and contains no $\ln(7/6)$. *(iii) $J_{\rm eff}^{\rm (ren)}$ cancels in the ratio.* The source amplitude $s_{\rm def}$ and the stiffness γ each carry one power of the renormalized edge coupling, so it cancels in the source-to-stiffness ratio κ/γ, leaving the lattice-geometric constant $3L_*/(4G_{\rm tet}(0)\kappa_m(L_*))$. The renormalized loop coupling therefore drops out of the observable normalization.

The absolute scale of S_∞ remains a fixed-epoch normalization convention in the bridge law, not a residual freedom in the source projection. In the cell-normalized gauge natural to the local source theorem, one may write

$$S_\infty^{\mathrm{cell}} = \frac{3\ln 2}{32\pi G_{\mathrm{tet}}(0)} = 0.0461482516\ldots.$$

In a horizon-normalized gauge, S_∞ instead carries the much larger apparent-horizon capacity. These are not two physical constants. A constant rescaling of the entropy field rescales S_∞ and κ/γ together, leaving $\kappa/(\gamma S_\infty)$ and hence G unchanged. Thus the weak-field source map is closed in the canonical branch once the mass–entropy map $\kappa_m(L_*)$, the tetrahedral Green constant, and the standard cell convention are specified.

C.6 Local susceptibility cross-check

The exact local moments of the admissibility-closed ensemble supply an independent non-degeneracy check on the source-side result. From the variance in Appendix B,

$$a_{\mathrm{UV}} := \frac{1}{\mathrm{Var}_{\eta_*}(K^2)} = 0.0637390269,$$

which is the local zero-mode inverse susceptibility of the closure scalar. Two features of this number matter for the source theorem in C.5. First, a_{UV} is finite and positive, confirming that the closed branch at η_* has a non-degenerate zero-mode response rather than a critical singularity that would invalidate the linear Green-function matching used to derive κ/γ. Second, the same local susceptibility controls the stability of the closure mode propagated into the shell and loop calculations, so the residual fractional discrepancy between the local benchmark and σ_* belongs to the loop sector rather than the local closure.

The actual closure of κ/γ in the canonical branch is the Green-matched theorem in Appendix C.5. The role of a_{UV} here is restricted to confirming non-degeneracy of the local mode being matched.

C.7 UV-to-IR payoff

At this stage the weak-field UV coefficient chain is explicit:

$$\Omega_{\mathrm{tet}} \to g_{\mathrm{share,eff}} \to L_* \to J_{\mathrm{bare}} \to J_{\mathrm{eff}}^{\mathrm{tree}} \to \Sigma_{\mathrm{ret}} \to J_{\mathrm{eff}}^{(\mathrm{ren})} \to \gamma.$$

The same chain feeds

$$a_0 = \frac{cH_0 g_{\mathrm{share,eff}}}{4\pi^2},$$

and the decorated support-to-rate action gives the substrate-induced scale

$$G_* = \frac{c^3 L_*^2}{\hbar} = \frac{9}{4}\frac{\hbar c}{m_e^2} Z_e^2 \ln^2\!\left(1 - e^{-7 g_{\mathrm{share,eff}}}\right).$$

The Green-matched source theorem fixes

$$\frac{\kappa}{\gamma} = \frac{3L_*}{4G_{\mathrm{tet}}(0)\kappa_m(L_*)}.$$

The weak-field bridge then converts this source-to-stiffness ratio into the observed Newtonian normalization through the invariant combination $\kappa/(\gamma S_\infty)$. The comparison with the same G_* assigned by the electron-anchored support-to-rate branch tests that this normalization propagates the substrate scale coherently; it is not a disjoint-input second derivation of G, since the matched route carries L_* within it.

The coefficients this appendix supplies to the main weak-field chain are J_{bare}, $J_{\mathrm{eff}}^{\mathrm{tree}}$, Σ_{ret}, $J_{\mathrm{eff}}^{(\mathrm{ren})}$, γ, and the Green-matched source projection κ/γ. The remaining uses of S_∞ belong to the fixed normalization of the weak-field bridge, not to the source sector itself.

Appendix D: Weak-Field Technical Derivations, Electron Anchor, and EFT Consistency

Appendix D collects the weak-field derivations that are central but too dense for the main line: the bridge law, Newtonian recovery, the electron anchor, and the EFT consistency checks.

D.1 Bridge law from the reduced action

The weak-field bridge is now fixed by an action equivalence rather than by extrapolating an exponential lapse. The Einstein scalar-constraint action reduces to

$$I_{\rm Newton}[\Phi] = \int dt\, d^3x \left[-\frac{(\nabla\Phi)^2}{8\pi G} - \rho\Phi\right].$$

The capacity action becomes the same functional, up to the constant $Z_S = 2\kappa S_\infty/c^2$, under

$$\delta S = -\frac{2S_\infty}{c^2}\Phi.$$

Hence

$$\frac{\Phi}{c^2} = -\frac{\delta S}{2S_\infty}$$

is the unique linear field redefinition matching both the kinetic and source terms with the UV normalization. The bounded nonlinear rule is instead $N^2 = q = 1 - \delta S/S_\infty$ in a static branch. An exponential lapse shares the first derivative at the vacuum point but is not used globally because it has no finite-capacity endpoint. The complete derivation is in Appendix N.

D.2 Point source, Newton limit, and lensing

In the renormalized static branch,

$$\nabla^2\delta S = -\frac{\kappa}{\gamma}\rho.$$

For a point source M,

$$\delta S(r) = \frac{\kappa M}{4\pi\gamma r}, \qquad g(r) = \frac{c^2\kappa}{8\pi\gamma S_\infty}\frac{M}{r^2} = \frac{GM}{r^2}.$$

For the ordinary longitudinal branch, variation of the unreduced Einstein scalar action gives

$$\Phi = \Psi$$

with asymptotically flat boundary conditions. This equality follows from the spatial metric constraint, not from a canonical capacity stress tensor. The effective-halo rewrite of the additional galactic response is

$$\rho_{\rm halo}(r) = \frac{1}{4\pi G r^2}\frac{d}{dr}\left[r^2(g_{\rm obs} - g_{\rm bar})\right].$$

Thus the same metric potential controls orbital dynamics and light bending in the baseline Einstein branch. Whether the transverse thermal correction obeys $\Delta\Phi_\perp = \Delta\Psi_\perp$ is not fixed by this result; it must follow from the transverse metric influence functional. A viable galactic completion must reproduce support and lensing with that one conserved metric response.

D.3 Electron anchor and composite matter

The canonical fermionic entropy increment is

$$\Delta S_f = \ln 2.$$

The UV mass normalization is

$$\kappa_{m,\mathrm{UV}} = \frac{\hbar}{cL_*}\frac{1}{\ln 2},$$

and the running law in the closed branch is

$$\kappa_m(\ell) = \kappa_{m,\mathrm{UV}}\left(\frac{L_*}{\ell}\right)^{1+\alpha_{\mathrm{cl}}}, \qquad \alpha_{\mathrm{cl}} = 0.$$

At the electron Compton scale $\ell = \lambda_e$ this gives

$$\kappa_m(\lambda_e) = \frac{m_e}{\ln 2},$$

which is the elementary anchor used here. Composite hadrons are not reduced to a bare constituent count. Their mass budget is assigned to a dressed bound-state entropy

$$m_{\mathrm{hadron}} = \kappa_m(\ell_H) S^{\mathrm{dressed}}_{\mathrm{ent},H},$$

whose microscopic decomposition must include confinement, gluonic structure, trace-anomaly contributions, and chiral vacuum reorganization.

The electron is an elementary one-bit defect and provides the dimensional anchor. Hadronic inertial content instead belongs to a dressed entropy budget that includes binding and vacuum structure; a bare constituent count is insufficient.

D.4 Faithful sector resolution and induced G_*

The electron anchor enters the gravitational normalization through the absolute substrate length. The exact inputs are the seven-channel entropy, the memoryless kernel selected by faithful resolution, channel factorization on the lightest branch, and the decorated marked-transfer vertex of Appendix H. The calculation below separates the determinant recurrence, positive survival gap, and finite marked response before combining them in the physical cell length.

The recurrence derivation of the dictionary. Appendices H.2, H.4, and H.5 show that faithful full-support resolution selects memoryless replacement and that the lightest branch has seven factorized channels. On the commutative boundary algebra define $(\mathsf{R}f)(b) = p_*(b)f(b)$ and $\tau_p(f) = \sum_b p_b f(b)$. The state-weighted determinant is

$$\Delta_{\tau_p}(\mathsf{R}) = \exp[\tau_p(\ln \mathsf{R})] = \exp\left(\sum_b p_b \ln p_b\right) = e^{-g_{\mathrm{share,eff}}}.$$

Multiplicativity gives $e^{-7g_{\mathrm{share,eff}}}$ on the seven-channel product. This is the quenched geometric transfer rate of the record-conditioned likelihood operator. It is not the Born probability of a specified microstate or the collision probability of two renewed blocks.

The corresponding history statement follows from the exact recurrence theorem. For a stationary ergodic finite-alphabet process, if R_n is the first return time of a length-n block, then

$$\lim_{n\to\infty} \frac{1}{n} \ln R_n = h \qquad \text{almost surely,}$$

where h is the Shannon entropy rate [64, 65]. This theorem selects Shannon entropy over raw state count and Rényi-2 entropy for long typical blocks. It does not by itself identify the seven distinguishable sector layers with a long temporal block, and it gives no exact finite-n return probability. For the present one-layer distribution,

$$H_1 = 7.4198000, \qquad H_2 = -\ln \sum_b p_b^2 = 7.4125486, \qquad H_\infty = -\ln \max_b p_b = 7.1343850.$$

The distinctions are numerically consequential after seven layers. The refresh projector $|\sqrt{p}\rangle\langle\sqrt{p}|$ carries the participation scale e^{H_2}, while the determinant and almost-sure Lyapunov rate use H_1. They are different observables. With

$$p_{\rm rec} \equiv e^{-7g_{\rm share,eff}},$$

the positive survival operator has nonzero eigenvalue $1 - p_{\rm rec}$. The decorated marked vertex supplies the electron factor Z_e, so the dressed electron gap is

$$E_e = -\frac{3\hbar Z_e}{2\tau_*} \ln(1 - p_{\rm rec}).$$

Identifying the lowest charged gap with $m_e c^2$ fixes the cadence from the already-declared electron anchor,

$$\tau_* = -\frac{3}{2} Z_e \tau_e \ln(1 - p_{\rm rec}), \qquad L_* = c\tau_*.$$

Thus

$$L_* = -\frac{3}{2} Z_e \lambda_e \ln\big(1 - e^{-7g_{\rm share,eff}}\big) = \frac{3}{2} Z_e \lambda_e e^{-7g_{\rm share,eff}} \left[1 + O\big(e^{-7g_{\rm share,eff}}\big)\right].$$

The logarithmic survival correction is $p_{\rm rec}/2 \simeq 1.4 \times 10^{-23}$. The finite closure-response correction is $Z_e - 1 = 0.00530828$. No second dimensional input, maximum-throughput clock, or tetrahedral-diameter equality is introduced.

The transfer-support dual. A spatial version uses the same dressed support and assigns the leading step count

$$N_{\rm eff} = \frac{2}{3Z_e} e^{7g_{\rm share,eff}}.$$

The temporal grammar gives $\tau_e = N_{\rm eff}\tau_*$, while the spatial grammar gives $\lambda_e = N_{\rm eff} L_*$. Their common factor cancels and returns $L_* = c\tau_*$. This is a consistency identity of the same electron phase mode; the positive transfer spectrum supplies its frequency.

Internal locality and the native vertex. The selected replacement kernel reaches the full ensemble. The tested single-label move class does not: shifts preserving injectivity split each parity copy into $4! = 24$ ordering sectors of $\binom{7}{4} = 35$ states because two slots cannot exchange order without a collision. Its strength-weighted walk saturates at entropy 7.374, below $g_{\rm share,eff} = 7.4198$. This excludes that implementation. Appendix H.8 gives the whole-tetrahedron replacement gate, and H.9 decorates it with the fresh-state amplitude and charged event. A geometric GFT still has to realize that native gate and select a stable condensate.

The reduced electron Compton wavelength is

$$\lambda_e = \frac{\hbar}{m_e c},$$

and the admissibility-closed sharing entropy is

$$g_{\rm share,eff} = 7.41980002357.$$

The closure fixed point also gives

$$\langle K^2\rangle_{\eta_*} = \frac{3}{2\eta_*}, \qquad \eta_* = 0.0298668443935,$$

so the closure-saturation factor is

$$C_{\rm cl} := \eta_* \langle K^2\rangle_{\eta_*} = \frac{3}{2}, \qquad C_{\rm cl}^{-1} = \frac{2}{3}.$$

Separating the baseline recurrence from the finite marked response gives

$$\ln\!\left(\frac{\lambda_e}{L_*}\right) = 7g_{\rm share,eff} - \ln\!\left(\frac{3}{2}\right) - \ln Z_e + O(e^{-7g_{\rm share,eff}}).$$

Equivalently,

$$\boxed{L_*^{\rm leading} = \frac{3}{2}\, Z_e \lambda_e e^{-7g_{\rm share,eff}}}.$$

The exact expression is $L_* = -(3/2)Z_e\lambda_e \ln(1-e^{-7g_{\rm share,eff}})$. It gives

$$L_* = 1.6162537014 \times 10^{-35}\ \text{m},$$

which differs from the CODATA Planck length by about 8.2×10^{-7} in fractional terms, or 8.2×10^{-5} percent.

The induced gravitational scale follows from the same algebra used in the stiffness matching:

$$G_* := \frac{c^3 L_*^2}{\hbar}.$$

Substituting the exact marked-transfer relation gives the closed form

$$\boxed{G_* = \frac{c^3}{\hbar}\left[-\frac{3}{2}\, Z_e\lambda_e \ln\!\big(1-e^{-7g_{\rm share,eff}}\big)\right]^2 = \frac{9}{4}\,\frac{\hbar c}{m_e^2}\, Z_e^2 \ln^2\!\big(1-e^{-7g_{\rm share,eff}}\big)}.$$

Numerically,

$$G_* = 6.6742890772 \times 10^{-11}\ \text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2},$$

or -0.073σ relative to CODATA. The same ζ_* gives the muon and tau ratios at -0.535σ and $+0.481\sigma$. One finite action therefore replaces the three leading deficits with one correction whose coefficient and routing are fixed jointly rather than adjusted independently for the three observables. The discrepancies were already known while the vertex was being constructed, so these are high-precision postdictions. Their evidential content is the common action, the absence of a continuous fit, and the nearby kernels rejected by the finite audits.

The factor $3/2$ is fixed structurally by the tetrahedral transverse-export geometry. The tetrahedral identity gives the transverse export fraction

$$\frac{1}{4}\sum_{i=1}^{4}(\hat{n}_i \cdot \hat{u})^2 = \frac{1}{3}, \qquad f_\perp = 1 - \frac{1}{3} = \frac{2}{3},$$

so $f_\perp^{-1} = 3/2$. The admissibility fixed point returns the same value,

$$C_{\rm cl}^{-1} = \big(\eta_* \langle K^2\rangle_{\eta_*}\big)^{-1} = \frac{2}{3},$$

but it is built in. The condition defining η_* is $\langle K^2\rangle_\eta = 3/(2\eta)$, so $C_{\rm cl} = \eta_*\langle K^2\rangle_{\eta_*} = 3/2$ holds by construction. It is not an independent cross-check. The decorated scale action reuses its reciprocal $2/3$ as the transverse export and inserts it once as a global factor, not seven times.

A discrete sensitivity audit enumerates 576 nearby formulas obtained by varying the response dimension, pair count, direction factor, determinant power, statistics, and singlet overlap. Only the decorated-vertex formula lies within one muon standard deviation. This count is not a probability distribution over theories. It shows that the landing is sensitive to the field content and graph, which is why the action and its adversarial alternatives are displayed explicitly. A shared edge mode, directed complex determinant, stable bosonic determinant, or six-state symmetric response misses the muon respectively by approximately $1.15 \times 10^4\sigma$, 13.8σ, 56.7σ, and $7.64\times10^4\sigma$. Appendix H derives the selected entries before performing the observable trace.

The electron anchor carries three related roles. Its reduced Compton wavelength λ_e is the non-gravitational length used in faithful sector resolution. Its status as the lightest one-bit fermionic defect identifies which elementary excitation calibrates the seven-channel dressing block. Its mass fixes the mass–entropy map through

$$\kappa_m(\lambda_e) = \frac{m_e}{\ln 2}.$$

The same elementary defect enters the two normalization channels in distinct roles: its Compton length sets the UV cell scale, while its mass per one-bit defect entropy sets the source normalization.

The seven-sector scale-setting relation is not a counting heuristic; it has a concrete finite-dimensional realization on a transfer-operator history space, which we now construct. The face label algebra is

$$\mathcal{A}_7 = \bigoplus_{m=-3}^{3} \mathbb{C}E_m, \qquad E_mE_n = \delta_{mn}E_m, \qquad \sum_{m=-3}^{3} E_m = I_7.$$

The admissibility-closed tetrahedral state space has 1680 oriented injective states, with stationary weight

$$p_{\eta_*}(b) = Z^{-1}e^{-\eta_* K^2(b)}.$$

For a single sector, the refresh kernel

$$P_{\eta_*}(b,b') = p_{\eta_*}(b')$$

has Perron stationary entropy $g_{\rm share,eff}$. The electron does not simultaneously occupy seven mutually exclusive face labels in one tetrahedron; the labels are sector channels in the dressing history of the one-bit defect. The appropriate support object is therefore a history space, with one admissibility-closed sector layer for each $m = -3,\ldots,3$.

Let

$$\mathcal{H}_{\rm hist} = \bigotimes_{m=-3}^{3} \mathcal{H}_B^{(m)}, \qquad \dim\mathcal{H}_B = 1680,$$

and let $\mathcal{T}_m$ act as P_{η_*} on the mth factor and as the identity on the others. A representative one-pass dressing operator is

$$\mathcal{D}_e^{(0)} = \mathcal{T}_{-3}\mathcal{T}_{-2}\cdots\mathcal{T}_3.$$

The displayed order is only a representative of the symmetrized one-pass class. It introduces no physical ordering or extra factor of 7!. The one-pass prescription visits each simple sector once. A history with omitted sectors is unresolved, while additional visits belong to a multi-pass dressing history rather than to repeated fermionic occupation.

A sector-resolution step should not be identified with conditioning the 1680-state ensemble on boundary states containing a given label m. Such conditioning changes the entropy and does not reproduce $g_{\text{share,eff}}$. The sector label instead specifies which simple face-algebra channel is being resolved while the admissibility cloud sampled in that step remains the full closed boundary ensemble.

With the effective dimension defined by the stationary Shannon entropy of the positive history kernel, each sector contributes $g_{\text{share,eff}}$, so

$$\dim_{\text{eff}}(\mathcal{D}_e^{(0)}) = \exp(7g_{\text{share,eff}}) = 3.60286052 \times 10^{22}.$$

Only the transverse part of this support is exported into the weak-field scalar channel. The normalized export weight is

$$w_\perp = \frac{2}{3},$$

the same fraction fixed by the tetrahedral transverse projection and by the reciprocal closure-saturation factor. Hence

$$\dim_{\text{eff}}(\mathcal{D}_{e,\perp}) = w_\perp \dim_{\text{eff}}(\mathcal{D}_e^{(0)}) = \frac{2}{3}\, e^{7g_{\text{share,eff}}} = 2.40190701 \times 10^{22}.$$

The decorated marked-transfer action gives the exact support-scale relation

$$\boxed{\frac{\lambda_e}{L_*} = \frac{2}{3Z_e\left[-\ln\!\left(1-e^{-7g_{\text{share,eff}}}\right)\right]} = \frac{1}{Z_e}\dim_{\text{eff}}(\mathcal{D}_{e,\perp})\frac{r}{-\ln(1-r)}}.$$

Thus $\lambda_e/L_* = Z_e^{-1}\dim_{\text{eff}}(\mathcal{D}_{e,\perp})[1 - r/2 + O(r^2)]$. Appendix H derives the memoryless replacement, determinant recurrence, positive gap, and decorated charged vertex. Fermionic exclusion and subadditivity select $k = 7$ and $\Delta_7 = 0$, while the marked graph fixes Z_e. A stable geometric GFT embedding, durable history capacity, and the full source-coupled condensate spectrum remain open.

The relation is fixed inside the displayed finite transfer action and the electron anchor. Its remaining conditionality is the broader claim that this transfer action is the charged sector of the eventual geometric GFT, not an unfixed number in the finite calculation.

This is the microscopic content of the length formula above: the elementary one-bit fermionic defect is supported over one complete transverse-exported dressing block. The matched weak-field gravitational constant then follows from the gauge-invariant bridge

$$G = \frac{c^2}{8\pi}\frac{\kappa}{\gamma S_\infty},$$

with the entropy-unit normalization handled by the rescaling convention described in Appendix C.5.

D.5 EFT consistency checklist

The ordinary longitudinal branch is a rewriting of the Einstein constraint sector, so it introduces no independent scalar propagator to which a separate ghost or tachyon test could be applied. Its consistency checks and those of the optional transport completion must be stated separately.

- *No extra longitudinal degree of freedom.* The Einstein parent supplies the Cauchy data and propagating tensor modes; δS is the static scalar-constraint coordinate.
- *Correct-sign sourcing.* Positive mass produces a capacity deficit and attractive Newtonian response in the reduced action.

- *Positive static quadratic form.* The Euclidean capacity functional has positive stiffness $\gamma > 0$ after the overall gravitational-sign convention is fixed.
- *Conditional causal transport.* The phenomenological telegrapher completion has finite characteristic speed when $D/\tau_0 = c^2$ with $D, \tau_0 > 0$.

These statements do not establish a UV completion or quantize an additional scalar. They show that the controlled static representation is consistent with its Einstein parent and that the separately proposed transport equation is linearly stable in its stated parameter range.

The checklist is intentionally modest. It verifies the reduced Einstein representation and the signs of the separate transport model; it does not establish an independently quantized scalar EFT.

The one place where an explicit formula is worth recording is linear vacuum stability in the time-dependent sector. Writing a small perturbation δs about the vacuum branch, the linearized telegrapher equation is

$$\tau_0 \ddot{\delta s} + \dot{\delta s} - D\nabla^2 \delta s = 0.$$

For a plane-wave mode $e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}$, this gives the dispersion relation

$$\tau_0 \omega^2 + i\omega - Dk^2 = 0.$$

With $\tau_0 > 0$ and $D > 0$, the corresponding mode frequencies have non-growing time dependence, so the phenomenological transport vacuum is linearly stable.

D.6 Quadratic fluctuations and weak-field stability

Before imposing the Einstein scalar constraint, a formal covariant quadratic representation is

$$I^{(2)}_{\text{formal}}[\delta S] = -\int d^4x \sqrt{-g}\, \frac{\gamma}{2}\, g^{\mu\nu} \partial_\mu \delta S\, \partial_\nu \delta S.$$

After constraint reduction only its spatial Poisson quadratic form remains. The absence of a mass term expresses the unscreened constraint and does not prove an additional propagating boson. The transverse mode of Section 15 belongs to the separate conditional condensate completion of Appendix N.8.

Appendix D provides the technical support layer for the weak-field bridge, Newton limit, electron anchor, substrate length branch, and EFT consistency audit.

Appendix E: Transport, Cosmology, and Hubble-Tension Implementation

Appendix E collects phenomenological time-dependent and homogeneous extensions of the static capacity variable. These additions are not implied by the Einstein constraint reduction and must recover it in their static limit. The transport relation is fixed only in its preferred branch, while the homogeneous and perturbation sectors remain open.

E.1 Telegrapher equation and causal closure

The time-dependent deficit field obeys

$$\tau_0 \partial_t^2 \delta S + \partial_t \delta S = D\nabla^2 \delta S + A\chi, \qquad \frac{A}{D} = \frac{\kappa}{\gamma}.$$

Causality requires
$$\frac{D}{\tau_0} = c^2.$$
In the canonical no-new-IR-scale branch,
$$\tau_0^{-1} = H_0, \qquad D = \frac{c^2}{H_0}.$$

The telegrapher form is the minimal causal completion of the static Poisson sector. It supplies propagation and relaxation while preserving the static weak-field law as its exact late-time limit.

E.2 Static-limit recovery for galaxies

For a Fourier mode k, the telegrapher characteristic equation
$$\tau_0 s^2 + s + Dk^2 = 0$$
has the roots
$$s = -\frac{1}{2\tau_0} \pm i\omega_k, \qquad \omega_k \simeq ck$$
whenever $4\tau_0 Dk^2 \gg 1$. Galactic wavelengths are far below the critical scale
$$\lambda_c = \frac{4\pi c}{H_0} \approx 54 \text{ Gpc},$$
so galactic modes are deeply underdamped. Time-averaging the sourced solution over intervals large compared with $2\pi/\omega_k$ returns the static Poisson branch exactly, and the residual ponderomotive correction scales parametrically as
$$\frac{\delta F_{\text{pond}}}{F_{\text{static}}} \sim e^{-T/(2\tau_0)} \left(\frac{\omega_{\text{orb}}}{\omega_k}\right)^2 \lesssim 10^{-6}$$
for representative orbital speeds, with the precise value depending on the averaging interval and system scale. The estimate keeps the correction below the near-stationary weak-field branch but does not supply a universal 10^{-8} bound.

E.3 Homogeneous mode and cosmological sourcing

The cosmological split is
$$S(x,t) = \overline{S}(t) + s(x,t),$$
with $\overline{S}(t)$ the homogeneous mode and $s(x,t)$ the inhomogeneous weak-field sector. The background capacity is normalized by the apparent horizon,
$$S_\infty(t) = \pi\frac{R_A(t)^2}{L_*^2}, \qquad R_A(t) = \frac{c}{\sqrt{H^2 + kc^2/a^2}}.$$

Because the field couples to the trace of the stress-energy tensor, the homogeneous mode is suppressed during radiation domination and turns on near matter–radiation equality.

This timing is the central cosmological virtue of the mechanism. The homogeneous mode is quiet when it must be quiet, then becomes relevant close to the epoch where a sound-horizon shift is most useful.

E.4 Sound-horizon shift and shear lock

In the conditional cosmological proposal, the trace-sourced homogeneous mode acts as a transient early-energy contribution. It is intended to reduce the sound horizon without rewriting the local static Poisson law. Demonstrating that separation together with the committed component requires the open joint Boltzmann calculation.

The homogeneous mode can alter the cosmological background while leaving the coefficients of the local weak-field branch unchanged. A quantitative perturbation calculation remains open.

The transport relation and preferred branch are closed; the cosmological sector remains structurally supported but not yet Boltzmann-closed.

Appendix F: Strong-Field Spherical Reduction

Appendix F records the strongest action-level statement currently available in the black-hole sector. The bounded capacity rule is realized invariantly in spherical symmetry by the areal-radius gradient $q_{\rm geo} = (\nabla R)^2$. The Schwarzschild exterior and the location $q_{\rm geo} = 0$ then follow from the reduced Einstein action. Treating that surface as the physical end of the substrate EFT, however, is an additional domain postulate whose boundary microphysics remains open.

F.1 Bounded capacity and the unique lapse map

The strong-field order parameter is the surviving-capacity fraction

$$q(x) = \frac{S_{\rm ent}(x)}{S_\infty} \in [0, 1].$$

This bound follows directly from finite local channel capacity. If the vacuum channel count is finite and $S_{\rm ent}$ is the logarithmic coarse entropy of the surviving local ensemble, then no physical branch can have either negative capacity or more than the asymptotic vacuum capacity.

In a static exterior, the lapse associated with the asymptotic Killing time is determined by the local surviving capacity. Let

$$N = f(q).$$

The conditions are:

$$f(1) = 1, \qquad \lim_{q\to 0^+} f(q) = 0.$$

The substrate-level composition axiom is that independent serial capacity losses compose multiplicatively on the lapse:

$$f(q_1 q_2) = f(q_1) f(q_2).$$

This is the assumption that extends the linear weak-field match to a nonlinear lapse map. With continuity, the positive solutions on $(0, 1]$ are $f(q) = q^\alpha$. Expanding near $q = 1 - \epsilon$ gives

$$N = q^\alpha = 1 - \alpha\epsilon + O(\epsilon^2).$$

The weak-field bridge gives

$$N = 1 - \frac{\epsilon}{2} + O(\epsilon^2),$$

so $\alpha = 1/2$ and therefore

$$N = \sqrt{q}, \qquad N^2 = q.$$

Equivalently, if one writes $N^2 = F(q)$, the unique continuous multiplicative completion is $F(q) = q$. The nonlinear static lapse rule is therefore fixed by capacity composition and weak-field matching; it is not a freely chosen black-hole ansatz. Because a lapse depends on foliation, this is a static constitutive statement. Its covariant spherical content is derived next.

F.2 Einstein action reduced to the capacity-adapted invariant

Take the most general spherically symmetric line element

$$ds^2 = h_{ab}(x)dx^a dx^b + R^2(x)d\Omega^2.$$

Here $x^0 = ct$, so the two-dimensional measure is $d^2x = dx^0 dr$. The same action written as $dt\,dr$ acquires one additional factor of c. The four-dimensional curvature decomposes as

$${}^{(4)}R = {}^{(2)}R + \frac{2}{R^2}\left[1 - (\nabla R)^2 - 2R\Box R\right].$$

After the angular integral, the Einstein–Hilbert plus GHY action becomes, up to the asymptotic and corner terms retained in $I_\partial^{(2)}$,

$$I_{\rm sph} = \frac{c^3}{4G}\int d^2x\,\sqrt{-h}\left[R^{2\,(2)}R + 2(\nabla R)^2 + 2\right] + I^{(2)}_{\rm matter} + I^{(2)}_\partial.$$

The integration by parts that converts $-4R\Box R$ into $+4(\nabla R)^2$ is legitimate only together with the reduced GHY contribution. The boundary term is therefore part of the statement. For $\partial\mathcal{M}_4 = \partial\mathcal{M}_2 \times S^2$ one has

$$K^{(4)} = K^{(1)} + \frac{2}{R}n^a\nabla_a R.$$

The second term cancels the surface term from the integration by parts, leaving

$$I_\partial^{(2)} = \varepsilon\,\frac{c^3}{2G}\int_{\partial\mathcal{M}_2} dy\,\sqrt{|\gamma_{(1)}|}\,R^2K^{(1)} + I_{\rm joint} + I_{\rm ref},$$

with ε the standard orientation sign.

Define

$$q_{\rm geo} \equiv (\nabla R)^2 = h^{ab}\partial_a R\partial_b R.$$

Varying R and h^{ab} in vacuum gives

$$R\,{}^{(2)}R - 2\Box R = 0,$$

$$2R\left(h_{ab}\Box R - \nabla_a\nabla_b R\right) + h_{ab}(q_{\rm geo} - 1) = 0.$$

These equations imply

$$\nabla_a M_{\rm MS} = 0, \qquad M_{\rm MS} = \frac{c^2 R}{2G}(1 - q_{\rm geo}).$$

Therefore

$$q_{\rm geo} = 1 - \frac{2GM_{\rm MS}}{c^2 R}.$$

The variational status is now unambiguous: $q_{\rm geo}$ is a composite of the two-dimensional metric and the areal-radius dilaton, and its vacuum profile is a first integral. No scalar has been added to the Einstein degrees of freedom.

Audit of the discarded multiplier form. If instead one writes an ADM term $\sqrt{h}\,\lambda(N^2 - q)$ while giving q no other bulk dependence, the q equation sets $\lambda = 0$. The constraint then only identifies an arbitrary scalar with a foliation-dependent lapse. It does not reproduce the capacity source equation and it has no invariant content away from a specified static slicing. The spherical reduction above supplies the invariant result that construction was trying to capture and supersedes it.

F.3 Variational status of the $q_{\rm geo} = 0$ boundary

For $\epsilon > 0$, let $\mathcal{B}_\epsilon$ be the timelike level surface $q_{\rm geo} = \epsilon$. The exterior Einstein problem is well posed with the standard term

$$I_{\rm GHY}[\mathcal{B}_\epsilon] = \varepsilon\,\frac{c^3}{8\pi G}\int_{\mathcal{B}_\epsilon} d^3y\,\sqrt{|\gamma|}\,K$$

and fixed induced metric on $\mathcal{B}_\epsilon$, together with the reference term at infinity and any required joints. The limit $\epsilon \to 0^+$ is null and must be taken using the corresponding null-boundary and joint prescription; a bare timelike GHY expression cannot simply be evaluated at the null surface.

This construction fixes the universal gravitational variation. It does not make

$$q_{\rm geo}\big|_{\partial\mathcal{M}_q} = 0$$

a new Euler–Lagrange boundary condition. The zero is the level set at which the spherical invariant becomes marginal. Declaring that level set to be the end of the physical substrate domain,

$$\mathcal{M}_q = \{q_{\rm geo} > 0\},$$

is the bounded-capacity postulate.

Any additional functional

$$\Gamma_{\partial q}[\sigma_{AB}, \text{boundary channels}]$$

would describe genuine substrate physics: absorption, partial reflection, relaxation, entropy, or a moving-boundary stress. Its variation may contain a boundary stress and a response conjugate to the limiting capacity, but neither its form nor its spectrum follows from Einstein–Hilbert reduction. The static exterior can be solved without inventing this term; claims about excision, infalling evolution, or finite reflectivity cannot.

F.4 Static spherical vacuum exterior

In static spherical vacuum, the result follows immediately. On $\mathcal{M}_q$, the bulk equations are the vacuum Einstein equations. The unique asymptotically flat static spherical solution is the Schwarzschild exterior,

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)c^2dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1}dr^2 + r^2 d\Omega^2,$$

so the spherical geometric representative of the capacity variable is

$$q_{\rm geo}(r) = N^2(r) = 1 - \frac{2GM}{c^2 r}, \qquad r > r_h,$$

with

$$r_h = \frac{2GM}{c^2}.$$

This also agrees with the weak-field capacity deficit:

$$\frac{\delta S(r)}{S_\infty} = \frac{2GM}{c^2 r}, \qquad q(r) = 1 - \frac{\delta S(r)}{S_\infty}.$$

The exterior $r > r_h$ is exactly the standard Schwarzschild exterior, and $q_{\rm geo} = 0$ at $r = r_h$. The geometric invariant becomes negative in the trapped region. Interpreting that sign change as exhaustion of a nonnegative substrate capacity motivates restricting the capacity EFT to $q_{\rm geo} \geq 0$, but this restriction is not implied by the Einstein equations. The absence of a physical classical interior is therefore a conditional substrate claim, not an action-level theorem.

The exterior-domain result does not decide what an infalling observer experiences at the $q_{\rm geo} = 0$ surface. Whether a capacity-exhaustion boundary is smooth, dissipative, anomalous, or absent requires the open functional $\Gamma_{\partial q}$, not merely the exterior Schwarzschild solution.

Geometric identification of the capacity variable. The spherical reduction supplies a normalization-independent gradient invariant on the orbit space,

$$q_{\rm geo} = h^{ab}\partial_a R\, \partial_b R = |\nabla R|^2,$$

which in Schwarzschild gives

$$q_{\rm geo} = 1 - \frac{2GM}{c^2 r} = N^2,$$

which can be identified with the substrate fraction $q = S_{\rm ent}/S_\infty$ on the exterior. In spherical dynamical collapse with Misner–Sharp mass $M_{\rm MS}(R,t)$,

$$q_{\rm geo}(R,t) = 1 - \frac{2GM_{\rm MS}(R,t)}{c^2 R}.$$

The marginal-trapped-surface condition $\theta_+\theta_- = 0$ coincides with $q_{\rm geo} = 0$ independently of null normalization: positive, zero, and negative $q_{\rm geo}$ label untrapped, marginal, and trapped spherical regions. Standard collapse in the metric parent can therefore produce the geometric zero. Equating that zero with substrate saturation and refusing the negative branch are the additional bounded-capacity interpretation to be tested by the transport and boundary theory.

Rotating and charged stationary exteriors. The baseline metric parent also admits Kerr, Reissner–Nordström, and Kerr–Newman exteriors when the appropriate conserved gauge sector is present. Their standard horizon entropy

$$S = \frac{k_B A}{4L_*^2}$$

retains its form with A the appropriate horizon area, and the Hawking temperature follows from the surface gravity as usual,

$$T_H = \frac{\hbar \kappa_{\rm sg}}{2\pi k_B c}.$$

A nonspherical covariant capacity scalar whose zero selects the outer horizon and whose sign defines a physical domain has not yet been derived. The absence of inner or trapped regions therefore cannot be inferred from the spherical invariant alone.

F.5 Horizon thermodynamics and boundary capacity

Because the exterior geometry is unchanged, semiclassical quantities depending only on the exterior near-horizon saddle are unchanged. The Euclidean continuation is used here as an exterior-saddle calculation: the resulting periodicity depends on regularity of the near-horizon exterior geometry, not on adopting the bounded-domain interpretation past $q_{\rm geo} = 0$. The Euclidean regularity argument therefore gives the standard Hawking temperature [10, 11],

$$T_H = \frac{\hbar c^3}{8\pi GM k_B}.$$

For the GR exterior saddle, the same Euclidean calculation gives the Bekenstein–Hawking area law [9, 10],

$$S_{\rm BH} = \frac{k_B A}{4L_P^2}.$$

Two substrate inputs fixed by the bulk EFT combine to give the 1/4 area coefficient exactly. The identity does not supply the microscopic count that converts boundary channels to physical area. That conversion remains an explicit open step below.

Per-channel cut entropy from fermionic face exclusion. If the bounded-capacity domain terminates at the spherical surface $q_{\rm geo} = 0$, a tetrahedral cell on the exterior side whose outward face would be paired with a neighbor across that face instead has an unfilled pairing slot. By Postulate II the elementary face slot is fermionic and admits only the occupied (paired) state or the excluded (unpaired) state. A horizon cut face is then an instance of the same elementary face-exclusion defect that anchors the one-bit fermionic sector in the bulk. The seven-state $j_{\rm eff} = 3$ channel belongs to the paired bulk link $j_0 \otimes j_0 \to j_{\rm eff}$, not to the cut face itself: with the partner cell absent, the paired representation is never formed. The seven-state structure enters the boundary count through the bulk graph response and hence through the channel density below, not through the per-channel entropy. Conditional on the domain postulate, the entropy carried by an elementary cut defect is consequently the same primitive fermionic increment that anchors the electron at $\kappa_m(\lambda_e) = m_e/\ln 2$,

$$\Delta S_f = \ln 2.$$

Channel density from the transverse bulk graph response. Because the local graph Green tensor is isotropic,

$$\mathcal{G}^{ab}_{\rm loc} = \frac{G_{\rm tet}(0)}{3}\delta^{ab},$$

a codimension-one horizon cut with local normal $\hat{n}$ exports only the transverse two-plane component,

$$G_\perp = (\delta^{ab} - \hat{n}^a\hat{n}^b)\mathcal{G}^{ab}_{\rm loc} = \frac{2}{3}G_{\rm tet}(0).$$

The horizon channel density is therefore not a new boundary response coefficient; it is the transverse projection of the same bulk Green response already fixed in Appendix C. The channel weight per outward angular direction is $G_\perp/\ln 2$, and integrating over the horizon two-surface (the spherical angular measure in Schwarzschild, the smooth axisymmetric horizon for Kerr, with isotropic transverse response in either case) gives

$$n_{\rm hor} = 4\pi\frac{G_\perp}{\ln 2} = \frac{8\pi G_{\rm tet}(0)}{3\ln 2}.$$

This $n_{\rm hor}$ is a dimensionless Green-response number integrated over the angular measure. It is not yet a count of independent boundary channels per unit physical area: the conversion from channels to A/L_*^2 requires a microscopic area operator or graph-flux count on the embedded cut — diamond-lattice bonds crossing a surface can be counted once a bond length and surface prescription are chosen, but that count is new structure the framework does not currently derive, and its numerical test is the graph-ensemble Monte Carlo listed among the closure tests of Appendix F.7.

Closure as an exact identity. Using the cell-normalized capacity baseline from Appendix C,

$$S_\infty^{\rm cell} = \frac{3\ln 2}{32\pi G_{\rm tet}(0)},$$

the product of the two substrate inputs is

$$n_{\rm hor} S_\infty^{\rm cell} = \frac{1}{4},$$

an exact identity in which the Joyce diamond-lattice constant $G_{\rm tet}(0)$ and the fermionic increment $\ln 2$ cancel between the two factors. In the cell-normalized horizon convention — one independent channel per cell area L_*^2 of the cut — a horizon of area A then carries the dimensionless entropy

$$\frac{S_{\rm hor}}{k_B} = \frac{A}{4L_*^2},$$

and the Bekenstein–Hawking coefficient is recovered after the gravitational scale is matched so that $L_P(G_*) = L_*$. This is a horizon normalization identity. Within the stated convention, the bulk Green-response number and capacity normalization combine algebraically to 1/4. The per-channel $\ln 2$ from fermionic face exclusion and the transverse projection 2/3 are substrate inputs; the channel-to-area conversion remains conventional. A microscopic area operator that derives this conversion would complete the count. The nonuniversal boundary spectroscopy, including the microscopic Hamiltonian behind relaxation, stretched-layer corrections, and transient response, also remains open.

Composition of the coefficient. Written as a product of its surviving factors, the coefficient is

$$\frac{1}{4} = \frac{2}{3} \times \frac{3}{8},$$

and the 2/3 is the transverse projection $G_\perp/G_{\rm tet}(0)$ — the same export factor that sets the electron's leading relation $\lambda_e/L_* = (2/3Z_e)e^{7g_{\rm share,eff}}[1+O(e^{-7g_{\rm share,eff}})]$ (Section 13.3) and enters the loop self-energy through the edge map $(2/3)(1/3) = 2/9$ (Section 8). The marked factor Z_e dresses the charged support without changing this geometric projection. A change to the transverse export would still displace the electron, stiffness, and horizon normalizations together.

Coherence with the continuum information results. The identity supplies a normalization consistency check. With the exterior-saddle temperature, the Clausius relation at entropy density $1/(4L_*^2)$ returns the induced constant $c^3L_*^2/\hbar = G_*$ through the Jacobson construction [21]. The horizon and bulk routes share the same normalization conventions, so their agreement is not independent evidence. A microscopic channel-to-area count would upgrade the identity to a derivation.

The three external constructions address distinct continuum steps. Chandrasekaran et al. make the empty de Sitter static patch the maximum-entropy state of an observer-dressed type II_1 algebra and recover generalized entropy for semiclassical states [23]. Dorau and Much equate the relative entropy of a coherent scalar excitation on a bifurcate Killing horizon with its Killing-weighted energy flux; their derivation of Einstein's equation assumes the entropy–area relation and does not supply the substrate count [22]. Bianconi starts with a local volumetric metric-mismatch entropy, recovers Einstein gravity at low curvature, and obtains $S_{\rm dS} \propto H^{-2}$ after integration over a causal diamond [24]. These results respectively support the maximum-capacity reference state, the information–energy–curvature relation, and the emergence of area scaling from a bulk relative-information density. None derives the channel-to-area conversion or its coefficient.

One structural distinction must travel with any composition of these results. Ordinary regional relative entropy is unbounded, a type II_1 algebra supplies an upper entropy bound without a finite-dimensional local state space, and the substrate variable is both bounded and assigned to a finite cell. The identification $\delta S_{\rm substrate} \propto S_{\rm rel}$ can therefore hold only in the weak, local regime, with the microscopic finite capacity providing the saturating continuation as $q \to 0$.

F.6 Absorption, ringdown, and echoes

The exterior Regge–Wheeler/Zerilli operators are unchanged. With $q_{\text{geo}} = 0$ at the marginal surface, the tortoise coordinate $r_* \sim r_h \ln q_{\text{geo}}$ sends the surface to $r_* \to -\infty$, and the near-horizon wave equation reduces to $(c^{-2}\partial_t^2 - \partial_{r_*}^2)\psi \simeq 0$. If the usual GR future-horizon regularity condition is retained, it selects

$$\psi \sim e^{-i\omega(t+r_*/c)},$$

and therefore

$$\mathcal{R} = 0.$$

The standard greybody factors and quasinormal spectrum then follow [12, 13]. But if the substrate EFT truly terminates at the marginal surface, future-horizon regularity is a boundary choice, not a consequence of the exterior differential operator alone. The open functional $\Gamma_{\partial q}$ must determine whether that choice is correct.

If the microscopic boundary has finite reflectivity or lies on a stretched layer

$$q = \epsilon > 0,$$

the region between the exterior potential barrier and that layer behaves as a cavity. A typical echo delay then scales as

$$\Delta t_{\text{echo}} \sim \frac{2r_h}{c}|\ln \epsilon| + \tau_{\text{ch}},$$

where τ_{ch} is a boundary-channel relaxation time. Thus $\mathcal{R} = 0$ is the GR-matching boundary condition, while $\mathcal{R}(\omega)$ is a boundary observable to be calculated rather than assumed.

F.7 Dynamical formation as a free-boundary problem

Standard collapse in the metric parent can form a marginal surface $q_{\text{geo}} = 0$. It does not prove that a substrate capacity field saturates there or that the physical evolution terminates. That identification requires a bounded causal transport law for a capacity variable q_{cap}, together with a constitutive relation showing $q_{\text{cap}} = q_{\text{geo}}$ in the regime of overlap. A candidate test system is

$$\partial_t q_{\text{cap}} + D_i J^i = -\Gamma(q_{\text{cap}})\Sigma[T_{\mu\nu}],$$

$$\tau_J(\partial_t + \mathcal{L}_v)J^i + J^i = -D(q_{\text{cap}})D^i q_{\text{cap}}.$$

Here J^i is the capacity flux, $\Sigma[T_{\mu\nu}]$ is a positive depletion source built from the collapsing stress-energy, $D(q_{\text{cap}})$ is a bounded mobility, $\Gamma(q_{\text{cap}})$ is a bounded depletion rate, and $\tau_J > 0$ is a relaxation time. Eliminating J^i gives a telegrapher-type equation with finite characteristic speed

$$v_{\text{cap}} \sim \sqrt{\frac{D_0}{\tau_J}}.$$

Choosing trial constitutive functions such as

$$D(q_{\text{cap}}) = D_0 q_{\text{cap}}(1 - q_{\text{cap}}), \qquad \Gamma(q_{\text{cap}}) = \Gamma_0 q_{\text{cap}}$$

makes $q_{\text{cap}} = 0$ and $q_{\text{cap}} = 1$ invariant sets, preventing overshoot. These functions are examples, not a derived action.

If q_{cap} first reaches zero on a two-surface and the microscopic theory licenses domain termination, that surface becomes a moving boundary

$$\partial\mathcal{M}_q(t) = \{x \mid q_{\text{cap}}(t, x) = 0\}.$$

The level-set kinematics are fixed by differentiating $q_{\rm cap}(t, X(t)) = 0$ along the moving surface:

$$V_n = -\frac{\partial_t q_{\rm cap}}{|\nabla q_{\rm cap}|}\bigg|_{q_{\rm cap}\to 0^+}.$$

In spherical symmetry this becomes

$$\frac{dr_f}{dt} = -\frac{\partial_t q_{\rm cap}}{\partial_r q_{\rm cap}}\bigg|_{r=r_f(t)}.$$

During continued infall the exterior should be Vaidya-like with a slowly varying mass parameter, settling to the Schwarzschild exterior after the front stabilizes. This is a well-posed program, but not yet a closed derivation: the transport coefficients, boundary action, and channel relaxation spectrum must be computed from the graph ensemble or constrained by simulation.

The dynamical system is presumed to preserve the usual covariant conservation of the combined matter-plus-capacity stress-energy, with any local matter depletion balanced by flux, boundary work, or capacity-sector stress. Showing that this conservation structure follows from a graph-derived transport action, rather than imposing it as a constitutive condition, is part of the dynamical closure work.

The concrete closure tests are correspondingly specific. A spherical collapse simulation should show formation of the first $q = 0$ surface without overshoot into $q < 0$. A coupled matter-plus-capacity run should approach a Vaidya exterior during accretion and a Schwarzschild exterior after settling while satisfying the combined conservation law. Exterior perturbation simulations with an absorbing boundary should reproduce standard Schwarzschild greybody factors and ringdown, while partial-reflectivity runs should produce controlled echo delays. Finally, a microscopic boundary-action calculation or a graph-ensemble Monte Carlo of saturated boundary channels should reproduce the channel-counting rule that yields $n_{\rm hor} S_\infty^{\rm cell} = 1/4$. These are not new fit knobs; they are the numerical and microscopic tests that would close the dynamical and boundary sectors.

F.8 Weak-field boundary and closure statement

In the weak-field Solar-System regime, the metric-only parent yields

$$\gamma_{\rm PPN} = \beta_{\rm PPN} = 1,$$

with the remaining standard PPN coefficients vanishing under the usual assumptions. This follows because the ordinary longitudinal capacity functional is a reduced representation of Einstein gravity, not because a scalar correction happens to be small. The separate transverse galactic influence functional must still be shown to decouple sufficiently in the high-acceleration regime.

The weak-field capacity coordinate ceases to be adequate when

$$\frac{|\Phi|}{c^2} = O(1), \qquad \frac{\delta S}{S_\infty} = O(1),$$

which is the regime where the spherical invariant $q_{\rm geo}$ provides the controlled nonlinear description.

Appendix F therefore closes one precise strong-field statement: the spherically reduced Einstein action makes $q_{\rm geo} = (\nabla R)^2 = 1 - 2GM_{\rm MS}/(c^2 R)$ a composite first integral and returns the Schwarzschild exterior. The bounded capacity interpretation of $q_{\rm geo}$, the exclusion of the negative branch, and any physical boundary dynamics are additional hypotheses. The $1/4$ coefficient is an exact normalization identity within the cell-area convention, while the microscopic channel-to-area count remains open. Rotating and charged solutions belong to the baseline metric parent, but their capacity-variable completion is also open.

Appendix G: Many-Pasts, Operational Closure, Branch Realization, and the Arrow of Time

Appendix G separates the three probability objects used by Many-Pasts: amplitudes for unresolved alternatives, probabilities for decoherent record histories, and conditional probabilities for histories compatible with one present record. This repairs the ambiguity in the earlier notation $P(H \mid P) \propto e^{-D(H,P)}$. The operational construction is standard decoherent-histories quantum mechanics [66]; Many-Pasts supplies its record-conditioned ontology.

G.1 What is a history of the entanglement network?

A coarse projective history $h = (\alpha_1, \ldots, \alpha_n)$ is a sequence of alternatives at ordered substrate times. The alternative α_k is represented by a projector $\Pi^{(k)}_{\alpha_k}$. With unitary evolution $U_{k,k-1}$ between times, its class operator is

$$C_h = \Pi^{(n)}_{\alpha_n} U_{n,n-1} \Pi^{(n-1)}_{\alpha_{n-1}} \cdots U_{2,1} \Pi^{(1)}_{\alpha_1} U_{1,0}.$$

This operator retains amplitudes. It is defined before any classical probability is assigned to the individual history.

Given an initial state ρ_0, the decoherence functional is

$$\mathcal{D}(h, h') = \mathrm{Tr}\Big(C_h \rho_0 C^\dagger_{h'}\Big).$$

A family admits ordinary probabilities when its off-diagonal terms are negligible at the required accuracy,

$$\mathcal{D}(h, h') \simeq 0 \qquad (h \neq h').$$

The diagonal entries $p(h) = \mathcal{D}(h, h)$ are then nonnegative and additive under coarse-graining. When alternatives do not decohere, their class operators must be added before the probability is evaluated. For $C_A = \sum_{h \in A} C_h$,

$$p(A) = \mathrm{Tr}\Big(C_A \rho_0 C^\dagger_A\Big),$$

which retains the interference terms. Many-Pasts places no classical distribution over unresolved fine-grained paths.

G.2 What is the present coarse configuration P?

The present P is a macroscopic record represented by a final projector Π_P. Let $\mathcal{H}_P$ be a decoherent family of histories whose final alternatives refine that record. Exhaustiveness and decoherence give

$$p(P) = \sum_{h \in \mathcal{H}_P} p(h) = \mathrm{Tr}(\Pi_P \rho_{\mathrm{now}}).$$

The conditional Many-Pasts measure is

$$\boxed{p(h \mid P) = \frac{p(h)}{p(P)}, \qquad h \in \mathcal{H}_P.}$$

The common measure over all possible records is normalized first; conditioning on the realized record comes afterwards. Normalizing a new set of histories separately for each already-selected present would leave the probabilities of the alternative presents undefined.

G.3 What is the distance $D(H, P)$?

For a decoherent history ending in P, define

$$D(h, P) = -\ln p(h),$$

with $D = +\infty$ when $p(h) = 0$. Then

$$p(h \mid P) = \frac{e^{-D(h,P)}}{\sum_{h' \in \mathcal{H}_P} e^{-D(h',P)}}.$$

The distance notation is shorthand for the diagonal decoherence-functional weight, not a second probability law. The earlier expression $-\ln \mathrm{Tr}(\Pi_P \rho_{H \to \mathrm{now}})$ is recovered when H already denotes a decohered preparation history and only the final record remains unresolved.

G.4 The operational Born branch

A laboratory setting x is represented by a quantum instrument $\{\mathcal{M}_{a|x}\}_a$. Each map is completely positive and trace non-increasing, while $\sum_a \mathcal{M}_{a|x}$ is trace preserving. For an initial state ρ,

$$p(a \mid x) = \mathrm{Tr}\big[\mathcal{M}_{a|x}(\rho)\big] .$$

If $C_{h,a|x}$ refines the histories ending in record a, then

$$p(h \mid a, x) = \frac{\mathrm{Tr}(C_{h,a|x} \rho C^{\dagger}_{h,a|x})}{p(a \mid x)}$$

for a decoherent refinement. The record marginal is the Born probability by construction. Its form is also unique under the hypotheses stated in Section 3.3: normalized noncontextual additivity on a sufficiently rich projector lattice gives $p(R) = \mathrm{Tr}(\rho R)$ by Gleason's theorem, and medium decoherence supplies generalized record projectors for pure branch states [60, 61]. The uniqueness belongs to the measure once quantum kinematics and record completeness are granted. It does not derive the Hilbert space, the state, or the decoherence functional from the substrate. A future substrate theory must reproduce that structure rather than merely fit its projective limit.

G.5 No-signaling

Let Alice and Bob act locally on ρ_{AB} with instruments $\{\mathcal{M}^A_{a|x}\}_a$ and $\{\mathcal{N}^B_{b|y}\}_b$. Their joint record probability is

$$p(a, b \mid x, y) = \mathrm{Tr}\Big[(\mathcal{M}^A_{a|x} \otimes \mathcal{N}^B_{b|y})(\rho_{AB})\Big] .$$

Summing over Bob's record gives

$$p(a \mid x, y) = \sum_b p(a, b \mid x, y) \tag{27}$$

$$= \mathrm{Tr}\Big[(\mathcal{M}^A_{a|x} \otimes \mathcal{N}^B_{y})(\rho_{AB})\Big] \tag{28}$$

$$= \mathrm{Tr}\Big[\mathcal{M}^A_{a|x}(\rho_A)\Big] = p(a \mid x), \tag{29}$$

where $\mathcal{N}^B_y = \sum_b \mathcal{N}^B_{b|y}$ is trace preserving. Alice's marginal is independent of y, and the same calculation applies to Bob. Global conditioning on a joint present record does not create a controllable signaling channel because the unconditioned local marginals remain those of standard quantum mechanics.

G.6 Branch realization

The postulate contains one realized present with its records. Its multiplicity lies among the admissible *pasts* of that present, with no forward branching into co-real macroscopic worlds and no collapse event selecting among them. An outcome is specified by the records that obtain in the present, and the Born weights give their statistics. This account of branch realization adds neither many worlds nor a collapse dynamics.

The present record algebra fixes the resolution at which past alternatives are physically distinguished. Let $\{R_i\}_{i\in I}$ be mutually orthogonal fine records that a later physical channel maps to one coarse record $\bar{R} = \sum_{i\in I} R_i$. For a decoherent joint refinement,

$$p(h, \bar{R}) = \sum_{i\in I} p(h, R_i), \qquad p(h \mid \bar{R}) = \frac{\sum_{i\in I} p(h, R_i)}{\sum_{h'} \sum_{i\in I} p(h', R_i)}.$$

This is the push-forward of the same joint quantum measure. Record loss needs no additional rule and does not alter the earlier unitary dynamics. It reduces the distinctions represented in the realized present. The history register of H.8 is one microscopic carrier of such distinctions; environmental and macroscopic records are others. The paper does not assume an inaccessible second archive after every physical carrier of a distinction has been erased.

The conditioning also has no forward dynamical role. The class operator C_h, the state, and the quantum instruments determine $p(h, P)$ before the ratio $p(h \mid P)$ is formed. A later record permits retrodictive conditioning once it exists; the conditional probability never appears as a force in the preceding evolution. Many-Pasts therefore preserves the causal and no-signaling content of the imported quantum channel structure.

This record-retention reading removes one apparent cosmological alternative under a precise condition. If a state has a trivial macroscopic record algebra with respect to an earlier era, two descriptions that differ only by whether that era occurred before the record-free state are represented by the same present state. A physically meaningful cycle count would require a record surviving the interval. The framework neither supplies such a meta-register nor derives that a record-free cosmological state is reached. The result is an ontological equivalence and makes no claim about cosmic recurrence.

G.7 Arrow of time from conditional typicality

The proposed arrow-of-time extension requires a further typicality statement. Let $h = \{M_t\}_{t_i \le t \le t_0}$ be a decoherent macrohistory conditioned on present records M_{t_0}. A coarse Markov description would assign

$$P(h \mid M_{t_0}) \propto \mu_i(M_{t_i}) \prod_{t_i \le t < t_0} T(M_{t+\Delta t} \mid M_t),$$

where μ_i is a boundary measure and T is the conditional macro-transition probability obtained only after summing the microscopic transitions compatible with each pair of macrostates. Multiplying a separate factor $e^{S(M_t)}$ at every time would generally count the same microscopic multiplicity twice. A neutral transition law is also insufficient: conditional counting without a low-entropy boundary condition is dominated by high-entropy pasts and Boltzmann-fluctuation histories. The required theorem must show that μ_i and T exponentially suppress those histories strongly enough for ordinary entropy-increasing histories to dominate. Neither that boundary measure nor the needed substrate transition law has been derived here. This equation defines the target without claiming a completed thermodynamic arrow.

The minimal missing boundary condition can be stated directly. A *Substrate Past Hypothesis* would require

$$\operatorname{supp} \rho_0 \subseteq \mathcal{H}_{M_{\text{low}}}, \qquad t_0 - t_i \ll \min(t_{\text{relax}}, t_{\text{rec}}),$$

where $\mathcal{H}_{M_{\mathrm{low}}}$ is a low-entropy macro-subspace and the elapsed time is short compared with equilibration and recurrence. Under a mixing substrate dynamics, standard large-deviation counting would then favor entropy growth away from that boundary. The paper has not derived this hypothesis or the required mixing estimate. The saturated phase of Section 20 supplies the capacity-sector half of the hypothesis by construction — the pinned state is a near-zero-entropy configuration of the capacity sector — while the matter-sector clause remains an assumption. Naming both premises isolates the remaining arrow-of-time problem from the already-closed operational probability branch.

The faithful-history-resolution theorem of Appendix H does not supply the missing boundary. Its stationary kernel satisfies

$$p(b)K_*(b,b') = p(b)p(b') = p(b')K_*(b',b),$$

so it is exactly detailed-balanced and time-reversal symmetric. The quantum dilation is globally reversible as well. Local export distinguishes a present register from a history register within the chosen update description, but the thermodynamic orientation still comes from the boundary condition and finite-time typicality theorem above.

G.8 Memoryless dressing and the connection to L_*

The history measure does not imply memorylessness merely because it is written as an exponential. The foundational faithful full-support principle selects it when applied to paths at the fixed admissibility marginal; maximum caliber is only the conventional name for that application. For any stationary dressing kernel with one-time marginal p_{η_*},

$$H(B_{t+1} \mid B_t) = g_{\mathrm{share,eff}} - I(B_t; B_{t+1}) \le g_{\mathrm{share,eff}}.$$

Equality holds only when consecutive configurations are independent, which fixes $K(b,b') = p_{\eta_*}(b')$ on the support. More generally, the entropy rate of any stationary process with this one-time marginal is bounded by $g_{\mathrm{share,eff}}$, with equality only when the present is independent of its complete past. The quantum replacement channel transfers the old local information into a dilation register rather than destroying it. Many-Pasts gives that register its history-space interpretation. The finite marked event is a separate statement from this renewal theorem and is supplied by the decorated vertex in H.9.

G.9 Relation to many-worlds, collapse, hidden variables, and decoherence

Many-Pasts is a form of history-space realism built on the decoherent-histories formalism. It posits one realized record-bearing present and no stochastic collapse term. Its probabilities are the diagonal entries of the standard decoherence functional, conditioned on that present only after the alternative records have been normalized. The distinctive claim lies in the ontology assigned to this conditional measure and in its proposed substrate realization. Born statistics and no-signaling are imported with the operational quantum structure; the substrate does not yet derive them.

G.10 Familiar quantum examples: double-slit, EPR/Bell, and measurement

In a double-slit experiment the two path alternatives remain combined in one class operator while no which-path record exists. The probability of a detection event therefore includes their interference term. A durable which-path record defines a decoherent refinement, after which the path histories admit separate conditional probabilities and the fringe term is suppressed.

In an EPR or Bell experiment the record is joint, so its history measure carries the standard nonclassical correlations. Summing over the remote record invokes a trace-preserving local map and returns a marginal independent of the remote setting, as shown in G.5. Conditioning on the observed joint record does not alter that prior no-signaling marginal.

Measurement creates a durable record and identifies a decoherent family. Operationally the calculation is ordinary quantum mechanics. The additional claim is that the realized present is supported by the conditional ensemble of compatible past histories.

These examples introduce no new laboratory predictions. They show where positive history probabilities are legitimate and where amplitudes must remain combined.

Appendix H: Microscopic Realization and Coarse-Graining

Appendix H examines a microscopic realization of the scalar stiffness and defect ontology. The weak-field appendices establish the coefficient chain independently.

H.1 GFT condensate realization and coarse-graining

The candidate microscopic realization is a GFT/condensate picture with bosonic tetrahedral quanta $\phi(g_1, \ldots, g_4)$ and fermionic defects ψ. In the condensate regime, the coarse field may be written as

$$\sigma(x) = \sqrt{n(x)}\, e^{i\theta(x)}.$$

The hydrodynamic identity

$$|\nabla_\mu \sigma|^2 = \frac{(\nabla_\mu n)^2}{4n} + n(\nabla_\mu \theta)^2$$

shows that if

$$S_{\rm ent}(x) = S_0 + \alpha \ln \frac{n(x)}{n_{\rm bg}},$$

then the coarse action contains a positive scalar stiffness

$$\gamma \sim \frac{Z_\sigma n_{\rm bg}}{2\alpha^2} > 0.$$

The coarse source channel arises from fermionic face exclusion. A localized condensate defect appears macroscopically as matter, and the surrounding reduction of available occupancy becomes the long-wavelength EFT field. The continuum kinetic term and source channel can therefore arise from the proposed substrate realization. Deriving every inhomogeneous continuum coefficient from the underlying kernel remains open.

This condensate argument supplies a microscopic realization compatible with the EFT ontology and sign choices. The coefficient derivation remains the one given earlier.

The condensate picture addresses the emergence of continuum geometry. The remainder of this appendix derives the defect dynamics behind faithful sector resolution from the admissibility ensemble of Part II together with the Many-Pasts postulate.

H.2 The dressing Hamiltonian and the lightest seven-channel branch

The faithful sector-resolution relation of Appendix D.4 was written there as a support-scale identity on the history space $\mathcal{H}_{\rm hist} = \bigotimes_{m=-3}^{3} \mathcal{H}_B^{(m)}$, using the refresh kernel $P_{\eta_*}(b, b') = p_{\eta_*}(b')$ on each sector layer. Its dynamical content is carried by an explicit defect Hamiltonian, which we now write down so that the choice of that kernel can be derived.

The seven face-label channels of the admissibility-closed ensemble define the orthogonal projectors E_m, $m = -3, \ldots, 3$, of the face algebra $\mathcal{A}_7$. By Postulate II the elementary defect is fermionic, so each channel carries an occupation number $n_m \in \{0, 1\}$ with fermionic creation and annihilation operators $c_m^\dagger, c_m$ and $n_m = c_m^\dagger c_m$. When channel m is occupied it is dressed by a cloud state described by a density operator ρ_m on the boundary ensemble $\mathcal{H}_B$. The defect Hamiltonian is

$$H = \underbrace{\sum_m \big(\varepsilon_0\, n_m - n_m\, F_m(\rho_m)\big)}_{\text{single-channel terms}} + \underbrace{\sum_{m<m'} V_{mm'}(\rho_m, \rho_{m'})}_{\text{inter-channel coupling}},$$

with ε_0 the bare cost to occupy a channel, F_m the free energy released by dressing channel m with its cloud, and $V_{mm'}$ the residual coupling between the clouds of distinct channels. Two properties of the ground state of H supply the two ingredients of the sector-resolution relation.

One-pass occupation and the exponent seven. A channel is energetically occupied when its dressing gain exceeds the fixed bare cost, $F_m(\rho_m) > \varepsilon_0$. Symmetry makes this inequality the same for all seven channels, so an all-bound configuration is a possible symmetric ground-state branch when the inequality holds. The static Hamiltonian alone cannot identify that branch with the lightest charged particle, because its ordering depends on the undetermined balance between ε_0 and the dressing free energy.

The adopted recurrence mass functional supplies the missing ordering. Let $k = \sum_m n_m$ be the number of resolved channels. Fermionic exclusion gives $0 \le k \le 7$, and with the admissibility-closed marginal fixed on every occupied layer the support relation generalizes to

$$\frac{\lambda_k}{L_*} = \frac{2}{3} \exp(k g_{\text{share,eff}} - \Delta_k)\,, \qquad m_k = \frac{3\hbar}{2cL_*} \exp(\Delta_k - k g_{\text{share,eff}})\,,$$

where $\Delta_k \ge 0$ is the total correlation among the occupied layers. For each k, the minimum occurs at $\Delta_k = 0$. Between those minima,

$$\frac{m_{k+1}}{m_k} = e^{-g_{\text{share,eff}}} \simeq 5.99 \times 10^{-4},$$

so every missing channel raises the minimum mass by $e^{g_{\text{share,eff}}} \simeq 1.67 \times 10^3$. The unique lightest resolved one-bit defect in this support-to-length branch therefore has $k = 7$ and $\Delta_7 = 0$. The factor seven and the product cloud are selected together; they need not be imposed as separate one-pass and factorization postulates. This theorem holds at fixed marginals p_{η_*}. Allowing the cloud to optimize a different marginal would change the ultraviolet ensemble rather than refine this branch.

Factorized cloud and the additive support. The dressing dimension multiplies across channels — so that the seven equal contributions $g_{\text{share,eff}}$ add rather than merge — precisely when the joint cloud state is a product, $\rho = \bigotimes_m \rho_m$. Whether the static ground state of H has this product form is controlled by the inter-channel term $V_{mm'}$. The recurrence mass functional supplies a separate ordering: at fixed marginals it selects the seven-channel product as the lightest branch even though the static Hamiltonian has not yet derived that ordering. Each selected channel must sample its whole ensemble, and the minimizing readout has no inter-layer correlation.

The static Hamiltonian identifies the channel and cloud variables but does not order all of their branches. The next subsections prove the entropy ceiling, the unique memoryless kernel that reaches it, and the joint $k = 7$, $\Delta_7 = 0$ minimum of the adopted recurrence mass functional.

H.3 Slot coupling, layer factorization, and the additivity theorem

Two distinct correlation structures appear in the closure data, and separating them is essential: conflating them leads to the false conclusion that the cloud cannot factorize.

The closure invariant is a pure pair coupling. Expanding $S^2 = \left(\sum_i m_i\right)^2 = \Sigma^2 + 2\sum_{i<j} m_i m_j$ in $K^2(b) = 48 - \frac{1}{3}(S^2 - \Sigma^2)$ cancels the self-terms exactly and leaves the identity

$$K^2(b) = 48 - \frac{2}{3}\sum_{i<j} m_i m_j.$$

The admissibility weight $e^{-\eta_* K^2(b)}$ therefore contains only cross-terms between distinct face slots of a single boundary state; it does not factorize over those four slots. This is the exact origin of the residual correlation between face slots on the closed ensemble,

$$I(\mathrm{slot}_0; \mathrm{slot}_1) = 0.1545 \text{ nats}, \qquad \frac{I}{H(\mathrm{slot})} = 0.079,$$

so the four faces of one tetrahedron are about eight percent correlated, a structural feature of the closure invariant.

Slot coupling does not obstruct layer factorization. The factorization the support relation requires is over the seven *sector layers* $b^{(-3)}, \ldots, b^{(3)}$ of $\mathcal{H}_{\mathrm{hist}}$, each layer being a full boundary state drawn from the entire 1680-state ensemble. The slot coupling $-\frac{2}{3}m_i m_j$ lives *inside* a single layer's boundary state: it relates the four faces of that one tetrahedron and never couples layer m to layer m'. The two structures act on different objects:

Structure	What it couples	Role
slot coupling $-\frac{2}{3}m_i m_j$	the four faces *within* one boundary state b	the eight-percent mutual information; lives inside each $\mathcal{H}_B^{(m)}$
layer coupling $V_{mm'}$	the seven sector layers $b^{(m)}$ of $\mathcal{H}_{\mathrm{hist}}$	controls factorization; acts *between* the factors

The substrate's intrinsic correlation, the most natural candidate obstruction to factorization, therefore acts at the wrong level to obstruct it: it is internal to a layer, not between layers.

Additivity of the decoherent preparation overlap. Appendix G.3 defines $D(h, P) = -\ln p(h)$ from the diagonal decoherence-functional probability. For the special case in which H already denotes a decohered preparation and only the final record remains unresolved, this reduces to $D(H, P) = -\ln \mathrm{Tr}(\Pi_P \rho_{H\to\mathrm{now}})$. If that preparation state and the resolution projector factorize over channels, $\rho = \bigotimes_m \rho_m$ and $\Pi_P = \bigotimes_m \Pi_m$, the trace factorizes and the logarithm converts the product into a sum,

$$\mathrm{Tr}(\Pi_P \rho) = \prod_m \mathrm{Tr}(\Pi_m \rho_m) \quad \Longrightarrow \quad D = \sum_m \left[-\ln \mathrm{Tr}(\Pi_m \rho_m)\right] = \sum_m D_m.$$

The overlap distance is therefore additive in this special factorized preparation. The effective-support statement used downstream follows independently from Shannon subadditivity: a product cloud with seven equal marginals has joint entropy $7g_{\mathrm{share,eff}}$ and perplexity $e^{7g_{\mathrm{share,eff}}}$. Whether the electron's dressing is that product is decided by the two conditions in the next subsections.

The ceiling and the two conditions. Subadditivity bounds the support from above. With each channel marginal fixed to the admissibility weight, $H(B_m) = g_{\mathrm{share,eff}}$, the joint entropy of a single readout obeys

$$H\big(B_{-3}, \ldots, B_3\big) \le \sum_{m=-3}^{3} H(B_m) = 7g_{\mathrm{share,eff}},$$

and the deficit

$$\Delta = \sum_m H(B_m) - H\big(B_{-3}, \dots, B_3\big) \geq 0$$

measures the total correlation among the seven channels. The full support $\dim_{\text{eff}} = e^{7g_{\text{share,eff}}}$ is reached precisely when each channel carries its full entropy $g_{\text{share,eff}}$ and the channels are mutually independent, $\Delta = 0$. The first is a condition on each layer in substrate time; the second is a condition on the seven layers at a single readout. The next two subsections establish the unique faithful-resolution kernel and the minimal-mass factorized branch, while separating those results from the still-open microscopic implementation.

H.4 Maximum caliber: complete renewal is uniquely selected

The closure calculation fixes the one-time marginal p_{η_*}. Applying faithful full-support resolution to entire paths maximizes their entropy rate, a condition conventionally called maximum caliber [62]. This is the temporal form of the existing foundational requirement. For an arbitrary stationary process, its entropy rate is

$$h_\mu := \lim_{n\to\infty} H(B_0 \mid B_{-1}, \dots, B_{-n}) \leq H(B_0) = g_{\text{share,eff}}.$$

Equality holds only when B_0 is independent of its complete past. Stationarity then makes the process independent and identically distributed, with every conditional distribution equal to p_{η_*}. Maximum path entropy therefore selects complete renewal uniquely. Finite capacity alone does not imply renewal. The selection follows from faithful full-support resolution, whose temporal formulation is maximum caliber.

The one-parameter family below illustrates the cost of retained memory:

$$K_a(b, b') = a\,\delta(b, b') + (1-a)\,p_{\eta_*}(b'), \qquad a \in [0, 1],$$

which repeats the current state with probability a and otherwise redraws from the stationary weight. Every member has p_{η_*} as its stationary distribution and the same single-time marginal, so the equilibrium ensemble cannot say which one governs the dressing. The per-channel conditional entropy $H(b' \mid b) = \sum_b p_{\eta_*}(b)\,H\big(K_a(b, \cdot)\big)$, evaluated on the exact 1680-state ensemble, nonetheless slides with the memory a:

a (memory)	per-channel $H(b' \mid b)$	status
0.0 (refresh)	$7.41980 = g_{\text{share,eff}}$	full entropy
0.1	6.99953	reduced
0.3	5.80153	reduced
0.5	4.40051	reduced
0.9	1.06642	reduced

Only the memoryless endpoint $a = 0$ — the replacement kernel $K_*(b, b') = p_{\eta_*}(b')$ of Appendix D.4 — returns the full $g_{\text{share,eff}}$. For any stationary Markov kernel with that marginal,

$$H(B_{t+1} \mid B_t) = H(B_{t+1}) - I(B_t; B_{t+1}) \leq g_{\text{share,eff}},$$

and equality holds if and only if $I(B_t; B_{t+1}) = 0$. The Markov corollary is therefore

$$\boxed{K_*(b, b') = p_{\eta_*}(b')}.$$

Single-label local moves conserve slot ordering, fracturing each parity copy into twenty-four sectors of thirty-five states, so no such local kernel reaches the full ensemble at any parameter value (Appendix D.4). The native-vertex construction of H.8 implements the selected nonlocal-in-label-space replacement as one operation on the complete local tetrahedral register.

The lightest-defect branch selects the same endpoint independently within the adopted support map. If a per-pass temporal mutual information I_t reduces the fresh entropy from $g_{\text{share,eff}}$ to $g_{\text{share,eff}} - I_t$, then at fixed L_*

$$\lambda(I_t) = \lambda(0)e^{-I_t}, \qquad m(I_t) = m(0)e^{I_t}.$$

Every retained temporal correlation raises the recurrence mass, so electron lightness again selects $I_t = 0$. Faithful full-support resolution selects the vacuum history process; mass minimization independently selects the same process in the defect sector.

A dimensionless mixing test shows that the selected kernel is dynamically available. Take any ergodic generator Q (a symbol used only within this mixing test; the survival complement of H.6 is a distinct operator) with p_{η_*} as its detailed-balance stationary weight. The test uses nonlocal transpositions of two occupied labels, not the injectivity-preserving single-label shifts excluded in the preceding paragraph. A continuous-time swap generator built from the same admissibility weights serves. The kernel $e^{Q\tau}$ loses endpoint mutual information,

$$I(\tau{=}0.1) \approx 3.4 \text{ nats}, \qquad I(\tau{=}1) \approx 0.030, \qquad I(\tau{=}2) \approx 1 \times 10^{-4},$$

so the replacement kernel is the late-τ limit of a broad class of dimensionless mixing dynamics. Faithful full-support resolution selects the exact endpoint rather than a particular approach to it. H.8 derives the quantum replacement channel and reversible update; H.9 supplies the fresh amplitude, marked event, and update interaction. Durable history capacity and the stable geometric GFT embedding remain open.

H.5 Independent channels: the electron as the lightest defect

The second condition is that the occupied channels factorize at one readout, so that the deficit Δ_k of H.3 vanishes. Appendix H.2 already showed that electron lightness selects both the maximum occupation $k = 7$ and $\Delta_7 = 0$ once the support-to-length dictionary is adopted. The calculation here isolates the factorization part of that joint minimum.

Keep the deficit explicit in the baseline support relation. With each channel at its full entropy, the joint entropy is $7g_{\text{share,eff}} - \Delta$, and

$$\frac{\lambda}{L_*^{(0)}} = \frac{2}{3}\, e^{\,7g_{\text{share,eff}} - \Delta}.$$

At a fixed substrate scale L_*, a one-bit charged defect whose dressing carries correlation Δ has spatial support $\lambda \propto e^{-\Delta}$, and through $m = \hbar/(c\lambda)$ a mass

$$m \propto e^{\Delta}.$$

Correlation therefore contracts the support and raises the recurrence mass. Because $\Delta \geq 0$, the lightest one-bit charged defect is the product dressing $\Delta = 0$; correlated dressings describe heavier branches within the same recurrence/support map. Independence is not inferred from the absence of an interaction term. It is forced by mass minimization in the theory's own definition of the electron anchor.

Within the marked-transfer branch, the information-theoretic selections read end to end. Faithful full support fixes memoryless refresh, while fermionic exclusion and electron lightness select $k = 7$ and $\Delta_7 = 0$. H.6 gives the baseline survival scale and H.9 its finite dressing:

$$L_* = -\frac{3}{2}\, Z_e \lambda_e \ln\!\left(1 - e^{-7g_{\text{share,eff}}}\right), \qquad G_* = \frac{c^3 L_*^2}{\hbar},$$

with no gravitational quantity used as input.

The decorated action realizes the nonlocal replacement, fresh state, and charged marked event. A geometric GFT must still realize that decoration in a stable condensate and determine the light fluctuation spectrum and inhomogeneous continuum coefficients; it need not re-select memorylessness, the clock conversion, or $\Delta = 0$.

H.6 Mass from the positive survival transfer operator

Faithful full-support resolution fixes the replacement process on history space but not a dimensional duration. The duration comes from the paper's existing electron anchor once the already-declared effective-support event is represented in the transfer space. No second clock principle or new premise is required.

The marked seven-channel transfer. On the factorized lightest branch, likelihood multiplication on each renewed cloud has determinant $e^{-g_{\text{share,eff}}}$. The product determinant is

$$r := \Delta_{\tau_p^{\otimes 7}}(\mathsf{R}^{\otimes 7}) = e^{-7g_{\text{share,eff}}}.$$

This is the scalar charged determinant line of the decorated transfer, not the probability of an arbitrarily chosen seven-layer microstate. Fermionic one-bit exclusion gives the positive no-loop transfer

$$T^{(0)}_{\text{surv}} = 1 - r,$$

equivalently the one-dimensional Grassmann determinant $\int d\bar{c}\, dc\, e^{-\bar{c}(1-r)c} = 1 - r$. One may therefore define

$$H^{(0)}_{\text{surv}} = -\frac{\hbar}{\tau_*^{(0)}} \ln T^{(0)}_{\text{surv}},$$

with exact raw gap

$$\boxed{E^{(0)}_{\text{raw}} = -\frac{\hbar}{\tau_*^{(0)}} \ln(1 - r)}.$$

The unit mixing gap on the renewed probability space is a different operator and is not identified with this exponentially small charged gap.

Electron calibration and exact cell length. Before the finite marked response is included, the transverse export gives $H_e^{(0)} = (3/2)H^{(0)}_{\text{surv}}$. The baseline electron calibration is

$$m_e c^2 = -\frac{3\hbar}{2\tau_*^{(0)}} \ln(1 - r).$$

It fixes

$$\boxed{\tau_*^{(0)} = -\frac{3}{2}\tau_e \ln(1 - r), \qquad L_*^{(0)} = c\tau_*^{(0)} = -\frac{3}{2}\lambda_e \ln(1 - r)}.$$

For $r = e^{-7g_{\text{share,eff}}}$,

$$L_*^{(0)} = -\frac{3}{2}\lambda_e \ln\bigl(1 - e^{-7g_{\text{share,eff}}}\bigr) = \frac{3}{2}\lambda_e e^{-7g_{\text{share,eff}}} \left[1 + \frac{1}{2}e^{-7g_{\text{share,eff}}} + O(e^{-14g_{\text{share,eff}}})\right].$$

The relative logarithmic correction to the leading expression is 1.4×10^{-23}. The baseline induced scale is

$$G_*^{(0)} = \frac{9}{4}\frac{\hbar c}{m_e^2}\left[\ln\bigl(1 - e^{-7g_{\text{share,eff}}}\bigr)\right]^2,$$

whose leading form is proportional to $e^{-14g_{\text{share,eff}}}$. H.9 derives the finite response and gives $\tau_* = Z_e\tau_*^{(0)}$, $L_* = Z_e L_*^{(0)}$, and $G_* = Z_e^2 G_*^{(0)}$.

Why this supplies the phase bridge. A positive Euclidean transfer operator defines an energy spectrum. Analytic continuation supplies the Lorentzian phase frequency of the same charged mode. An independent identification of an integer renewal count with a condensate winding is unnecessary. H.9 realizes the determinant recurrence and marked response in the decorated charged vertex.

The temporal and spatial grammars agree without referring to a literal tetrahedron diameter. If $N_{\rm eff}$ is the common leading support factor, $\tau_e = N_{\rm eff}\tau_*$ and $\lambda_e = N_{\rm eff}L_*$. The identity $\lambda_e = c\tau_e$ then gives $L_* = c\tau_*$. The graph calculation uses this L_* as its canonical displacement unit; the support size of a future geometric tetrahedron, if one is introduced, is a separate quantity constrained by locality rather than a coefficient that sets the tick.

Internal locality and the native vertex. The tested single-label move cannot implement replacement. Injectivity-preserving shifts fracture the 1680 states into

$$48 \text{ components} = 4! \text{ orderings} \times 2 \text{ orientations}, \qquad 35 \text{ states per component.}$$

H.8 constructs a one-layer reversible dilation when the complete tetrahedral register is the native local vertex. This establishes circuit depth one, not the physical duration; the duration is fixed spectrally by the electron equation above. H.9 decorates that gate with the fresh amplitude and marked charged event.

H.7 Conditional dressing functionals and the condensate-spectrum lemma

The dressing Hamiltonian named in Section 23,

$$H = \sum_m \big(\varepsilon_0\, n_m - n_m F_m(\rho_m)\big) + \sum_{m<m'} V_{mm'}(\rho_m, \rho_{m'}),$$

carries three named functionals ε_0, F_m, V. This subsection gives a conditional reconstruction of them from a proposed mean-field condensate. It does not derive the required condensate from a specified GFT action, and the two residuals of H.6 remain separate outputs of the same missing fluctuation calculation rather than collapsing automatically into one premise.

Mean-field reduction. In a GFT condensate the boundary-data order parameter is $\langle\hat\varphi(b)\rangle = \sigma(b)$. The admissibility calculation fixes a normalized modulus profile $p_{\eta_*}(b)$, so a candidate condensate may be written

$$|\sigma(b)|^2 = \bar n\, p_{\eta_*}(b).$$

This fixes neither the overall density $\bar n$ nor any of the phases. A general 1680-component condensate has a common phase and relative-phase directions, and their physical status is determined by the kinetic and interaction kernels of the specified GFT action. Likewise, η_*^{-1} may be used as a formal closure-temperature parameter, but the identity

$$\eta_*\langle K^2\rangle_{\eta_*} = \frac{3}{2}$$

is the stationarity condition generated by the three-component determinant weight, not a literal equipartition theorem for the bounded discrete spectrum. Conditional on a condensate realization, $T_* = 1/\eta_* = 33.48$ may nevertheless be read as the dimensionless substrate closure temperature, with $\langle K^2\rangle_{\eta_*} = \frac{3}{2}T_* = 50.22$; this is an interpretation of the same stationary point and introduces no new number.

The refresh projector. The memoryless transition matrix is $K_*(b, b') = p_{\eta_*}(b')$. Let $D_p = \mathrm{diag}(p_{\eta_*})$ and define the detailed-balance symmetrization

$$\widetilde{K}_* = D_p^{1/2} K_* D_p^{-1/2}.$$

With $v_b = \sqrt{p_{\eta_*}(b)}$, one obtains

$$\widetilde{K}_* = |v\rangle\langle v| \equiv P, \qquad \widetilde{L}_{\mathrm{mix}} = I - \widetilde{K}_* = P_\perp.$$

The classical replacement process has one stationary direction in the weighted probability space and removes every orthogonal probability mode after one discrete update. The vector $|v\rangle$ belongs to the symmetrized classical transfer representation; it is not automatically the quantum state of the cell. It does not follow that $P_\perp$ is a Hermitian mass matrix in a closed Lorentzian GFT. The Markov update, a quantum density operator, and the coherent fluctuation Hessian are different objects.

Conditional refresh–Hessian bridge. The minimal reversible bridge uses the positive refresh Dirichlet form as the internal Euclidean quadratic cost,

$$\mathcal{K}^{(2)}_{\mathrm{int}} = \Delta_{\mathrm{ref}} P_\perp, \qquad \Delta_{\mathrm{ref}} > 0.$$

A fixed-j Euclidean effective action realizing this bridge is

$$\Gamma_E[\sigma] = \int d^4x_E \left[Z\, \partial_\mu \sigma^\dagger \partial^\mu \sigma - \mu^2 \sigma^\dagger \sigma + \frac{\lambda}{2} (\sigma^\dagger \sigma)^2 + \Delta_{\mathrm{ref}} \sigma^\dagger P_\perp \sigma \right].$$

It has $\sigma_0 = \sqrt{\bar{n}}\, e^{i\theta_0} v$ with $\bar{n} = \mu^2/\lambda$. With canonically normalized real fluctuations, its Euclidean quadratic operator contains a radial singlet with $m_h^2 = 2\mu^2/Z$, a common phase with $m_\pi^2 = 0$ when number U(1) is exact, and 1679 complex relative modes with $m_\perp^2 = \Delta_{\mathrm{ref}}/Z$. Lorentzian propagation requires the usual analytic continuation and the global $(-,+,+,+)$ convention of Appendix A. The exclusion source couples at linear order to the density fluctuation h because $\delta(\sigma^\dagger \sigma) = 2\sqrt{\bar{n}}\, h + \cdots$. In the constant-phase realization displayed here, the common phase is derivatively coupled and has no linear zero-frequency overlap with that static density source. This is not a structural decoupling theorem for a relational GFT condensate. If $\sigma = \sqrt{n}\, e^{i\theta}$ and the background carries relational phase current $\partial_\chi \theta_0 \neq 0$, the Madelung term contains

$$n(\partial_\chi \theta)^2 \supset 2(\partial_\chi \theta_0)\, \delta n\, \partial_\chi \delta\theta,$$

so density and phase can mix. The complete coherent Hessian and defect/source vertex must determine whether a phase mode is gapped, remains light but source-orthogonal, or mediates an additional response. Simplicial interactions may break number U(1) and gap it.

An open-system completion may instead implement renewal as the quantum replacement channel

$$\mathcal{E}_*(\rho) = \rho_*\, \mathrm{Tr}\, \rho.$$

The classical closure ensemble fixes the diagonal probabilities of ρ_* in the boundary basis. H.9 adopts the diagonal maximum-entropy completion and constructs its reversible update. A dissipative representation with Lindblad generator $\dot{\rho} = \Gamma_{\mathrm{ref}}[\mathcal{E}_*(\rho) - \rho]$ gives every traceless input perturbation a relaxation gap. A geometric condensate embedding must determine how this transfer description appears in its coherent fluctuation spectrum.

Selection, mixing, and marked survival are distinct operators. The combinatorial Hamiltonian selects the admissibility profile. The mixing generator $\widetilde{L}_{\rm mix} = P_\perp$ controls decay toward that profile. The exponentially small electron scale comes from a third object: the scalar determinant transfer of the seven-channel likelihood operator,

$$r := \Delta_{\tau_p^{\otimes 7}}(\mathsf{R}^{\otimes 7}) = e^{-7g_{\rm share,eff}},$$

whose one-bit survival transfer is $T^{(0)}_{\rm surv} = 1 - r$. Over a baseline proper-time step $\tau_*^{(0)}$ its exact spectral gap is

$$\mathrm{gap}(H^{(0)}_{\rm surv}) = -\frac{1}{\tau_*^{(0)}}\ln(1-r), \qquad m_e c^2 = \frac{3}{2}\hbar\,\mathrm{gap}(H^{(0)}_{\rm surv}).$$

Solving gives $\tau_*^{(0)} = -(3/2)\tau_e \ln(1-r)$ and $L_*^{(0)} = c\tau_*^{(0)}$. H.9 derives the marked factor Z_e and the physical $L_* = Z_e L_*^{(0)}$. The unit mixing gap and charged survival gap are not identified.

ε_0, the bare cost. ε_0 is the condensate chemical potential $\mu_0 = \partial E/\partial N$, the energy to commit one cell against the mean field, of order $E_* = \hbar c/L_*$. Energies in H are measured in units of E_*, and $T_* = 1/\eta_*$ is dimensionless. The rest mass is read from the positive marked-survival operator $H_{\rm surv}$, not from the static spectrum of H or the unit mixing gap of $L_{\rm mix}$.

F_m, the dressing free energy. A bound channel carries a cloud ρ_m over the boundary ensemble. The proposed released free energy is $F_m(\rho_m) = T_* S[\rho_m]$. At fixed mean closure its maximum is the Gibbs state p_{η_*} with entropy $g_{\rm share,eff}$. On the selected lightest branch the seven clouds carry total entropy $7g_{\rm share,eff}$. The faithful state-weighted determinant of their likelihood operator is exactly $e^{-7g_{\rm share,eff}}$. This is a quenched multiplicative transfer rate, not a raw single-draw Gibbs probability.

Two energies, kept distinct. The separation cost and rest energy are different quantities. The separation cost is the entropic free energy required to dissolve a defect into the substrate; in this construction it is large and linear in $g_{\rm share,eff}$. The rest energy is $m_e c^2 = \hbar\Gamma_e$, with Γ_e set by the rare joint recurrence and therefore exponential in $-7g_{\rm share,eff}$. Reading the separation cost as the mass would place the electron near tens of E_* and remove the required suppression.

V, the inter-channel coupling. The permutation-invariant singlet

$$|s\rangle = \frac{1}{\sqrt{7}}\sum_m |m\rangle$$

is the natural channel through which a scalar capacity fluctuation couples uniformly to the seven sectors. Within the minimal constant-phase refresh–Hessian completion displayed above, the radial density fluctuation is the only linearly source-coupled singlet at static order and the relative sector is gapped. Integrating out that density mode gives

$$V_{mm'} = v\,\langle m|s\rangle\langle s|m'\rangle = \frac{v}{7},$$

a rank-one outer product. The per-incidence overlap is $1/7$; the separate orientation doubling gives the $2/7$ used in Appendix I.1. The implication is

$$\text{one source-coupled density singlet} \quad \Longrightarrow \quad \mathrm{rank}(V) = 1.$$

This result is derived inside the minimal bridge, not from the closure ensemble alone. A microscopic interaction that mixes P and $P_\perp$, closes the relative gap, or sources the common phase would add exchange channels and deform the uniform structure.

Positive-transfer phase and the exchanged mode. The projector calculation separates the static source from the transfer clock. The static exclusion source couples to the radial density singlet. Independently, the positive Euclidean survival operator defines the electron energy, and ordinary analytic continuation gives the Lorentzian phase frequency $\omega_e = E_e/\hbar$ of that same charged mode. No additional identification between a renewal count and an independently postulated condensate winding is needed. The decorated vertex supplies the finite charged routing. The remaining rank-one exchange question belongs to its geometric embedding: the coherent microscopic interaction must preserve the source projection and relative-mode gap.

The remaining microscopic lemma. The effective projector model narrows the missing calculation. For a specified fixed-j simplicial GFT action admitting $\sigma_b = \sqrt{\bar{n}\, p_{\eta_*}(b)}e^{i\theta}$, compute its complete Bogoliubov or Schwinger–Keldysh kernel and test

$$P_\perp H_{\rm full} P = 0, \qquad P_\perp H_{\rm full} P_\perp \geq \Delta_{\rm gap} P_\perp, \qquad \Delta_{\rm gap} > 0.$$

The decorated transfer vertex of H.9 fixes the charged recurrence and its edge Hessian. A geometric GFT must still determine whether the condensate realization produces a coherent mass, a relaxation gap, or both, and verify that no additional source-coupled light field survives. The closure weight fixes P exactly, while the refresh–Hessian bridge for the geometric condensate remains a selected effective completion.

H.8 Faithful history resolution and reversible Many-Pasts renewal

Earlier versions grouped four distinct statements under "refresh": the classical history process, the quantum channel, its reversible implementation, and the conversion of a positive charged spectrum into proper time. They are treated separately below. Section H.9 supplies the charged marked vertex. Maximum-caliber terminology and the clock construction add no foundational premise.

Maximum path entropy. Let $\mathcal{B}$ be the finite admissible boundary space and let $p_*(b) = p_{\eta_*}(b)$ be its fixed stationary marginal. For any stationary process $\{B_n\}$ on $\mathcal{B}$,

$$h_\mu = \lim_{N\to\infty} \frac{1}{N} H(B_1, \ldots, B_N) = H(B_0 \mid B_{-1}, B_{-2}, \ldots) \leq H(p_*).$$

Equality holds exactly when B_0 is independent of its complete past. Applying the same statement after every time translation factorizes every finite joint distribution,

$$P(B_1 = b_1, \ldots, B_N = b_N) = \prod_{n=1}^{N} p_*(b_n).$$

The unique maximum-caliber history process is therefore i.i.d. For a Markov representation its transition kernel is

$$\boxed{K_*(b, b') = p_*(b')}.$$

Here "maximum caliber" is terminology for faithful full-support resolution on history space [62]. It is not a sixth theory-defining input. The foundational requirement already says that an admissible resolution carries the full available entropy compatible with its fixed data; applying that same requirement to a path gives the equality case above. Finite capacity alone would not imply renewal, but the paper's stronger faithful full-support premise does.

The selected process carries no thermodynamic orientation. It obeys

$$p_*(b)K_*(b, b') = p_*(b)p_*(b') = p_*(b')K_*(b', b),$$

so its stationary path measure is exactly invariant under time reversal. Faithful history resolution derives local renewal, not a low-entropy past and not a physical cadence.

Proper time from the positive marked-transfer spectrum. The renewal theorem fixes the discrete process but says nothing about seconds per update. Proper time comes from the determinant survival operator of H.6,

$$r = \Delta_{\tau_p^{\otimes 7}}(\mathsf{R}^{\otimes 7}) = e^{-7g_{\mathrm{share,eff}}}, \qquad T^{(0)}_{\mathrm{surv}} = 1 - r.$$

Defining the baseline transfer over $\tau_*^{(0)}$ gives

$$H^{(0)}_{\mathrm{surv}} := -\frac{\hbar}{\tau_*^{(0)}} \ln T^{(0)}_{\mathrm{surv}}, \qquad E^{(0)}_{\mathrm{raw}} = -\frac{\hbar}{\tau_*^{(0)}} \ln(1 - r)$$

on its nonzero support. The baseline transverse identification $m_e c^2 = (3/2)E^{(0)}_{\mathrm{raw}}$ therefore yields

$$\boxed{\tau_*^{(0)} = -\frac{3}{2}\tau_e \ln(1 - r)}, \qquad \boxed{L_*^{(0)} = c\tau_*^{(0)} = -\frac{3}{2}\lambda_e \ln(1 - r)}.$$

The leading form is $L_*^{(0)} = (3/2)\lambda_e r[1 + O(r)]$. H.9 supplies the finite marked factor Z_e and hence the physical $L_* = Z_e L_*^{(0)}$. No update diameter, throughput maximization, or geometric conversion factor enters. The complete tetrahedron being a native gate is a locality and circuit-depth statement only; its physical embedding size is a separate consistency question.

A positive Euclidean transfer spectrum also supplies, by analytic continuation, the Lorentzian phase frequency. The clock and phase use the same charged spectral mode and require no independent winding postulate. H.9 derives the recurrence as the state-weighted determinant and implements its finite marked dressing.

Quantum replacement theorem. Let $\mathcal{H}_A$ be the replaceable closure-register Hilbert space and $\mathcal{E} : A \to A'$ a trace-preserving quantum channel. It does not include the retained position and internal fiber of a persistent charged defect. Complete local renewal means that the output contains no information about the input, including information visible through an arbitrary reference R:

$$(\mathrm{id}_R \otimes \mathcal{E})(\rho_{RA}) = \rho_R \otimes \rho_* \qquad \text{for every } \rho_{RA}.$$

Taking product inputs first shows that every normalized input has the same output ρ_*. Linearity then gives the unique channel on all operators,

$$\boxed{\mathcal{E}_*(X) = \rho_* \operatorname{Tr} X}.$$

Every traceless perturbation is annihilated after one update. As a superoperator, $\mathcal{E}_*$ has one stationary direction and zero on the traceless operator subspace. The closure ensemble fixes $\langle b|\rho_*|b\rangle = p_*(b)$ in the boundary basis. H.9 selects the diagonal maximum-entropy completion for the decorated transfer action; a different coherent GFT state would define a different microscopic completion and must be tested against that vertex. H.11 proves that this statement is compatible with particle interference only when the renewed output excludes the persistent marked-position fiber and the discarded free-renewal record is branch-independent.

Reversible history export. Local replacement is compatible with global unitarity by Stinespring dilation [63]. Introduce the old present register A, a fresh register F, its purifier R, and a history register H. For

$$|\Psi_*\rangle_{FR} = \sum_{b \in \mathcal{B}} \sqrt{p_*(b)}\, |b\rangle_F |b\rangle_R$$

in the diagonal closure-state realization, a register permutation can act as

$$|\psi\rangle_A |\Psi_*\rangle_{FR} |0\rangle_H \longmapsto |\Psi_*\rangle_{AR} |0\rangle_F |\psi\rangle_H.$$

Tracing out R, F, H leaves ρ_* on the new closure present and removes every dependence on the old replaceable state. The inverse unitary recovers that state from H, so no global information has been destroyed. Postulate III supplies the interpretation of the exported register as history structure; the dilation theorem alone calls it an environment. For an unmeasured freely propagating defect, the state exported by this map may not encode the cell address of the mark. The marked fiber is transported in the retained defect system, and only a genuine interaction may correlate it with a discarded environmental record. Repeating the construction indefinitely requires fresh and history capacity or a controlled recycling mechanism, which the present paper has not derived.

The tetrahedral circuit. For

$$\mathcal{B} = \{(s; m_1, m_2, m_3, m_4) : s = \pm,\ m_i \in \{-3, \dots, 3\},\ m_i \neq m_j\}, \qquad |\mathcal{B}| = 1680,$$

the register permutation above is one circuit layer if the complete tetrahedron is a native local gate and the fresh admissible register has already been prepared. Neither injectivity nor the collective function $K^2(m_1, m_2, m_3, m_4)$ then requires a sequence of face repairs; the gate replaces one admissible four-face object by another.

A depth-one layer of disjoint two-face gates cannot test all six injectivity relations. The direct pair-comparison construction uses the three perfect matchings

$$\{(1,2),(3,4)\}, \qquad \{(1,3),(2,4)\}, \qquad \{(1,4),(2,3)\},$$

and therefore has depth three. This proves the stated depth for the direct comparator and excludes depth one in a pairwise architecture without a jointly coupled ancilla. It is not a lower-bound proof for every possible ancilla-assisted state-preparation algorithm. H.9 takes the complete tetrahedron as the native decorated gate; a geometric GFT embedding must realize that locality and sustain the prepared state dynamically.

What closes downstream. The foundational faithful full-support condition closes the classical replacement kernel when applied to histories. Detailed-balance symmetrization then closes $P = |\sqrt{p}\rangle\langle\sqrt{p}|$ and $P_\perp = I - P$ in probability space. Reference decoupling closes the quantum replacement-channel form, and the native tetrahedral architecture supplies a one-layer reversible dilation. H.9 adds the marked recurrence, after which positivity and the electron anchor close the clock, length, and phase-frequency conversion. These results remove an arbitrary transition matrix, geometric clock factor, and separate winding assumption while showing how local memorylessness can coexist with a globally retained past.

They do not by themselves close the coherent GFT Hessian. The exclusion sufficiency lemma of C.5 proves that one scalar statistic contains all local classical information distinguishing the canonical defect from the vacuum. The replacement process removes orthogonal probability memory. A microscopic spectrum must still show that no additional conserved or coherent light field survives and that the source couples only to the capacity singlet. The bridge from $P_\perp$ to a Euclidean mass or Schwinger–Keldysh relaxation pole therefore retains its conditional grade. The stationary reset also leaves the macroscopic arrow-of-time problem in G.7 unchanged.

H.9 Decorated simplicial marked-transfer vertex

The renewed marginal, lightest seven-channel branch, determinant recurrence, and reversible present/history exchange now determine a finite charged vertex. Its marked sector is fully specified; the geometric gluing tensor and stable condensate remain part of the broader GFT embedding problem. The vertex realizes faithful full-support history resolution on the native tetrahedron, with the established one-bit fermionic defect supplying the hard-core mark, and adds no foundational postulate. Its field space and routing are physical choices: the countervertices tested below fail the finite audits.

The marked fiber from primitive fusion. The nine-state marked space also has an independent geometric derivation. A shared face begins with two primitive fermionic slots of spin $j_0 = 3/2$. Maximum-capacity fusion selects the unique maximal channel,

$$V_{3/2} \otimes V_{3/2} = V_3 \oplus \underbrace{(V_0 \oplus V_1 \oplus V_2)}_{Q}, \qquad \dim Q = 9.$$

The selected V_3 is the seven-state effective bulk link used by the closure ensemble. The nonmaximal information is the representation $Q = V_0 \oplus V_1 \oplus V_2$. The marked response was obtained from a different construction: the closure response is a vector V_1, and one response excitation on each of the present and history strands gives

$$\mathcal{H}_{\text{mark}} = V_1^P \otimes V_1^H = V_0 \oplus V_1 \oplus V_2.$$

Thus there is an $SU(2)$ intertwiner

$$\mathcal{I} : Q \longrightarrow \mathcal{H}_{\text{mark}}, \qquad \mathcal{I}\,|(j_0 j_0); J, M\rangle = |(1_P 1_H); J, M\rangle, \quad J = 0, 1, 2,$$

with $\mathcal{I}^\dagger \mathcal{I} = P_Q$ and $\mathcal{I}\mathcal{I}^\dagger = I_9$. Each irrep occurs once, so the intertwiner is unique up to one phase on each J block.

The match is more selective than the dimension count. For a response strand of spin s,

$$V_s \otimes V_s = \bigoplus_{J=0}^{2s} V_J,$$

whereas the complement of maximal fusion for two primitive spin-j slots is

$$Q(j) = (V_j \otimes V_j) \ominus V_{2j} = \bigoplus_{J=0}^{2j-1} V_J.$$

The two representations agree if and only if $j = s + 1/2$. The independently fixed closure-vector value $s = 1$ therefore selects $j = 3/2$. Appendix B proves the representation theorem, constructs the sharp complement isometry, and states its logical condition. If the marked present/history register is the faithful dynamical record of primitive nonmaximal fusion, this is a derivation of $j_0 = \frac{3}{2}$ from the existing three-dimensional response. Without that incidence identification it remains an exact structural match. The complete geometric vertex must still reproduce the equal weights, flags, phases, hard-core mark, and routing of the explicit vertex below.

Fresh-state amplitude from the closure action. Let $C_a(b)$ be the three-component closure vector, with $K^2(b) = C_a(b)C_a(b)$. The diagonal renewed state has amplitude

$$A_*(b) = Z^{-1/2} \exp\Big[-\frac{\eta_*}{2} K^2(b)\Big], \qquad |A_*(b)|^2 = p_*(b).$$

For a standard three-component Gaussian auxiliary ξ_a,

$$\exp\Big[-\frac{\eta_*}{2} C^2\Big] = \int \frac{d^3\xi}{(2\pi)^{3/2}} \exp\Big[-\frac{1}{2}\xi^2 + i\sqrt{\eta_*}\,\xi_a C_a\Big].$$

The amplitude-level closure incidence is therefore $\sqrt{\eta_*}$. The minimal charged decoration reuses this strand, so it introduces no second response coupling. Faithful full-support resolution is being applied on a marked record space with no further joint constraint. Adding a defect-only invariant would define a nonminimal countervertex; it is allowed as a different theory and is one of the falsifiers of the construction, not an extra premise needed by the selected action.

The reversible update has independent present and history Gaussian strands. Their first normalized Hermite excitations obey

$$\langle a|c\rangle_P = \delta_{ac}, \qquad \langle b|d\rangle_H = \delta_{bd}.$$

The ordered products $|a,b\rangle = |a\rangle_P \otimes |b\rangle_H$ form nine orthonormal states,

$$\langle a,b|c,d\rangle = \delta_{ac}\delta_{bd}.$$

They carry $\mathbb{R}^3 \otimes \mathbb{R}^3 = \mathbf{1} \oplus \mathbf{3} \oplus \mathbf{5}$. Introduce hard-core marked states $d^\dagger_{ab}|0\rangle$ and the stranded vertex

$$V_Q = \sum_{a,b=1}^{3} d^\dagger_{ab}\xi^P_a\xi^H_b + \text{h.c.}$$

Its reduced Gram matrix is $I_3 \otimes I_3 = I_9$. Equal marked weights follow from this two-strand contraction, not from rotational symmetry or the number nine alone. Trace and epsilon contractions would split the three rotation sectors and falsify the vertex.

Pair contraction and canonical dilation. For each unordered occupied pair $e = (m, m')$, the two directed scalar returns are orthogonal alternatives. Define

$$R_e = q\big(\langle m \to m'| + \langle m' \to m|\big), \qquad q = \frac{2}{7},$$

and $B_e = \sqrt{\eta_*}R_e$. Then

$$B_e B_e^\dagger = 2\eta_* q^2 = \frac{8\eta_*}{49} \equiv u, \qquad 0 < u < 1.$$

The canonical unitary dilation of this contraction is

$$U_e = \begin{pmatrix} \sqrt{I - B_e^\dagger B_e} & -B_e^\dagger \\ B_e & \sqrt{I - B_e B_e^\dagger} \end{pmatrix}.$$

On the occupied scalar-event support its no-event amplitude is

$$\alpha = \sqrt{1-u} = \sqrt{1 - \frac{8\eta_*}{49}}.$$

This identifies the origin of every entry: η_* comes from the same closure amplitude that prepares the vacuum, $2q^2$ is the norm of the directed return pair, and the square root follows from reversible dilation.

For an auxiliary Grassmann representation, introduce $\Gamma_e = (\gamma^P_e, \gamma^H_e)^T$ and $J = \left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)$. With $n_m = c^\dagger_m c_m$ and $n_d = \sum_{ab} d^\dagger_{ab} d_{ab}$, the marked action is

$$\boxed{\begin{aligned} S_{\text{mark}} = {} & g_{\text{share,eff}}\, n_d + \Lambda_d n_d(n_d - 1) \\ & + \frac{1}{2}\sum_{m<m'} \Gamma^T_{mm'}\left[1 + n_d n_m n_{m'}(\alpha - 1)\right] J\Gamma_{mm'}, \\ & \Lambda_d \to +\infty. \end{aligned}}$$

The quadratic coefficient is $\alpha = \sqrt{1-u}$, not the probability $1-u$. The Grassmann integral returns a Pfaffian amplitude, so inserting $1-u$ in each 2×2 Majorana block would count the no-event probability twice and produce $(1-u)^{21}$ instead of the required amplitude $(1-u)^{21/2}$.

The Majorana variables represent the antisymmetric present/history amplitude; they are not additional propagating particles. When the marked fiber or either incident channel is absent, the reference block is J and has Pfaffian one. On the established $n_m = 1$ seven-channel branch, all $\binom{7}{2} = 21$ blocks become αJ, so

$$Z_{\text{edge}} = \alpha^{21} = \left(1 - \frac{8\eta_*}{49}\right)^{21/2}.$$

These event records do not belong to the physical composite space $\Lambda^2\mathbb{C}^7$. The natural rank-one singlet lift on that composite space has the exact $6+15$ split derived in the audit; it does not act on the separately stranded internal present/history response records used here.

This distinction is why all 21 pair records survive even though the scalar source couples through one rank-one singlet. The singlet projector acts on physical channel amplitudes. The 21 Majorana blocks label mutually distinguishable internal present/history incidences, so quotienting them by the physical singlet would identify different incidences and violate faithful resolution. Their edge labels are internal to the marked response and carry no spatial cell address under the free vertex.

Complete decorated vertex. Let $\mathcal{G}_v$ denote the geometric fixed-spin gluing tensor and W_* the native update that prepares $A_*(b)$ and exports the old replaceable closure register. The position and nine-state marked fiber belong to the retained defect system, not to a cell-addressed environment chain. Under free transport W_* must therefore be translation-covariant and must leave the discarded renewal output independent of the defect's spatial branch. This is the no-which-path condition derived in H.11; it restricts the realization of the already-required quantum vertex and adds no foundational postulate. The incidence V_Q above identifies the nine marked basis vectors with the ordered Gaussian response products. The transfer space also contains the unmarked vacuum. Its projector is

$$\Pi_Q = |0\rangle\langle 0| + \sum_{a,b=1}^{3} |a,b\rangle\langle a,b|.$$

For $e = (m, m')$, define

$$P_e = n_d n_m n_{m'}, \qquad \widetilde{U}_e = (I - P_e) + P_e U_e.$$

The complete one-step decorated transfer is

$$\boxed{\mathcal{A}_v = \mathcal{G}_v W_* \Pi_Q \exp[-g_{\text{share,eff}} n_d - \Lambda_d n_d(n_d - 1)] \prod_{m<m'} \widetilde{U}_{mm'}.}$$

The empty state therefore has unit weight, while each of the nine one-mark states has fugacity $e^{-g_{\text{share,eff}}}$. The factors $\widetilde{U}_e$ act on distinct edge-history strands and commute. The Grassmann action above is the Pfaffian trace representation of these controlled pair factors; it is not an additional operator multiplied into $\mathcal{A}_v$. Faithful full-support resolution on this internal response space, which has no further joint edge constraint, selects the tensor product of its one-edge marginals. Taking its finite internal trace at fixed position branch gives

$$\boxed{\zeta_* = 9e^{-g_{\text{share,eff}}} \left(1 - \frac{8\eta_*}{49}\right)^{21/2} = 0.005123584484947.}$$

Complete edge-Hessian audit. On the 21-dimensional edge-label space, two distinct edges either share one endpoint or are disjoint. Let A_1 and A_0 be their adjacency matrices. The Johnson edge algebra has spectra

$$\mathrm{spec}(A_1) = \{-2^{\times 14}, 3^{\times 6}, 10^{\times 1}\},$$

$$\mathrm{spec}(A_0) = \{-4^{\times 6}, 1^{\times 14}, 10^{\times 1}\}.$$

A generic permutation-covariant edge kernel can therefore carry three eigenvalues. The conditioned vertex instead gives

$$\mathcal{K}^{(2)}_{\mathrm{edge}} = I_{21} \otimes \alpha J, \qquad \mathrm{spec}_{\mathrm{edge}} = \{\alpha^{\times 21}\},$$

with zero off-diagonal edge norm. Coherently averaging the induced edge permutation matrix over all $7! = 5040$ channel relabelings gives $\mathbf{1}\mathbf{1}^T/21$, of rank one. That operation would collapse the result to a shared singlet. It is not the charged gluing used here: $m = -3, \dots, 3$ are physical spin weights, K^2 is not invariant under arbitrary relabeling, and the defect Hamiltonian contains diagonal occupations and density interactions rather than hopping terms $c^\dagger_m c_{m'}$.

Primitive routing enumeration. Write $w = e^{-g_{\mathrm{share,eff}}}\alpha^{21}$, so $\zeta_* = 9w$. The one-step transfer block contains three finite state spaces. The universal sector has one empty state and nine marked states, hence

$$Z_\mu = 1 + 9w = 1 + \zeta_*.$$

The electron feedback sector has one empty state and states $|m; I_L, I_R\rangle$, with seven persistent charged labels and two independently renewed nine-state endpoints. Its dimension is

$$1 + 7 \times 9 \times 9 = 568,$$

and its trace is

$$Z_{e,\mathrm{return}} = 1 + 7 \times 81w^2 = 1 + 7\zeta_*^2.$$

The second-shell sector has one empty state and nine marked states with one scalar passage $q = 2/7$, giving

$$Z_{\tau,2} = 1 + 9qw = 1 + \frac{2}{7}\zeta_*.$$

The direct product contains $10 \times 568 \times 10 = 56{,}800$ configurations. Exhaustive enumeration reproduces the analytic product exactly. The finite routing factors are therefore

$$Z_\mu = 1 + \zeta_*, \qquad Z_e = (1 + \zeta_*)(1 + 7\zeta_*^2), \qquad Z_{\tau,2} = 1 + \frac{2}{7}\zeta_*.$$

One native update applies each controlled gate once. Hard-core unit charge restricts the marked return to vacuum plus one return motif; the charged label persists diagonally while the response endpoints renew; and the shell grading terminates after $N = 2$. These restrictions implement structures already present elsewhere in the paper: one native renewal pass, the one-bit fermionic anchor, the physical spin-weight labels, and the three-shell closure ladder. They add no separate foundational premise. Relaxing them gives explicit controls. A shared endpoint polarization changes the electron factor to $1+7\zeta_*^2/9$ and moves G to -14.6σ; off-diagonal charged propagation gives $1 + 49\zeta_*^2$ and moves it to $+98.1\sigma$.

Predictions and scope. The physical scale and charged-lepton ratios are

$$L_* = Z_e\left[-\frac{3}{2}\lambda_e \ln(1-e^{-7g_{\mathrm{share,eff}}})\right], \qquad G_* = \frac{c^3L_*^2}{\hbar},$$

$$\frac{m_\mu}{m_e} = 720\frac{2}{7}Z_\mu, \qquad \frac{m_\tau}{m_e} = 720^2\left(\frac{2}{7}\right)^4 Z_\mu Z_{\tau,2}.$$

They evaluate to $G_* = 6.6742890772 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$, $m_\mu/m_e = 206.768280237$, and $m_\tau/m_e = 3477.343310$. The residuals are -0.073σ, -0.535σ, and $+0.481\sigma$.

The finite marked-transfer action is now specified and audited. Its generic countermodels remain useful falsifiers: a collective edge mode, a physical pair-composite interpretation, persistent endpoint polarization, additional defect-only coupling, or off-diagonal charged propagator changes the answer. The remaining microscopic task is the geometric embedding: $\mathcal{G}_v$ must realize this oriented decoration, admit the required condensate, and produce no additional light source-coupled mode. Appendix H.10 proves the universal Gaussian determinant and relative-covariance functional that any successful realization would produce, and reduces the remaining embedding to three audits of the specified geometric gluing tensor. The marked vertex supplies two exact inputs to those audits — support preservation under record integration and the $V_0 \oplus V_1 \oplus V_2$ internal-fiber identity — but does not replace the condensate, covariance, Ward, or transfer-positivity calculations. The action does not solve durable history storage, the thermodynamic arrow, or the infrared transverse influence functional.

Reproduction. The supplementary scripts reproduce the ensemble invariants, state-weighted determinant, 48×35 locality fracture, Gaussian closure amplitude, unitary pair dilation, nine-state Gram matrix, 21-block edge Hessian, and 56,800-state routing trace. They also verify the dressed scale and corrected charged-lepton ratios of Appendix I.1. Appendix O contains the self-contained numerical-spine script; the complete Hessian and graph enumerations remain in the accompanying audit files.

H.10 Gaussian GFT–geometric-relative-entropy bridge and reduced embedding audits

At any finite regulator, positive dressed and reference Hessians on one gauge-reduced physical support determine a normalized Gaussian determinant and a positive relative-covariance functional uniquely. This algebraic result requires no geometric interpretation. Its physical use remains conditional on identifying the operators with decorated-GFT fluctuations, assembling their local fibers into a covariant metric response, and continuing the result to Lorentzian dynamics.

A. Physical Hessians and relative covariance. Let $\mathcal{H}'_{\mathrm{phys}}$ be a finite-dimensional regulated real fluctuation space after gauge fixing and removal of the same exact zero modes in the equilibrium and dressed backgrounds. For a complex GFT field this means its real Nambu form, with the conjugate components included in the physical trace. On that common support, take

$$S_0^{(2)}[\varphi] = \frac{1}{2}\langle\varphi, \mathbb{K}_0\varphi\rangle, \qquad S^{(2)}[\varphi] = \frac{1}{2}\langle\varphi, \mathbb{K}\varphi\rangle, \qquad \mathbb{K}_0 > 0, \quad \mathbb{K} > 0.$$

Whitening by the equilibrium Hessian defines

$$\Theta = \mathbb{K}_0^{-1/2}\mathbb{K}\mathbb{K}_0^{-1/2} > 0, \qquad \mathcal{G} = \Theta^{-1} = \mathbb{K}_0^{1/2}\mathbb{K}^{-1}\mathbb{K}_0^{1/2} > 0.$$

Equivalently, with $C_0 = \mathbb{K}_0^{-1}$ and $C = \mathbb{K}^{-1}$,

$$\mathcal{G} = C_0^{-1/2}CC_0^{-1/2}.$$

Every prime below means this identical gauge and zero-mode prescription. Equality of the two physical supports is a hypothesis to be tested in the microscopic spectrum; it is not supplied by the determinant identity.

B. Finite-regulator determinant theorem. The equilibrium-normalized Gaussian functional is

$$\boxed{\widehat{Z}_{\mathrm{G}} = \frac{\displaystyle\int_{\mathcal{H}'_{\mathrm{phys}}} D\varphi\, e^{-\frac{1}{2}\langle\varphi,\mathbb{K}\varphi\rangle}}{\displaystyle\int_{\mathcal{H}'_{\mathrm{phys}}} D\varphi\, e^{-\frac{1}{2}\langle\varphi,\mathbb{K}_0\varphi\rangle}} = (\det{}'\Theta)^{-1/2} = (\det{}'\mathcal{G})^{1/2}.}$$

Consequently,

$$\boxed{2\ln\widehat{Z}_{\mathrm{G}} = -\operatorname{Tr}'\ln\Theta = \operatorname{Tr}'\ln\mathcal{G}.}$$

The proof is just the finite Gaussian integral

$$Z[\mathbb{K}] = (2\pi)^{N_{\mathrm{phys}}/2}(\det{}'\mathbb{K})^{-1/2}.$$

Taking the ratio cancels the measure normalization, while multiplicativity of the determinant under the congruence defining Θ gives the displayed result. The proof requires neither $[\mathbb{K}_0,\mathbb{K}] = 0$ nor simultaneous diagonalization. Positivity is needed to define the real Gaussian measure and the principal operator logarithm. The real/Nambu convention fixes the exponent $-1/2$; a complex-coordinate calculation gives the same result only after its conjugate variables and Jacobian are counted consistently in N_{phys} and Tr'.

C. Exact quadratic-source Legendre transform and Gaussian KL divergence. Write the whitened dressed Hessian as

$$\Theta = I - U > 0, \qquad U = U^{\dagger},$$

and define the normalized connected functional

$$W[U] = \ln\widehat{Z}_{\mathrm{G}}[U] = -\frac{1}{2}\operatorname{Tr}'\ln(I-U).$$

Its matrix derivative is

$$2\frac{\delta W}{\delta U} = (I-U)^{-1} = \mathcal{G}.$$

The convex conjugate with the corresponding quadratic-source normalization is

$$\Gamma_{\mathrm{rel}}[\mathcal{G}] = \sup_{\substack{U=U^{\dagger}\\ I-U>0}} \left\{\frac{1}{2}\operatorname{Tr}'(\mathcal{G}U) - W[U]\right\}.$$

Stationarity gives $U = I - \mathcal{G}^{-1}$. Substitution therefore yields the exact identity

$$\boxed{\Gamma_{\mathrm{rel}}[\mathcal{G}] = \frac{1}{2}\operatorname{Tr}'\left[\mathcal{G} - I - \ln\mathcal{G}\right].}$$

If $g_i > 0$ are the eigenvalues of $\mathcal{G}$, then $g_i - 1 - \ln g_i \geq 0$, with equality only at $g_i = 1$. Hence

$$\Gamma_{\mathrm{rel}} \geq 0, \qquad \Gamma_{\mathrm{rel}} = 0 \Longleftrightarrow \mathcal{G} = I \Longleftrightarrow C = C_0$$

on the common support. In probabilistic language this is precisely

$$\Gamma_{\mathrm{rel}} = D_{\mathrm{KL}}(\mathcal{N}(0,C)\,\|\,\mathcal{N}(0,C_0)).$$

Thus the positive mismatch functional in Sections 25 and 29.9 is not an inserted potential. It is the convex conjugate of the normalized Gaussian trace logarithm.

D. Conditional relation to geometric relative entropy. Bianconi's geometric-relative-entropy construction uses a positive metric-response operator $G_B = \Theta_B^{-1}$ and the same operator-convex mismatch form [24]. If a specified decorated GFT gluing tensor derives the operator identification

$$\Theta_B = \Theta, \qquad G_B = \mathcal{G},$$

then the Bianconi trace-log and Legendre functionals are the Gaussian effective functionals of the dressed GFT covariance relative to equilibrium. Under that still-conditional identification, G_B can be read in this theory as a composite covariance or susceptibility ratio, slaved to the condensate, metric, matter, and relational boundary data. The functional identity proved above is exact; the microscopic gluing and operator map are not. Bianconi's independent derivative dynamics are not imported into the metric-only ordinary branch, and this paper does not attribute the composite reading to her construction itself.

E. Euclidean sign and continuation. The Euclidean one-loop contribution obtained by integrating out the Gaussian fluctuation is

$$\boxed{\Delta\Gamma_{E,\mathrm{eff}} = -\ln \widehat{Z}_{\mathrm{G}} = \frac{1}{2}\,\mathrm{Tr}'\ln\Theta = -\frac{1}{2}\,\mathrm{Tr}'\ln\mathcal{G}.}$$

This sign is not the sign of the positive Legendre/KL mismatch. The logarithm of a multiplicity or partition ratio and the Euclidean effective free-energy contribution carry opposite signs, while the convex conjugate measures the nonnegative cost of changing the covariance. A common Lorentzian parent must fix the analytic continuation and the sign of its retarded influence kernel. The determinant theorem and positive Legendre identity do not depend on that unresolved continuation.

F. Conditional local derivative expansion. Suppose the regulated operator is quasilocal in a geometric cell basis,

$$\Theta_{vv'} = \delta_{vv'}\Theta_v + \mathcal{R}_{vv'},$$

with $\mathcal{R}$ derivative-suppressed on fields varying slowly compared with L_*. Only under that assumption may the exact trace be organized as

$$2\ln\widehat{Z}_{\mathrm{G}} = \nu_* \int d^4x\,\sqrt{|g|}\,\big[-\mathrm{tr}_{\mathrm{phys}}\ln\Theta(x)\big] + \mathcal{O}\big(L_*\nabla\Theta, L_*^2\nabla^2\Theta\big)\,.$$

Here $\mathrm{tr}_{\mathrm{phys}}$ is the trace over the local gauge-reduced fiber and ν_* is its continuum block density. Neither ν_* nor the map from the finite cell labels to that covariant fiber is fixed by the 1680-state probability distribution. Both must follow from the geometric gluing and blocking calculation. The finite determinant is exact before locality is assumed; the local continuum density is conditional on this derivative expansion.

G. Low-curvature and static Einstein corollary. If the covariant operator supplied by the embedding has the small-response form

$$\Theta = I - \widetilde{U}, \qquad \widetilde{U} = \beta\widetilde{R} - \alpha\widetilde{M} + \mathcal{O}(R^2, RM, M^2, \nabla^2),$$

where $\widetilde{R}$ and $\widetilde{M}$ are the dimensionless curvature and matter insertions on the physical fiber, then

$$-\,\mathrm{Tr}'\ln(I-\widetilde{U}) = \mathrm{Tr}'\,\widetilde{U} + \frac{1}{2}\,\mathrm{Tr}'\,\widetilde{U}^2 + \cdots\,.$$

The linear trace lies in the Einstein–Hilbert plus matter universality class if, and only if, the covariant fiber trace, Ward identity, and coefficients supplied by the gluing calculation have the required form and normalization. The already-controlled longitudinal reduction fixes the target of that calculation:

$$I_{\rm cap}^{\rm static} = \int dt\, d^3x \left[-\frac{\gamma}{2}(\nabla\delta S)^2 + \kappa\rho\,\delta S\right], \qquad \delta S = -\frac{2S_\infty}{c^2}\Phi,$$

$$G = \frac{c^2\kappa}{8\pi\gamma S_\infty}, \qquad \boxed{I_{\rm cap}^{\rm static} = Z_S I_{\rm Newton}.}$$

If the full condensate spectrum contains one light source-coupled capacity-density singlet and all other coherent scalar modes are gapped or source-orthogonal, the longitudinal Gaussian bridge reduces to the Einstein scalar constraint rather than adding a new scalar force. That spectrum condition is an audit target, not a result of the trace-log theorem.

H. Marked-record support-preservation lemma. The pair contraction and the seven-channel recurrence are different quantities. From H.9,

$$u = \frac{8\eta_*}{49} = 0.00487621949\ldots, \qquad \alpha = \sqrt{1-u} = 0.9975589108\ldots,$$

$$\alpha^{21} = 0.9499693634\ldots, \qquad r = e^{-7g_{\rm share,eff}} = 2.77557\times 10^{-23}.$$

The positivity of the marked edge factor follows from $0 < u < 1$, not from the exponential smallness of r. On the established seven-channel one-mark branch, integrating out the 21 internal present/history Majorana blocks multiplies the remaining bosonic geometric transfer by the strictly positive scalar α^{21}. Taking the branch-independent internal trace of the nine-state hard-core marked fiber gives

$$\boxed{\zeta_* = 9e^{-g_{\rm share,eff}}\alpha^{21} = 0.00512358448\ldots > 0.}$$

Therefore the integrated marked-record sector, by itself, neither removes nor creates a bosonic geometric mode. This is a decoration-side support lemma only. It does not prove that the dressed geometric Hessian $\mathbb{K}$ and equilibrium Hessian $\mathbb{K}_0$ have the same gauge-reduced physical support. New condensate zero modes, altered constraints, or background-dependent degeneracies remain part of the spectrum audit.

I. Exact internal-fiber identity and limited ADM reading. The primitive fusion identity of H.9 is

$$V_{3/2}\otimes V_{3/2} = V_3\oplus V_0\oplus V_1\oplus V_2.$$

Removing the selected maximal channel gives

$$Q = (V_{3/2}\otimes V_{3/2})\ominus V_3 = V_0\oplus V_1\oplus V_2, \qquad \dim Q = 1+3+5 = 9.$$

Independently, the marked response fiber obeys

$$\mathcal{H}_{\rm mark} = V_1^P\otimes V_1^H = V_0\oplus V_1\oplus V_2.$$

The intertwiner already constructed in H.9 therefore proves equivalence of the two internal fibers, with one copy of each irrep. Under spatial rotations, the ADM shift and symmetric spatial metric decompose as

$$h_{0i}\in V_1, \qquad h_{ij}\in V_0\oplus V_2.$$

The nine marked states consequently have the same spatial-rotation content as the three shift components plus the six spatial-metric components. The lapse h_{00} is not in this nine-state fiber; it remains the separate scalar constraint multiplier. This is an internal $SU(2)$ representation identity, not a proof that the marked fiber is literally the metric. It does not establish inter-cell covariance, derive the lapse constraint or diffeomorphism Ward identity, imply $\Phi = \Psi$, or exclude scalar collective modes of the unmarked condensate. It supplies an exact kinematic target for the geometric gluing tensor, not the covariant assembly theorem itself.

J. Reduction from six physical requirements to three audits. Before the two exact inputs above, the embedding problem could be listed as six requirements: a stable condensate; positive dressed/reference Hessians on a common support; a local covariant fiber and derivative expansion; a unique light source-coupled capacity singlet; the correct continuum normalization and Ward identity; and a reflection-positive transfer with a Lorentzian continuation. The support lemma and representation identity reduce the independent work but do not close those physical conditions. The correct compression is

$$\boxed{\text{six physical requirements} \longrightarrow \text{three audits of one specified } \mathcal{G}_v + \text{ two exact kinematic inputs.}}$$

The exact inputs are that integrated marked records preserve bosonic support on the established branch and that $Q = V_0 \oplus V_1 \oplus V_2$ supplies the ADM shift-plus-spatial-metric representation target. The three remaining audits are as follows.

Audit 1: condensate, support, spectrum, and source overlap.

1. Specify the full decorated geometric GFT action, including $\mathcal{G}_v$.
2. Solve its mean-field equation for a condensate satisfying $|\sigma_b|^2 = \bar{n}\, p_{\eta_*}(b)$.
3. Determine the allowed condensate phases and relational-clock dependence.
4. Gauge-fix the equilibrium and dressed backgrounds identically.
5. Construct $\mathbb{K}_0$ and $\mathbb{K}$ and identify every gauge, global, relational-clock, and conserved zero mode.
6. Verify that the two primed operators act on the same physical support.
7. Compute the complete Bogoliubov or Schwinger–Keldysh spectrum.
8. Compute the defect/source vertex and its overlap with every scalar eigenmode.
9. Test whether only the capacity-density singlet is both light and sourced.

The relational-phase warning is essential. For $\sigma = \sqrt{n}\, e^{i\theta}$ and $\partial_\chi \theta_0 \neq 0$,

$$n(\partial_\chi \theta)^2 \supset 2(\partial_\chi \theta_0)\, \delta n\, \partial_\chi \delta\theta,$$

so density–phase decoupling is not structurally guaranteed. The audit passes only if one light scalar has nonzero source overlap while each other scalar satisfies $m^2 \geq \Delta_{\text{gap}} > 0$ or is exactly source-orthogonal in the physical retarded kernel. A light phase or Goldstone mode is acceptable only if it is source-orthogonal and mediates no second static gravitational force.

Audit 2: covariant assembly, Ward identity, and normalization.

1. Show how neighboring tetrahedra glue the local fibers, whose rotation content is $V_0 \oplus V_1 \oplus V_2$. Identify the lapse/constraint channel.
2. Derive the continuum metric-response vertex and the local or quasilocal derivative expansion of Θ.
3. Show that the response is independent of an arbitrary coordinate foliation.
4. Derive the physical diffeomorphism Ward identity
$$\nabla_\mu \frac{\delta \Gamma_{\text{eff}}}{\delta g_{\mu\nu}} = 0$$
on the equations of motion.
5. Determine the physical trace multiplicities and continuum block density ν_*.

6. Recover the already-fixed Newton normalization

$$G = \frac{c^2\kappa}{8\pi\gamma S_\infty} = G_*.$$

7. Introduce no new continuously fitted normalization coefficient.

8. Preserve the metric-only ordinary branch and introduce no independent Cauchy data.

This audit also applies Appendix N.6: the near-equilibrium covariance operator may approximate the bounded, quasilocal capacity observable only locally. It may not globally identify the unbounded relative entropy with the bounded variable $q_{\mathrm{cap}} \in [0, 1]$.

Audit 3: transfer positivity, reflection structure, and continuation. Self-adjointness alone is insufficient:

$$T = T^\dagger \quad \not\Longrightarrow \quad T \geq 0.$$

A sufficient normalized target is

$$0 < T \leq I,$$

together with the appropriate reflection invariance and a compatible positive Euclidean measure. Then

$$H = -\frac{1}{a_\chi} \ln T$$

is self-adjoint and bounded below. The audit must construct the full transfer including $\mathcal{G}_v$, renewal, and marked decoration; test positivity of that complete transfer rather than only its marked blocks; establish Osterwalder–Schrader-type reflection positivity or its relational analogue; derive the Lorentzian retarded kernel; verify closed-time-path unitarity; preserve the Ward identity after continuation; and fix the sign relation among $\ln \widehat{Z}_{\mathrm{G}}$, $\Delta\Gamma_E$, and Γ_{Lor}. Complete positivity of the local replacement channel is not equivalent to reflection positivity of the spacetime measure.

K. Status. The result is therefore

$$\boxed{\begin{array}{c} \text{Gaussian determinant bridge: exact,} \\ \text{quadratic-source Legendre/KL mismatch: exact,} \\ \text{marked support preservation: exact on the established branch,} \\ \text{internal } V_0 \oplus V_1 \oplus V_2 \text{ identity: exact,} \\ \text{full GFT–geometric-relative-entropy operator map: conditional,} \\ \text{covariant continuum embedding: open.} \end{array}}$$

The open problem is no longer to invent a viable continuum relative-information functional. That functional follows from any positive Gaussian dressed/reference Hessian pair on common physical support. The remaining problem is to determine whether one specified decorated geometric gluing tensor produces the required stable condensate, physical spectrum, covariant metric vertex, normalization, and reflection-positive transfer. Success would identify Bianconi's geometric-relative-entropy functional as the Gaussian covariance-effective description of this finite-capacity substrate. Failure would falsify that particular continuum embedding without undoing the finite marked-transfer, state-counting, or static reduced-action calculations.

H.11 Coherence under renewal and the conditional information-geometric QED bridge

The question. The renewed register contains no information about its input, whereas quantum interference is carried by off-diagonal operators. Coherence can survive only if the interfering degrees of freedom remain in the retained system and the discarded complement carries no path record. The calculation below states this division exactly and then gives the limited role of entropic dynamics in a continuum quantum bridge. It adds no fourth foundational postulate.

Exact renewed-output theorem. For

$$\mathcal{E}_*(X) = \rho_* \operatorname{Tr} X,$$

the Heisenberg dual is fixed by

$$\operatorname{Tr}[\mathcal{E}_*(X)O] = \operatorname{Tr}\Big[X\mathcal{E}_*^\dagger(O)\Big].$$

Substitution gives

$$\boxed{\mathcal{E}_*^\dagger(O) = \operatorname{Tr}(\rho_* O)I.}$$

Every later observable on the renewed output therefore pulls back to a multiple of the identity, and every traceless input operator is annihilated. Equivalently, for any reference R,

$$(\mathrm{id}_R \otimes \mathcal{E}_*)(\rho_{RA}) = \rho_R \otimes \rho_*.$$

The replacement channel is entanglement-breaking on the renewed register. This does not mean that a global dilation destroys the old state: its complementary output can retain it, exactly as the history register in H.8 does. The theorem locates where coherence cannot remain — in the replaced output — rather than proving that no coherence exists globally.

Complementary-record coherence theorem. Let X denote the defect position and D its retained internal marked fiber. For two freely transported branches, write the relevant part of a one-step isometry as

$$V\big(|x\rangle_X|q\rangle_D|0\rangle_E\big) = |F(x)\rangle_X U_D|q\rangle_D|e_x\rangle_E,$$

and similarly for y. The environment E denotes only what is discarded when the defect state is read. Directly tracing E gives

$$|x\rangle\langle y| \longmapsto \underbrace{\langle e_y|e_x\rangle}_{\gamma_{xy}} |F(x)\rangle\langle F(y)| \otimes U_D|q\rangle\langle q|U_D^\dagger.$$

Thus

$$\boxed{\mathcal{C}_{xy}(1) = \gamma_{xy}\mathcal{C}_{xy}(0), \qquad \gamma_{xy} = \langle e_y|e_x\rangle.}$$

For independent record factors over n updates,

$$\mathcal{C}_{xy}(n) = \left(\prod_{k=1}^{n} \gamma_{xy}^{(k)}\right) \mathcal{C}_{xy}(0).$$

The equation separates the two possible microscopic readings. If the vertex leaves a cell-addressed record, $|e_x\rangle = |x\rangle_E$ and $|e_y\rangle = |y\rangle_E$, then $\gamma_{xy} = 0$ for $x \neq y$: position coherence dies in one update. If the persistent internal fiber remains inside $X \otimes D$ and the discarded free-renewal output is branch-independent, $|e_x\rangle = |e_y\rangle$, then $\gamma_{xy} = 1$ and renewal produces no intrinsic position decoherence. Intermediate overlaps produce ordinary partial decoherence.

Calling a record "worldline-attached" is not by itself sufficient. A spatially localized record that moves along two different worldlines is still a which-path record if it is discarded. In this paper the phrase means the stronger structure: the marked response fiber is retained as part of the coherently transported defect, while the discarded renewal register contains no branch address. Genuine interactions with detectors or an environment may, and ordinarily do, make the discarded states distinguishable.

What $Q \perp V_3$ does and does not prove. Primitive fusion gives the orthogonal decomposition

$$V_{3/2} \otimes V_{3/2} = V_3 \oplus Q, \qquad Q = V_0 \oplus V_1 \oplus V_2,$$

and hence

$$P_Q P_3 = 0, \qquad \operatorname{Tr} P_Q = 9, \qquad \operatorname{Tr} P_3 = 7.$$

If a discarded record actually resolves these two sectors, the same overlap theorem sets every Q–V_3 off-diagonal to zero. This gives exact marked/unmarked sector dephasing under a sector-resolving export. It is compatible with charge-sector superselection, but it is not yet a derivation of the full electric-charge superselection rule. That stronger statement requires the mark to be identified with the generator of a local $U(1)$ gauge symmetry and requires the physical observable algebra to preserve its Gauss-law sectors. Orthogonality of two representation subspaces alone does not force the environment to measure them.

Forced vertex constraint, not a new premise. The complete microscopic Hilbert space at a marked vertex must therefore be read as

$$\mathcal{H}_v = \mathcal{H}_{\text{ren}} \otimes \mathcal{H}_X \otimes \big(\mathbb{C}|0\rangle \oplus Q\big),$$

where $\mathcal{H}_{\text{ren}}$ is replaced by ρ_*, while $\mathcal{H}_X \otimes Q$ is transported coherently. In the free sector the dilation of W_* must satisfy

$$\boxed{\langle e_y | e_x \rangle = 1 \quad \text{for unmeasured position branches } x, y}$$

after the canonical translation identifying their background renewal states. The exact equality defines the minimal ideal free vertex; electron-interference experiments impose a finite lower bound on the product of overlaps over their actual coherence time [27]. A cell-fixed alternative is not another interpretation of the same action. It is an experimentally excluded countervertex.

Sections 3.3 and 22 already require the operational branch to reproduce ordinary quantum interference. The overlap theorem converts that existing requirement into an explicit microscopic test of W_*, so the condition adds no ontological premise. Electron interference would falsify an action that violates it. The finite calculation verifies the dual-channel identities, the Q–V_3 projector orthogonality, and the zero, unit, and intermediate record-overlap cases to machine precision.

Fast/slow architecture. The result separates the roles that renewal had previously blurred. The fast closure register supplies the objective replacement process and its exported history correlations. The persistent marked fiber carries internal phase and position coherence. The fast output cannot feed old off-diagonal data back into present-cell observables because $\mathcal{E}_*^\dagger(O)$ is scalar; the slow coherent sector can nevertheless evolve unitarily because it is not the argument of $\mathcal{E}_*$. Renewal and interference are therefore compatible provided the complete vertex respects the tensor split above.

Hamilton–Killing condition. Caticha's information-geometric reconstruction makes the remaining quantum-dynamical target precise [25, 26]. Let z denote the retained slow configuration, with probability functional $P[z]$ and conjugate phase functional $\Phi[z]$. If coarse-graining of the marked transfer derives the canonical symplectic form and Fisher-compatible metric

$$\Omega = \int \mathcal{D}z\, \delta P[z] \wedge \delta\Phi[z],$$

$$ds^2 = \int \mathcal{D}z \left[\frac{\hbar}{4P[z]}(\delta P[z])^2 + \frac{P[z]}{\hbar}(\delta\Phi[z])^2\right], \qquad \Psi[z] = \sqrt{P[z]}e^{i\Phi[z]/\hbar},$$

then flows preserving both structures are Hamilton–Killing flows. On the normalized phase quotient, the Caticha reconstruction turns their bilinear generators into self-adjoint operators and their evolution into a linear Schrödinger equation. This supplies a theorem-level route from an information geometry to complex amplitudes and unitary dynamics; it does not prove that the present transfer action generates the displayed metric or flow.

Caticha's local-gauge extension then adds charged nonrelativistic point particles and a radiation field. With a local $U(1)$ redundancy, the corresponding gauge constraint becomes the quantum Gauss law,

$$\big(\boldsymbol{\nabla}\cdot\widehat{\mathbf{E}} - \widehat{\rho}_q\big)\Psi = 0,$$

and the resulting Hamilton–Killing evolution reproduces charge conservation, the Maxwell equations, the Coulomb potential, and the usual nonrelativistic QED Hamiltonian in Coulomb gauge [26]. Thus the required probability–phase and gauge structures are jointly sufficient once obtained. This theorem does not derive them from $\mathcal{A}_v$, fix the electromagnetic coupling from η_*, or supply relativistic Dirac QED, particle creation, or vacuum polarization.

The remaining requirements are consequently explicit:

1. derive a gapless persistent local $U(1)$ connection and identify the hard-core mark with its conserved charge;
2. derive the slow (P, Φ) statistical manifold, its Fisher–Kähler metric, and Hamilton–Killing flow from the decorated transfer or its controlled continuum limit;
3. derive the local radiation and charged-defect kernels, including the Gauss constraint, rather than inserting the nonrelativistic QED Hamiltonian;
4. fix the electromagnetic normalization from the microscopic action; and
5. for full physical QED, extend the result to a relativistic spinor field with pair creation and the required renormalized interactions.

Failure of any requirement falsifies this proposed quantum completion without changing the exact renewal or record-overlap theorems.

Closure statement.

$$\boxed{\begin{gathered}\text{renewed-output coherence blindness: exact,}\\ \text{record-overlap multiplier: exact,}\\ Q \perp V_3:\ \text{exact,}\\ \text{cell-addressed marked export: excluded by interference,}\\ \text{branch-independent retained marked fiber: required vertex condition,}\\ \text{Hamilton–Killing and nonrelativistic-QED bridge: conditional,}\\ \text{microscopic phase/gauge dynamics and relativistic QED: open.}\end{gathered}}$$

Appendix I: Mass and Gauge Extensions

Appendix I develops possible particle and gauge extensions of the entanglement logic. The closed weak-field derivation does not depend on them.

I.1 Charged-lepton spectrum from the shell algebra

The electron is the ground-state one-bit charged defect — the $N = 0$ configuration whose seven-channel dressing sets the substrate length (Appendix D.4). The muon and tau are heavier shell excitations of the same defect. Each adds one radial entanglement shell, realized microscopically as the exclusion of one face label from the seven-state alphabet of Appendix B. The closure machinery loses its nondegenerate weighting after the third branch, so the construction contains three charged leptons. Their baseline masses follow from the shell entropy and singlet overlaps; the decorated marked-transfer vertex supplies the finite response that earlier versions left in the residuals. Because the ratios carry no dimensionful input, they test the same marked action that enters G_*.

With that picture fixed, the spectrum is written as a shell expansion,

$$\log \frac{m_N}{m_e} = B_0 N + A_0 N^2, \qquad N = 0, 1, 2,$$

with the electron the $N = 0$ ground state and the muon and tau the $N = 1, 2$ radial entanglement-shell excitations of the same core structure. Each shell excitation is the exclusion of one face label from the seven-state alphabet of the boundary ensemble in Appendix B.

Three-generation termination from closure-spectrum collapse. Combinatorial injectivity alone allows up to $N = 3$ shells, since the reduced alphabet must retain at least four distinct labels to support an injective face assignment ($7 - N \geq 4$). The sharper structural bound comes from the admissibility-closure machinery. For the reduced ensemble at shell N, the closure invariant K^2 inherits a discrete spectrum whose dispersion gives the admissibility kernel $e^{-\eta K^2}$ real weight. For $N = 0, 1, 2$ the K^2 spectrum has multiple distinct values, and the closure equation $\langle K^2 \rangle_\eta = 3/(2\eta)$ has a non-degenerate solution.

At $N = 3$ the spectrum collapses. With only four labels left, every injective assignment to the four faces uses all of them, and K^2 depends on the labels only through the permutation-invariant sums S and Σ^2, so every microstate of the reduced four-label ensemble carries the same value of K^2. With a single spectral value K_0^2 the closure equation $\langle K^2 \rangle_\eta = 3/(2\eta)$ still fixes $\eta = 3/(2K_0^2)$ formally; what fails at $N = 3$ is selection, not solvability. The kernel $e^{-\eta K^2}$ acts trivially when every microstate carries the same K^2, so it weights nothing and leaves no admissibility dispersion to define a branch. Shell $N = 3$ therefore does not define a non-degenerate admissibility-closed branch in the same sense as the earlier shells: the cutoff rests on the collapse of the K^2 dispersion, not on the closure equation admitting arbitrary η.

The number of charged-lepton generations is consequently the number of shells with a non-degenerate closure branch,

$$N \in \{0, 1, 2\} \qquad \Longrightarrow \qquad \text{three generations},$$

a structural bound stronger than the combinatorial count. The closure-spectrum collapse fixes the generation count at three; injectivity alone would allow four.

Linear coefficient from the first-shell reduced-alphabet entropy. The linear coefficient is the entropy cost of adding one shell excitation. With one face label excluded, the reduced ensemble has

$$\Omega_1 = 2\,P(6, 4) = 720 = 6!,$$

i.e., the full symmetric group S_6 with orientation degeneracy. Direct admissibility evaluation on the reduced ensemble gives $g_1 = \ln \Omega_1 = \ln 720$ to within 0.1%, since the admissibility correction is small on the reduced ensemble where the alphabet symmetry is nearly intact. Hence

$$B_0 = \ln \Omega_1 = \ln 720 = 6.579\ldots,$$

compared with the empirical fit $B_0 = 6.586$ (agreement at 0.1%).

Why the same 720 at every shell. The per-shell factor is the *same* $\Omega_1 = 720$ for each added shell, not a decreasing sequence $720 \cdot 240 \cdots$ in which each shell excludes a further label. At the tau, cumulative exclusion would give $m_\tau/m_e \simeq 1151$ (-67%), whereas the repeated factor gives 3454.6 (-0.65%), so the data require the same reduction at each shell. The structural reason is the rank of the inter-shell coupling. The shells couple only through the permutation-invariant singlet projector $P_{\rm sing}$, whose spectrum is $\{1, 0, 0, 0, 0, 0, 0\}$: rank one. A rank-one projector pins exactly one shared direction, and it is the same direction however many shells are already stacked, so each added shell surrenders only that one shared singlet direction and retains the other six — the same direction at every shell, not a fresh label removed cumulatively. The injective count realizes this exactly: $P(6, 4)/P(7, 4) = 360/840 = 3/7$, so $1680 \to 720$ once and again at each subsequent shell. This also separates the two reduced-alphabet notions that otherwise look inconsistent. The *generation cutoff* above counts $7 - N$ surviving labels because it asks whether a non-degenerate closure *branch* still exists; the *mass factor* here is the rank-one singlet pinning one shared direction per shell. They are different operations, which is why the cutoff scales with $7 - N$ while the ladder carries a fixed 720.

Quadratic coefficient from singlet-projection shell algebra. The quadratic coefficient follows from three premises already in the framework: (i) mass arises from scalar capacity response (Postulate II); (ii) the scalar EFT response is quadratic in the total source; (iii) the coarse scalar branch projects onto the permutation-invariant singlet of the seven-state face alphabet.

Let

$$\mathcal{H}_7 = \mathrm{span}\{|m\rangle : m = -3, \ldots, 3\}, \qquad |u\rangle = \frac{1}{\sqrt{7}} \sum_{m=-3}^{3} |m\rangle, \qquad P_{\rm sing} = |u\rangle\langle u|.$$

A shell excitation that marks the face state $|a_r\rangle$ contributes to a coherent shell source

$$|J_N\rangle = \sum_{r=1}^{N} |a_r\rangle.$$

The coarse scalar field δS is permutation-symmetric on face labels and therefore couples only through $P_{\rm sing}$. The scalar-channel response is quadratic in the total source, so the relevant object is the source norm

$$\langle J_N|P_{\rm sing}|J_N\rangle = \sum_{r,s=1}^{N} \langle a_r|P_{\rm sing}|a_s\rangle.$$

Because $\langle a|P_{\rm sing}|b\rangle = 1/7$ for any a, b, this reduces to $N^2/7$ independent of which specific face labels are excluded. Including the binary orientation degeneracy of the boundary ensemble, the orientation-summed scalar transfer weight per ordered shell incidence is

$$\mathcal{M}_{rs} = 2\,\langle a_r|P_{\rm sing}|a_s\rangle = \frac{2}{7}.$$

The ordered-pair count decomposes as $N^2 = N + 2\binom{N}{2}$: N self-incidences along the diagonal and $2\binom{N}{2}$ directed cross-incidences. Two distinct steps enter here, and only the first is the quadratic

EFT response. The quadratic singlet response fixes that there are N^2 ordered incidences, each carrying the per-incidence overlap $\mathcal{M}_{rs} = 2/7$; it does not by itself make the mass multiplicative. The passage from these N^2 additive incidences to a multiplicative factor is the log-overlap mass map: the mass is the exponential of a dressing free energy whose pairwise term is $\sum_{r,s} \ln \mathcal{M}_{rs} = N^2 \ln(2/7)$, the same log-overlap form that carries the gravitational dressing (Appendix H.3) and stated explicitly below. With that map, the N-shell dressing is

$$\prod_{r,s=1}^{N} \mathcal{M}_{rs} = \left(\frac{2}{7}\right)^{N^2},$$

so

$$A_0 = \ln\frac{2}{7} = -1.253\ldots,$$

compared with the empirical fit $A_0 = -1.255$ (agreement at 0.15%).

The ordered-bilinear structure is not a free assumption. It is the unique scalar-channel response to a multi-source state under the three premises just stated: the source norm $\langle J_N | P_{\text{sing}} | J_N \rangle$ runs over both indices independently because the integrated-out scalar contribution is quadratic in J_N.

Mass ladder and numerical accuracy. Combining the two coefficients,

$$\boxed{\frac{m_N^{(0)}}{m_e} = 720^N \left(\frac{2}{7}\right)^{N^2}, \qquad N = 0, 1, 2.}$$

Equivalently,

$$\log\frac{m_N}{m_{N-1}} = \ln 720 + (2N-1)\ln\frac{2}{7},$$

so each added shell contributes $\ln 720$ plus an odd number of new scalar-overlap incidences $(1, 3, 5, \ldots$, summing to $N^2)$. The second difference is fixed at

$$\Delta^2 \log m_N = 2\ln\frac{2}{7} = -2.506\ldots,$$

compared with the empirical -2.509 (0.15%). The marked vertex of H.9 then supplies

$$Z_\mu = 1 + \zeta_*, \qquad Z_{\tau,2} = 1 + \frac{2}{7}\zeta_*.$$

Thus

$$\frac{m_\mu}{m_e} = 720\frac{2}{7} Z_\mu = 206.768280237 \qquad (-0.535\sigma),$$

$$\frac{m_\tau}{m_e} = 720^2 \left(\frac{2}{7}\right)^4 Z_\mu Z_{\tau,2} = 3477.343310 \qquad (+0.481\sigma).$$

The dressing map, explicitly. The three factors follow from one construction. Write the N-th charged lepton as a coherent source raised on the electron ground dressing by exciting N shells, $\mathcal{S}_N = \sum_{k=1}^{N} s_k$, where each s_k excludes one face label from the seven-state alphabet. Its dressing free energy is a log-overlap functional of the kind used for the gravitational dressing in Appendix H.3, and it separates into a per-shell term and a pairwise term. Each shell carries the reduced-alphabet entropy $\ln \Omega_1 = \ln 6! = \ln 720$, the permutation count of the six surviving labels, which builds the linear factor 720^N. Each ordered pair of shells contributes a singlet log-overlap: a single label projected on the closure singlet $u = \frac{1}{\sqrt{7}}(1, \ldots, 1)$ of Section 8 has weight $|\langle u|m\rangle|^2 = 1/7$, and orientation summation doubles it to $2/7$. The N shells form N^2 ordered pairs, so the pairwise term is $N^2 \ln(2/7)$, and exponentiating the free energy returns

$$\frac{m_N^{(0)}}{m_e} = \Omega_1^N \left(\frac{2}{7}\right)^{N^2} = 720^N \left(\frac{2}{7}\right)^{N^2}.$$

The coefficients are fixed: $720 = 6!$, the N^2 pair count is combinatorial, and $2/7$ is the orientation-summed singlet projection behind $\Omega_{\text{tet}} = 2\,P(7,4) = 1680$. The marked factors are finite traces of the same decorated vertex used in the electron scale.

The support exponent, tested against the spectrum. The ladder also tests the exponent in the support-to-length identification of Appendix D.4, which reads the physical ratio as the first power of the entropy support, $\lambda/L_* \propto e^H$. Letting that power float, $\lambda/L_* \propto e^{\alpha H}$, turns each lepton ratio into a measurement of α, since $m_N/m_e = \left[720^N (2/7)^{N^2}\right]^{\alpha}$:

$$\alpha_N = \frac{\ln(m_N/m_e)_{\text{PDG}}}{\ln\left[720^N (2/7)^{N^2}\right]}, \qquad \alpha_\mu = 1.0010, \qquad \alpha_\tau = 1.0008.$$

The leading shell values give $\alpha_\mu = 1.0010$ and $\alpha_\tau = 1.0008$, while square-root and quadratic support maps miss by orders of magnitude. The decorated action does not float this exponent. It keeps the first-power support map and computes the finite response factors separately.

Status. The three-generation termination, baseline shell coefficients, and marked response are fixed inside the displayed finite action. The collapse at $N = 3$ is the permutation-invariance of K^2; 720, $2/7$, and N^2 are reduced-alphabet, singlet-projection, and ordered-pair counts; and Z_μ, $Z_{\tau,2}$ are the enumerated marked traces. The remaining particle-sector work concerns other defect classes, not an unfixed charged-lepton coefficient.

The electron remains the elementary weak-field anchor; the heavier charged leptons are correlated outputs of the same marked shell algebra, not closure-defining inputs. Composite hadrons remain part of the dressed bound-state entropy program. The neutrino and quark sectors are not treated here. Extending the closure-spectrum-collapse argument to those defect classes remains a separate task.

I.2 Gauge structure and the conditional projective color sector

The framework's primary target remains the gravitational and dark sector. The Abelian and weak sectors below are substrate-natural hosting arguments whose physical Standard Model identification remains external. Color now has a stronger status. One conditional microscopic identification — a colored defect carries one persistent open tetrahedral-routing leg, transported reversibly until singlet closure — fixes the projective qutrit transport algebra and the adjoint form of the local pure-gauge action. A minimal kinetic completion then fixes its primitive adjoint transfer. This does not derive the global $SU(3)$ endpoint lift, chiral quarks, hypercharge assignments, quark masses, confinement, or the finite matching to a continuum QCD scheme.

Baseline-redundancy template. The same logic that organizes the gravity sector extends to any conserved-charge sector. For a conserved charge Q, introduce an entropy-like potential $S_Q(x)$ and require that physical observables depend only on differences of that potential, not on its absolute baseline. Localizing that redundancy requires a compensating connection. In the Abelian case,

$$D_\mu S_Q = \partial_\mu S_Q - qA_\mu, \qquad S_Q \to S_Q + \alpha(x), \qquad A_\mu \to A_\mu + \frac{1}{q}\partial_\mu \alpha,$$

giving Maxwell-type dynamics for A_μ. The non-Abelian generalization, for multiplet-valued entropic potentials transforming under a compact Lie group G, gives Yang–Mills covariant derivatives and field strengths in the standard form.

The template tells us what form a gauge interaction takes once a conserved-charge sector with baseline redundancy is present. It does not select which specific groups are physically realized.

Substrate-natural components. Two ingredients of Standard Model gauge structure are naturally accommodated by the substrate.

$U(1)_Y$. The baseline-redundancy template naturally accommodates an Abelian conserved-charge sector of the hypercharge type. The associated entropy potential is conserved-charge in character, and localizing its baseline redundancy produces the corresponding Maxwell-type dynamics. The argument is independent of the gravity-sector derivation, applying the same template to a separate conserved-charge potential.

$SU(2)_L$. The substrate naturally carries an $SU(2)$-representation structure through its half-integer face data. On the marked-fusion incidence branch, the three-dimensional vector response and the complement theorem select the primitive spin $j_0 = \frac{3}{2}$ (Section 5 and Appendix B); without that identification, the same spin is the surviving discrete branch. The static K^2 ensemble breaks this $SU(2)$ to its $U(1)$ Cartan through the magnetic-quantum-number projection, but the full $SU(2)$ remains present in the quantum face data discarded by that projection. Identifying this representation-theoretic $SU(2)$ with the internal weak group $SU(2)_L$, including chirality and doublet assignments, remains external to the present derivation.

Three tetrahedral routes and the closed-cell obstruction. Four faces admit exactly three opposite-face pairings,

$$A = (12\,|\,34), \qquad B = (13\,|\,24), \qquad C = (14\,|\,23).$$

Tetrahedral permutations act transitively on this set, so no route is preferred. On a closed effective cell define

$$R_A = m_1 m_2 + m_3 m_4, \quad R_B = m_1 m_3 + m_2 m_4, \quad R_C = m_1 m_4 + m_2 m_3.$$

Their sum is fixed by the closure invariant,

$$R_A + R_B + R_C = \frac{1}{2}\left[\left(\sum_i m_i\right)^2 - \sum_i m_i^2\right] = 72 - \frac{3}{2}K^2.$$

The symmetric route is therefore the scalar closure mode already read by gravity. The exact 1680-state sum at η_* gives the connected covariance

$$C_R = \begin{pmatrix} 27.45571280 & -7.84449077 & -7.84449077 \\ -7.84449077 & 27.45571280 & -7.84449077 \\ -7.84449077 & -7.84449077 & 27.45571280 \end{pmatrix}.$$

With

$$P_{\mathbf{1}} = \frac{1}{3}\mathbf{1}\mathbf{1}^T, \qquad P_{\mathbf{2}} = I - P_{\mathbf{1}},$$

this becomes

$$\boxed{C_R = 11.76673126\, P_{\mathbf{1}} + 35.30020358\, P_{\mathbf{2}}}.$$

The singlet eigenvalue is fixed analytically,

$$\lambda_{\mathbf{1}} = \frac{1}{3}\,\mathrm{Var}(R_A + R_B + R_C) = \frac{3}{4}\,\mathrm{Var}(K^2).$$

A closed cell thus supplies $\mathbf{3}_{\text{route}} = \mathbf{1}_{\text{closure}} \oplus \mathbf{2}_{\text{imbalance}}$. It cannot furnish an irreducible color triplet as a closed scalar observable.

Primitive route geometry. The route carrier belongs to the primitive spin layer, before neighboring slots fuse into the effective $j_{\rm eff} = 3$ link. Couple the four primitive $j_0 = 3/2$ slots according to A, B, or C, taking each pair through its maximal $J = 3$ channel and closing the two intermediate spins to a singlet. Standard $SU(2)$ recoupling gives

$$\langle A|B\rangle = \langle A|C\rangle = \langle B|C\rangle = -\frac{1}{20}.$$

The route Gram matrix is therefore

$$G = \begin{pmatrix} 1 & -1/20 & -1/20 \\ -1/20 & 1 & -1/20 \\ -1/20 & -1/20 & 1 \end{pmatrix} = \frac{9}{10}P_{\mathbf{1}} + \frac{21}{20}P_{\mathbf{2}}.$$

Its eigenvalue ratio is

$$\boxed{\frac{\lambda_{\mathbf{2}}}{\lambda_{\mathbf{1}}} = \frac{7}{6}},$$

the same exact ratio that supplies the isotropically averaged one-label-exclusion source benchmark $\ln(7/6)$ in Appendix C.5. This equality follows before any color datum is introduced. It is a cross-sector recoupling identity, not a determination of the strong coupling.

The layer ordering is fixed by the construction. Two primitive shared-face slots first form $V_{3/2} \otimes V_{3/2}$, maximum-capacity fusion selects V_3, and the four resulting effective links carry the K^2 closure ensemble. The color route therefore resides in an open primitive record that survives between fusion events. Appendix H.9 gives the complementary result that the nonmaximal primitive fusion channels $V_0 \oplus V_1 \oplus V_2$ are exactly the nine-state representation already required by the present/history marked fiber.

Persistent open routing and the gauge algebra. The conditional color identification is precise: a colored fermionic defect carries one unclosed tetrahedral-routing leg, and transport preserves its correlations with an arbitrary reference until the leg closes into a singlet. The three unresolved route alternatives span the qutrit

$$\mathcal{H}_{\rm route} \simeq \mathbb{C}^3.$$

A reversible quantum channel between equal finite-dimensional systems is unitary conjugation,

$$\mathcal{E}_U(X) = UXU^\dagger, \qquad U \in U(3).$$

The channel is unchanged under $U \mapsto e^{i\theta}U$, so its physical transport group is

$$PU(3) = U(3)/U(1) \simeq SU(3)/Z_3,$$

with Lie algebra $\mathfrak{su}(3)$. This fixes the local connection algebra and its adjoint representation. It does not choose the global lift of the projective channel to endpoint amplitudes. A fundamental $SU(3)$ endpoint and Z_3 triality require such a lift together with an identification of the central $U(1)$ phase; those are part of the open matter-sector completion.

The endpoint observable algebra provides the same result in a complementary form. A route-resolving endpoint supplies $P_A = |A\rangle\langle A|$; tetrahedral permutations generate P_B and P_C. If S_{ab} carries route b to route a, then

$$P_a S_{ab} P_b = |a\rangle\langle b| \equiv E_{ab}.$$

The nine matrix units span $M_3(\mathbb{C}) = \mathbf{1} \oplus \mathbf{8}$. The identity is the common phase direction and the traceless Hermitian operators are the eight connection directions. The route endpoint is physically resolved, while its basis in transit is gauge covariant. This closes the earlier gap between a discrete S_3 triple and a continuous gauge algebra; S_3 alone would generate only $\mathbb{C} \oplus M_2(\mathbb{C})$.

Choosing route bases independently at the cells gives the link transformation

$$U_{yx} \longmapsto V_y U_{yx} V_x^\dagger.$$

Around a loop based at x, $U_p = \prod_{\ell \in p} U_\ell$ therefore transforms by conjugation, $U_p \mapsto V_x U_p V_x^\dagger$. This is the local gauge redundancy of the transported route frame.

Adjoint Wilson form from channel-return fidelity. The transported object fixed above is a channel on route density operators, so its return cost must be unchanged by $U_p \mapsto e^{i\theta} U_p$. The difference of two normalized route projectors is traceless and Hermitian; on that irreducible adjoint space, the positive frame-invariant quadratic form is unique up to an overall scale. The scale is absorbed into κ_{open}. For a normalized route state q, set $P_q = |q\rangle\langle q|$ and define the resulting Hilbert–Schmidt channel-return failure

$$\Delta_p(P_q) = \left\| P_q - U_p P_q U_p^\dagger \right\|_{\text{HS}}^2 = 2\left(1 - |\langle q|U_p|q\rangle|^2\right).$$

For an unbiased qutrit, the Haar second moment gives

$$\overline{|\langle q|U_p|q\rangle|^2} = \frac{\operatorname{Tr}(U_p U_p^\dagger) + |\operatorname{Tr} U_p|^2}{3(3+1)} = \frac{3 + |\operatorname{Tr} U_p|^2}{12}.$$

Using $\chi_{\mathbf{8}}(U) = |\operatorname{Tr}_{\mathbf{3}} U|^2 - 1$,

$$\overline{\Delta_p} = \frac{3}{2}\left(1 - \frac{|\operatorname{Tr}_{\mathbf{3}} U_p|^2}{9}\right) = \frac{4}{3}\left(1 - \frac{1}{8}\chi_{\mathbf{8}}(U_p)\right).$$

A cost κ_{open} per unit return failure therefore gives the adjoint Wilson form [6],

$$\boxed{S_{\text{gauge}} = \beta_{\text{adj}} \sum_p \left(1 - \frac{1}{8}\operatorname{Tr}_{\mathbf{8}} \operatorname{Ad} U_p\right), \qquad \beta_{\text{adj}} = \frac{4}{3}\kappa_{\text{open}}.}$$

This action is well defined on $PU(3)$ and is blind to the center, exactly as the channel construction requires. A fundamental-trace Wilson term would require the unresolved $SU(3)$ endpoint lift. The group argument alone does not fix κ_{open}.

Minimal primitive kinetic normalization. The primitive recoupling matrix fixes the missing local coefficient after one named microscopic choice: geometric gluing uses the identity pairing on the primitive boundary Hilbert space before an independent kinetic operator is added. In the nonorthogonal route basis, the identity pairing has matrix G. A direct branch $K = sG$ admits a trace-preserving reversible dilation only when $K^\dagger K \le I$. Maximum faithful throughput saturates the largest eigenvalue, so

$$s = \frac{20}{21}, \qquad K = \frac{20}{21}G = P_{\mathbf{2}} + \frac{6}{7}P_{\mathbf{1}}, \qquad L = \sqrt{I - K^\dagger K} = \frac{\sqrt{13}}{7}P_{\mathbf{1}}.$$

Together, K and L define an isometry and therefore admit a reversible unitary dilation. Only the symmetric closure component enters the complementary branch; the primitive imbalance doublet has unit direct amplitude.

Transport through the vacuum carries no preferred absolute route frame. Replacing $P_{\mathbf{1}}$ by an arbitrary rank-one projector $P = |n\rangle\langle n|$ and averaging the equivalent frames gives

$$K_P = I - \frac{1}{7}P, \qquad L_P = \frac{\sqrt{13}}{7}P.$$

The qutrit Haar identities

$$\int dP\, P = \frac{I}{3}, \qquad \int dP\, PXP = \frac{X + \mathrm{Tr}(X)I}{12}$$

then yield the covariant channel

$$\begin{aligned} \mathcal{E}(X) &= \int dP \left(K_P X K_P^\dagger + L_P X L_P^\dagger\right) \\ &= \frac{13}{14}X + \frac{1}{42}\mathrm{Tr}(X)I = 1_{\mathbf{1}} \oplus \frac{13}{14} I_{\mathbf{8}}. \end{aligned}$$

Thus

$$\boxed{t_8 = \frac{13}{14}}, \qquad -\ln t_8 = \ln\frac{14}{13}.$$

For generators normalized by $\mathrm{Tr}(T^a T^b) = \delta^{ab}/2$, one primitive transfer interval τ_{r} has the isotropic adjoint generator

$$\mathcal{L}(X) = -\frac{\ln(14/13)}{3\tau_{\mathrm{r}}} \sum_{a=1}^{8} [T^a, [T^a, X]].$$

In the heat-kernel coordinate $t_R = \exp[-g_{\mathrm{HK}}^2 C_R/2]$ with $C_A = 3$,

$$\boxed{g^2_{\mathrm{HK,step}} = \frac{2}{3}\ln\frac{14}{13} = 0.04940531477.}$$

This value was obtained from the primitive geometry without a QCD target. It is conditional on the identity kinetic pairing, maximum faithful throughput, and route-frame isotropy.

Causal blocking on the diamond regulator. The paper already uses the four-valent diamond graph as the embedded adjacency realization of the tetrahedral bond frame. Its minimal product causal completion is $D_{\mathrm{diamond}} \times \mathbb{Z}$, with one temporal successor at each spatial site and no diagonal shortcuts. Let d be the spatial bond length, a_t the temporal spacing, $r = a_t/d$, and let ρ be the ratio of temporal to spatial primitive heat times. The shortest mixed time–space loops have four links; the shortest spatial loops are diamond hexagons with six links. Their per-site oriented area tensors are

$$M_{\mathrm{E}}^{ab} = \frac{2}{3} r^2 d^4 \delta^{ab}, \qquad M_{\mathrm{M}}^{ab} = \frac{32}{9} d^4 \delta^{ab}.$$

The diamond primitive translations $a_1 = d(n_1 - n_2)$, $a_2 = d(n_1 - n_3)$, $a_3 = d(n_1 - n_4)$ have determinant $16d^3/(3\sqrt{3})$ and contain two sites, fixing the embedded volume per site to

$$V_s = \frac{8}{3\sqrt{3}} d^3.$$

Matching the electric and magnetic area sums to one isotropic continuum action gives

$$g_{\mathrm{E}}^2 = \frac{8\sqrt{3}}{3r}(1+\rho)t_s, \qquad g_{\mathrm{M}}^2 = \frac{3\sqrt{3}}{2} r t_s,$$

where t_s is the primitive spatial heat time. The condition $g_{\mathrm{E}}^2 = g_{\mathrm{M}}^2$ fixes

$$\frac{a_t}{d} = \frac{4}{3}\sqrt{1+\rho}, \qquad g_{\mathrm{iso}}^2 = 2\sqrt{3(1+\rho)}\, t_s.$$

The equal-transfer completion $\rho = 1$ treats spatial route transfer and present/history transfer as uses of the same primitive instrument. It gives

$$\boxed{\frac{a_t}{d} = \frac{4\sqrt{2}}{3}}, \qquad \boxed{\frac{g_{\rm iso}^2}{g_{\rm HK,step}^2} = 2\sqrt{6}}.$$

The corresponding projective pure-gauge measure is

$$Z_{\rm gauge} = \int [dU] \prod_{p_{\rm E}} K_{4t_s}(U_{p_{\rm E}}) \prod_{p_{\rm M}} K_{6t_s}(U_{p_{\rm M}}),$$

and its graph heat-kernel coordinate is

$$\boxed{g_{\rm graph,HK}^2 = 2\sqrt{6}\, g_{\rm HK,step}^2 = \frac{4\sqrt{6}}{3} \ln \frac{14}{13} = 0.24203562353.}$$

Here $U_\ell \in PU(3)$ and K_t is the heat kernel for the projective group; locally it has the same $\mathfrak{su}(3)$ Casimir normalization used above. This is a graph heat-kernel coordinate, not an $\overline{\rm MS}$ coupling. Conversion to a Wilson or continuum scheme requires the corresponding class integral and finite matching; neither is supplied by the blocking identity. The dimensionless result also does not identify the embedded bond length d with the electron-calibrated L_*; that geometric dictionary belongs to the full GFT embedding.

Color and generation factorize. The shell and open-route spaces are independent in the minimal colored-defect extension,

$$\mathcal{H}_{\rm defect} = \mathcal{H}_{\rm shell} \otimes \mathbb{C}^3_{\rm route}, \qquad K_{\rm total}^2 = K_{\rm shell}^2 \otimes I_3, \qquad T^a_{\rm color} = I_{\rm shell} \otimes T^a.$$

It follows that $[K_{\rm total}^2, T^a_{\rm color}] = 0$. The shell cutoff is also alphabet-independent: every one- and two-label exclusion leaves a nondegenerate closure branch, while every three-label exclusion makes K^2 constant. A colored realization would therefore carry three route components at each of the same three shell depths. The factor of three is gauge multiplicity and is not an additive $\ln 3$ contribution to an isolated particle mass; physical states require their color-flux dressing and singlet constraints.

Status of the color result. The persistent-open-route identification is the conditional physical premise. Given it, reference-preserving reversible transport fixes the $PU(3)$ channel group, $\mathfrak{su}(3)$ connection algebra, and eight adjoint directions; phase-invariant channel-return fidelity fixes the adjoint Wilson form. The identity primitive kinetic pairing, maximum-throughput rule, and route-frame isotropy fix $t_8 = 13/14$. The embedded-diamond, equal-transfer causal completion fixes the dimensionless blocking factor $2\sqrt{6}$ and places the microscopic projective pure-gauge branch in a weak-coupling regime.

Physical QCD requires further structure that this derivation does not contain: a global $SU(3)$ endpoint lift and central $U(1)$ identification, triality-carrying chiral quark defect classes, two weak species per generation, hypercharge assignments, a lattice fermion action, finite graph-to-$\overline{\rm MS}$ matching, and nonperturbative scale setting. The projective pure-gauge action does not prove confinement or predict a hadron mass. The result is a conditionally closed microscopic local pure-gauge sector, not a derivation of the Standard Model matter sector.

Hadronic matter and the mass–entropy bridge. The mass–entropy bridge $m = \kappa_m(\ell)\,\Delta S$ applies to fermionic defects regardless of their color label. For charged leptons, the defect is color-singlet and the marked shell action of Appendix I.1 gives both measured ratios within one standard deviation. For hadrons, the dressed entropy is dominated by the QCD-internal contributions of confinement-scale gluonic flux, trace-anomaly structure, chiral vacuum reorganization, and quark binding. The mass–entropy bridge then organizes the full dressed bound-state entropy budget,

$$S_{\mathrm{ent,H}}^{\mathrm{dressed}} = S_{\mathrm{defect}} + S_{\mathrm{bind}} + S_{\mathrm{conf}} + S_{\chi\mathrm{SB}},$$

without requiring the framework to re-derive QCD. The structural claim is compatibility: the dressed entropy budget is the quantity QCD calculates, and the mass–entropy bridge converts it to inertial mass through the same running $\kappa_m(\ell)$ used for elementary sectors.

Scope summary. The gauge sectors have distinct grades. The baseline-redundancy template supplies a natural Abelian hosting mechanism. Primitive spin data supply an $SU(2)$ representation, while its identification with the chiral weak group remains external. The persistent-open-route premise gives a conditional microscopic $PU(3)$ projective pure-gauge sector with a fixed minimal adjoint transfer channel and causal regulator. Its global $SU(3)$ lift, the Standard Model quotient, chiral representations, matter multiplets, and renormalized couplings have not been derived. The gravitational and charged-lepton results do not depend on this color extension.

The sectors in this appendix are structurally linked to the same entanglement logic but are not part of the controlled ordinary Einstein/capacity branch.

Appendix J: Lattice Microstructure Calculations

Appendix J records the technical content of Section 24: the host ensemble and the checks that its geometry remains connected, simplicial, foliated, thermalized, and volume-matched; the coupled measure and its declared modeling forks; the control methodology; the exact cell-order calculations; and the measured compatibility, defect, and capacity-transport results. It then states what a continuum critical-surface calculation would have to establish. Where Appendix K recomputes the coefficient chain from its definitions, this appendix couples that chain to a dynamical substrate and reports what the finite-lattice system does. The report, analyzer outputs, run configurations, verification scripts, and raw tables are maintained with the manuscript materials at `jaigp.org`; the report is generated from the archived CSV and log files rather than transcribed by hand.

J.1 Host ensemble and engine discipline

The host is a standard causal-dynamical-triangulations ensemble [75, 76]: triangulations of $S^1 \times S^3$ sampled by local Pachner-class moves under the Regge action, with the $(4,1)$-simplex number pinned by a quadratic penalty. Early demonstration calculations used eighty spatial slices near $(k_0, \Delta) = (2.2, 0.6)$. The largest-volume calculations use the directly mapped point $(2.0, 0.4)$ and auto-tune k_4 separately for the 40,000 and 100,000 volumes and for the forty- and eighty-slice realizations; the final values are 0.7560, 0.7608, and 0.7700, respectively. The action is tracked incrementally and checked against full recomputation throughout every run, and every quoted final triangulation is verified to be a connected simplicial manifold with the required foliation.

The host's phase location has been mapped directly. A (k_0, Δ) grid with k_4 auto-tuned to pseudo-criticality at every point — the volume-pin residual serving as the feedback error, as in standard CDT practice where k_4 is retuned per coupling point — finds an extended region at $k_0 = 1.0$–2.0, with slice-volume coherence peaking at $(k_0, \Delta) = (2.0, 0.4)$ and a dead row by

$k_0 = 3.5$: a visible boundary. A fixed $k_4 = 0.9$, which an untuned grid had used throughout, was the obstruction — every untuned cell was fighting the pin rather than sampling its phase. Before the largest-volume data were examined, the expected finite-volume trend at the selected couplings was stated explicitly: both the Hausdorff estimator and the concentration of volume in an extended blob should rise with volume. They do. The $N_{41} \simeq 42{,}112$ ensemble gives $d_H = 3.31$, blob score 1.46, and 23 active slices; the $N_{41} \simeq 102{,}002$ ensemble gives $d_H = 3.56$, blob score 1.57, and 24 active slices. This supports the placement of the selected couplings in the extended finite-volume region. It does not determine the order of the visible boundary or establish a continuum critical trajectory.

Validation of the cell identification in Section 24.1 exposed a defect in the original move set. The moves preserved the four-dimensional manifold conditions but could fold a spatial slice, leaving a few percent of slice triangles in violation of the required closed-3-manifold condition. The corrected engine rejects every proposal that would break the foliation. Its local test is complete because the link of an affected triangle necessarily passes through a newly created pentachoron, and it preserves detailed balance in the same way as the other locality filters. Every ensemble reported here uses this engine and passes the global audit: each slice triangle belongs to exactly two spatial tetrahedra, and every pentachoron is correctly threaded between its slices.

No result below infers more from that comparison than finite-volume host suitability. Every substrate claim compares ensembles at the same regulator: coupled against bare, failure against closed control, or one source strength against another. Establishing a continuous geometric transition and taking the decorated common-parent continuum limit are the separate tasks of J.9.

J.2 The coupled measure and its fidelity anchors

The coupled measure is the joint Gibbs distribution of Section 24.2, with the per-face label sum normalized so that each label carries base measure 1/7. Under that base measure two properties hold exactly: at $\beta = 0$ the label entropy cancels and the geometry marginal is identically the bare host, so the uncoupled ensemble reproduces the host by construction; and uniform label assignment at geometry moves is exactly detailed-balanced, so no proposal correction is needed for label births. The centering constant μ is the per-cell label free energy,

$$\mu(\beta) = -\frac{1}{\beta N_{\text{cells}}} \ln \mathbb{E}_{m\sim\text{unif}}\left[e^{-\beta \sum_c E_c}\right] = \frac{1}{\beta}\int_0^\beta \langle \bar{E} \rangle_s \, ds,$$

computed by thermodynamic integration on the thermalized base configuration and logged with each ensemble, so the extensive part of the closure term is neutralized rather than allowed to masquerade as a host coupling. Total volume is nevertheless checked directly and matched among comparisons rather than assumed from the centering. Equivalently, the normalized factor is $\widehat{Z}_{\text{cap}} = e^{\beta\mu N_{\text{cells}}} Z_{\text{cap}}$. It is exactly invariant under a common energy-origin shift $E_c \mapsto E_c + C$, $\mu \mapsto \mu + C$, which is the finite-regulator identity used in Section 25. Because $N_{\text{cells}} = N_{41}/2$, the omitted centering is exactly an N_{41} source. In the standard (κ_4, Δ) action basis this means $\delta\kappa_4 = -\beta\mu/2$ and $\delta\Delta = +\beta\mu/2$, not a pure k_4 displacement; Section 25 separates its physical volume component from the regulator-asymmetry component.

Every run opens by rebuilding the single-cell tables and checking the Part II anchors ($\Omega_{\text{tet}} = 1680$, $\eta_* = 0.0298668$, $\langle K^2 \rangle_{\eta_*} = 50.223$, $g_{\text{share,eff}} = 7.4198$), and refuses to sample if any fails.

Three modeling forks are declared. Injectivity is imposed as a penalty λ per colliding pair ($\lambda = 3$ in the quoted runs) because a hard seven-color constraint at the maximal conflict degree does not guarantee an ergodic single-face heat bath; the hard constraint is the $\lambda \to \infty$ limit, the residual collision fraction is reported in every table so the softness stays visible, and the physical $\beta = 1$

weighting is read at the ordered end. The closure sum is evaluated in the unoriented reading, in which the two cells sharing a face see the same label; the oriented alternative, in which they see opposite signs, requires oriented-slice bookkeeping through all moves and is recorded as an open fork. Parity contributes exactly $\ln 2$ per cell, extensive at pinned volume, and drops out of the coupled dynamics.

J.3 Controls and the reading discipline

Four validity conditions precede any physical reading. Every quoted triangulation must remain a connected simplicial manifold with the required foliation; its action and volume time series must be stationary over the measured window; compared ensembles must have matched total-simplex number, because volume differences can masquerade as dimension differences; and the Markov chain must continue moving, because a frozen chain is a failed calculation rather than a result. The interpretation of each possible outcome, including which outcomes would license no claim at all, was written before the corresponding data were examined.

The main control shuffles the closure energies. The closure energy takes 210 distinct values on the sorted-label orbits of the cell; the control reassigns those values among the orbits, preserving the value pool and permutation symmetry while destroying the closure structure, and is evolved at the same $\beta = 1$ coupling. Only a difference from this matched shuffled control is attributed to closure; anything it reproduces is sampling machinery.

An earlier EPRL calculation exposed the need for this control. A vertex amplitude [79] at frozen boundary spin $j = 3$, computed with the `sl2cfoam-next` library [80], was coupled to the same host. An analysis specified before reading the data found five misspecifications: off-equilibrium centering shifted the bare cosmological coupling; the label heat bath used the wrong exponent; label births lay outside the acceptance ratio; the result depended at order unity on an arbitrary slot convention; and sampling required a positivization of the sign-alternating amplitude. Every effect vanished at $\beta = 0$ and grew with β, so an uncoupled comparison could not reveal them. An entry-shuffled vertex tensor reproduced the apparent geometric steer at matched coupling and identified it as sampler machinery. The sampler was rebuilt as an exact joint Gibbs measure, and the later closure calculations include the same shuffled-energy control. This episode is reported as methodology. The frozen-spin truncation and positivization already limit its physics scope, and the EPRL intertwiner space is distinct from the seven face labels of the present ensemble (Section 29.8).

J.4 Exact results at cell order

Six calculations, independent of any Monte Carlo run, fix what the substrate can and cannot do for the vacuum before the coupled measurements are read.

Induced curvature coupling. The geometry marginal of the coupled measure weights a slice by the free energy of its labels, and the leading geometric dependence enters through the coordination q of slice edges, the discrete curvature variable. A transfer-matrix evaluation of the label free energy on the ring of cells around an edge gives, at the physical $\beta = 1$ weighting, an induced coupling $c_0 = +0.019$ per slice edge, against the bare $k_0 \simeq 2.2$: about one percent. The label correlation length is about half a lattice step, and the beyond-quadratic dependence on q is negligible at physical coordinations. The closure weighting alone therefore cannot supply the host's geometric stiffness, and its predicted effect on the coupled sweep is a steer too small to see at single-seed resolution, a prediction the calculation of J.5 confirms.

The refresh at cell order. Modeling the selected memoryless kernel as whole-boundary redraws arriving at unit rate per cell, the single-cell stationary state is exact: the probability that k faces retain their last self-drawn values is $k!(4-k)!/120$, vacuum admissibility is 0.536, and $\langle K^2 \rangle = 48.74$ against the ensemble value 50.22. The state is geometry-blind to exponential accuracy and induces no vacuum curvature action. It does couple geometry updates to the local density of closure failure when their rates depend on that density. This calculation tests the effective refresh model, not a derivation of its microscopic operator.

The history-weighting ceiling. Two computations bound the strongest vacuum role a Many-Pasts tilt over closure-class observables could play. Amplifying the closure link interaction to 128 times the physical $\beta = 1$ weighting saturates the link-agreement ratio near 0.55 with correlation length 1.7 links and a curvature-sector signal near 0.1: a structural ceiling, set by the ground-state degeneracy of the closure-plus-injectivity interaction, twenty times below the bare coupling. Independently, the exact Doob-transform solution of a closure-maintaining history tilt (paths weighted by time-integrated closure quality) on a ring of six seven-state faces drives the nearest-neighbor label correlation from -0.12 through zero to only $+0.13$ across the full tilt range, with longer-range correlations near 0.03 throughout: no long-range order at any tilt strength. Together with the two paragraphs above this closes the label sector: static weight, refresh, and closure-class history weighting each fail, quantitatively, to supply vacuum geometric stiffness at cell order.

The free conserved field, integrated out. A conserved Gaussian field back-coupled to the geometry induces, on integration, the geometric action $\frac{1}{2} \ln \det' L(g)$ per channel and slice: the spanning-tree entropy of the slice graph, by the matrix-tree theorem. Computed on degree-matched graph classes, the entropy per cell is 0.607–0.613 on diamond-cubic lattices (the smooth extended proxy), 0.609 on random 4-regular graphs (the crumpled proxy), and 0.56–0.57 on real engine slices: nearly universal at fixed coordination, with the genuinely nonlocal soft-mode part diluting as $\ln N/N$ per cell. The per-cell differential between extended and crumpled geometry is a few times 10^{-4}; with all seven channels the phase-tipping force is of order 10^{-3} against $k_0 \simeq 2.2$.

The budget-bounded field. The finite budget of Postulate I makes the field non-Gaussian, with saturation curvature $m^2 = 2/g_{\text{share,eff}} \simeq 0.27$, and the massive determinant differential between the same proxies shrinks from 1.2×10^{-3} per cell at zero mass to 4×10^{-4} at the budget mass and toward zero beyond: the theory's own non-Gaussianity suppresses the modes where the sensitivity lived. What survives, measured on eight engine slices, is a local coupling renormalization of scale $\simeq 0.08$ per cell in the seven-channel count, absorbed into the host's bare-coupling scan; it moves phase boundaries and supplies no phase of its own. The two containers no branch covers are named in Section 24.3: long-range field kernels and non-equilibrium geometry–field co-evolution, both unspecified by the theory.

The fracture construction. Under unit label shifts preserving injectivity, the 1680-state move graph has 48 connected components of 35 states, labeled by slot ordering and parity. This proves that the tested move class cannot equilibrate the full admissibility ensemble. More general microscopic dynamics are not excluded.

J.5 Compatibility at matched volume

The closure comparison starts from common tuned geometries at $N_{41} \simeq 100{,}000$. Three eighty-slice ensembles use respectively the uncoupled value $\beta = 0$, the physical weighting $\beta = 1$, and $\beta = 1$ after the closure energies have been shuffled among label orbits. The physical weighting is also repeated with forty slices. In every case the final triangulation is connected, simplicial, correctly foliated, and stationary over the measurement window. Across the three eighty-slice ensembles the total-simplex spread is 0.6%.

ensemble	N_4	N_{41}	collision fraction	d_H	centering μ
$\beta = 0$ (80-slice control)	323,514	100,216	0.654	3.55	3.9913
$\beta = 1$ (80-slice physical)	323,232	99,998	0.077	3.59	2.4990
$\beta = 1$ (80-slice shuffled)	325,346	99,944	0.736	3.59	3.1420
$\beta = 1$ (40-slice repeat)	316,238	100,020	0.078	3.74	2.4983

The uncoupled ensemble remains on the exact uniform-measure benchmark $1 - 840/2401 = 0.650$ within sampling error. The physical weighting suppresses collisions by nearly an order of magnitude, and the shuffled control remains high despite carrying the same energy-value pool. At the same time the matched eighty-slice d_H values occupy the narrow interval 3.55–3.59, their extended volume profiles persist, and the forty-slice repetition reproduces the ordered collision fraction. Thus the microscopic closure structure is strongly active while the host geometry is stable at this resolution. This supersedes the earlier 20,000-volume preview; its conclusion is unchanged and its finite-volume caveat is smaller.

J.6 The Regge bridge: a predicted volume-sector dressing

The bridge is an ordinary statistical-mechanical statement made quantitative. Integrating the labels out at fixed geometry defines an effective action for the host,

$$e^{-S_{\text{eff}}[g]} = e^{-S_{\text{host}}[g]}\, Z_{\text{label}}[g], \qquad S_{\text{eff}}[g] = S_{\text{host}}[g] - \log Z_{\text{label}}[g],$$

with S_{host} the Regge action plus the volume pin. The label free energy $-\log Z_{\text{label}}[g]$ is a function of the geometry alone. Its extensive part is $\beta\mu(\beta)\, N_{\text{cells}}$, with μ the per-cell label free energy of J.2; its geometry-dependent remainder is the induced curvature-sector action priced by the ring expansion of J.4. The centered ensemble subtracts the extensive part by construction; left in, it is the leading coefficient of the effective action, and the host's response to it can be predicted before it is measured.

The centering constant μ of J.2 therefore gives a direct prediction. In the centered ensemble, the subtraction of μ removes the extensive label free energy from the geometry marginal. If that subtraction is omitted, the geometry sees a residual positive linear contribution $+\beta\mu N_{\text{cells}}$ to the effective action. Here N_{41} denotes the total number of $(4, 1)$ and $(1, 4)$ pentachora, so each spatial cell, shared by one pentachoron of each orientation, is counted once by the pair and $N_{\text{cells}} = N_{41}/2$. The positive linear tilt competes with the quadratic volume pin $\varepsilon(N_{41} - \bar{N})^2$, and the shift follows in two lines. The volume-dependent part of the effective action is

$$S_{\text{vol}}(N_{41}) = \varepsilon\,(N_{41} - \bar{N})^2 + \beta\mu\, N_{\text{cells}} = \varepsilon\,(N_{41} - \bar{N})^2 + \frac{\beta\mu}{2}\, N_{41},$$

and setting $dS_{\text{vol}}/dN_{41} = 2\varepsilon(N_{41} - \bar{N}) + \beta\mu/2 = 0$ gives the equilibrium displacement

$$\Delta N_{41} = N_{41}^{\text{eq}} - \bar{N} = -\frac{\beta\,\mu}{4\,\varepsilon}.$$

Here μ is computed by annealed thermodynamic integration of the label ensemble on the base geometry, before any coupled dynamics run; the zero-lattice reference — the exact single-cell free energy from direct enumeration of the 7^4 label states, computable from η_* and the injectivity penalty alone — is 2.477 at this point, within half a percent of the lattice value, the difference being the shared-face correlations. At the demonstration point ($\beta = 1$, $\mu = 2.488$, $\varepsilon = 0.01$) the prediction is $\Delta N_{41} = -2.488/0.04 = -62.2$. Three ensembles from a common equilibrated base at $N_{41} = 2000$ measure it:

ensemble	mean N_{41}	shortfall
bare ($\beta = 0$)	1993.2	-6.8
centered ($\beta = 1$)	1969.8	-30.2
uncentered ($\beta = 1$)	1907.0	-93.0

The uncentered-minus-centered difference is -62.8 against the predicted -62.2. The subtraction isolates the μ-term, since the two $\beta = 1$ ensembles share every other piece of machinery; the centered ensemble's own -30 is the second-order non-neutrality of the thermodynamic-integration centering. It cancels in the difference and is kept out of other comparisons by matching their volumes directly. A number computed from the ultraviolet specification thus appears, at the predicted magnitude, as an extensive N_{41} contribution to the dynamical host action. This is a dressing of the host, not a derivation of its continuum limit. In the standard CDT basis it is a correlated (κ_4, Δ) displacement; only projection with the two-simplex Regge volume operator identifies its physical volume component. The measurement is at demonstration volume and uses one seed. The corresponding curvature-sector measurement remains open.

The demonstration point is one member of a predicted family whose functional form was recorded before sampling. At fixed geometry-side settings the displacement obeys $\Delta N_{41}(\beta, \varepsilon) = -\beta\mu(\beta)/(4\varepsilon)$, and each factor is separately testable. In β the curve is concave, because $\beta\mu(\beta) = \int_0^\beta \langle \bar{E} \rangle_s \, ds$ and the Gibbs mean falls as the labels order, with local slope $d(\Delta N_{41})/d\beta = -\langle \bar{E} \rangle_\beta/(4\varepsilon)$ — an Ehrenfest-type relation in which the same calculation supplies both sides, since $\langle \bar{E} \rangle_\beta$ is measurable in the ensemble whose displacement it predicts. In ε the law is exactly inverse at fixed β; departures from $1/\varepsilon$ measure the curvature of the background free energy that the matched-pair subtraction cancels at first order, so the ε family doubles as a control on the derivation's one assumption. The shuffled table carries its own computable free energy: at $\beta = 1$ the single-cell values are $\mu_{\text{shuf}} = 3.17$ against $\mu = 2.48$, so the shuffled ensemble is predicted to displace by $-\beta\mu_{\text{shuf}}(\beta)/(4\varepsilon)$ — not by zero, and not by the physical value, a 28% separation. The measured number must track the table from which it was computed, so the shuffled calculation is a second quantitative prediction rather than merely a null control.

The family has been measured at demonstration volume, with every prediction written to disk before sampling began. Eight of nine planned coupling–volume-penalty combinations produced readable data; the ninth failed at launch and contributes no datum. Across a fifteen-fold range of predicted displacements, from -10.5 to -157.6, the measured-to-predicted ratios are 0.96–1.07 with one point at 0.89. The β-curve is concave as required: -43, -78, -121 measured against -43, -76, -125 predicted — not the straight line that would have falsified the free-energy integral. Halving ε doubles the displacement to within four percent. The shuffled pair lands on its own line, displaced thirty percent from the physical-table results and at ratio 1.00 to the prediction computed from its shuffled table. A repetition of this separate bridge calculation at the largest volume has not yet been performed.

J.7 The pinned-defect experiment

Two independent matched comparisons resume from thermalized $N_{41} \simeq 100{,}000$ ensembles with the physical $\beta = 1$ weighting and run 3000 further sweeps. For each random seed, one ensemble pins one hundred cells at maximal closure failure (all four faces $m = 0$: six collisions, $K^2 = 48$), while the control pins one hundred cells at the best-closed injective configuration ($m = \{0, 1, 2, 3\}$, $K^2 = 40.67$). Pinned faces are excluded from the label heat bath, and the carrier cells are protected from removal by the geometry moves identically. Every final triangulation remains connected and correctly foliated, and every ensemble retains all one hundred pins. The final analyzer pools the two seeds. When an observable–shell cell is present in both seeds its uncertainty is set by across-seed scatter; otherwise it uses the Sokal-windowed integrated-autocorrelation standard error. The largest measured integrated autocorrelation time is 349.5 measurement intervals.

condition	shell	mean E	collision fraction	mean coordination	cells per shell
failure	1	1.6802 ± 0.0026	0.0698 ± 0.0009	5.8452 ± 0.0024	4
	2	1.7299 ± 0.0006	0.0783 ± 0.0002	6.0869 ± 0.0281	7.7289 ± 0.2255
	3	1.7269 ± 0.0002	0.0786 ± 0.0001	6.1986 ± 0.0097	12.7960 ± 0.4518
closed control	1	1.7260 ± 0.0008	0.0783 ± 0.0002	5.8429 ± 0.0146	4
	2	1.7288 ± 0.0003	0.0788 ± 0.0001	6.0816 ± 0.0204	7.8633 ± 0.2143
	3	1.7277 ± 0.0002	0.0787 ± 0.0001	6.2384 ± 0.0022	12.5920 ± 0.4737

The failure-minus-closed-control differences make the reading unambiguous. At shell one, the mean closure energy changes by -0.0458 (16.8σ) and the collision fraction by -0.0085 (9.2σ). At shell two their separations are 1.7σ and 2.1σ, respectively. At shell three the mean coordination changes by -0.0398 (4.0σ), while its first- and second-shell separations are only 0.2σ. Shell-cell counts separate by at most 0.4σ anywhere in the measured range. Thus persistent failure produces a replicated, short-range capacity disturbance and a statistically resolved local coordination response. The earlier preview's shell-volume signal does not survive the larger-volume autocorrelation and seed treatment and is not retained as a result.

J.8 The conservation law and the transport instrument

The dichotomy of Section 24.7 in operator form. If free capacity is a per-cell budget re-equilibrated by maximum entropy — link shares $c_l \geq 0$ with $\sum_{l \ni x} c_l = C - m_x$ — stationarity gives one multiplier per cell, and the linearized constraint reads $(2z\mathbb{1} - L)\,\delta\mu = -m/\chi$ with $z = 4$ and L the slice Laplacian. The operator has no small-momentum pole; on a measured 169-cell slice its Green function falls eight orders of magnitude within thirteen steps.

The transport instrument tests a different object. Its pinned commitment level m_x is a fixed source label, while Q_x is the maintenance throughput the source draws. On a closed slice the simulated free field obeys

$$f_x(t+dt) - f_x(t) = -\sum_{\text{faces } xy} J_{xy} - Q_x + \overline{Q}, \qquad J_{xy} = -J_{yx}, \qquad \sum_x f_x = \text{const},$$

where $\overline{Q}$ is the uniform return required by the compact zero mode. This is steady service, not $f \to m$ stock conversion. Any local flux law linearizes to $J \propto -D\nabla f$ and the steady state solves $L\,\delta f = (Q/D)(\delta_{\text{source}} - \text{uniform return})$: the massless graph-Poisson equation, whose Green function on a three-dimensional slice falls as $1/r$. Section 24.7 derives the corresponding incidence-matrix Ward identity and identifies the continuum junction coefficient still to be measured.

The instrument implements the conservation reading with the sink left to emerge. A field on the slice cells is initialized at the vacuum anchor $g_{\text{share,eff}} = 7.4198$ and transported by antisymmetric face fluxes; the code asserts conservation at machine precision every sweep and aborts on any violation. The pin levels are fixed source labels, so the conserved quantity during this test is the free-field sum $\sum_x f_x$. Absorption is proportional to a cell's *persistent excess* closure failure above the measured vacuum baseline. A pinned defect therefore draws maintenance only through the failure generated by the ordinary label dynamics: its flux is measured, not set. Pins are placed at the five source values $m = 0, 1, 2, 3, 6$. The zero value measures commitment-independent boundary dressing; the positive values test source additivity and the deficit per unit emergent charge. At large distance the calculation tests whether the response follows a graph-Coulomb power law and whether a nonzero screening mass is detected. This first implementation freezes geometry while labels and capacity remain dynamical, isolating the transport law; conservative transport through geometry moves is the specified extension.

On small validation volumes the measured deficit increases with commitment, recovers monotonically with distance, and carries an emergent flux that rises with source value while the total field remains exactly conserved. The highest-commitment pin draws less flux than the linear extrapolation because it depletes its own neighborhood, a self-screening the model produces without being told.

Two failed definitions are retained as diagnostics rather than results. Absorption on total failure makes the vacuum itself a sink and produces the predicted four-to-seven-step screening. Absorption on instantaneous excess rectifies the vacuum's mostly-zero collision process and also creates a screening mass. Both failures follow quantitatively from their own update laws. The adopted definition therefore uses a 200-sweep persistence average, equilibrates the field before measurement, and subtracts the same-slice far-field reference to remove the compact zero mode. These analysis choices were fixed before the largest-volume data were generated.

The transport calculation uses an ordered forty-slice realization at $N_{41} \simeq 100{,}000$. It places thirty-two pins at each of the five source values, measures through shell eight, and runs 4000 sweeps. The mean field remains 7.4198 throughout. Per-pin time series are corrected with Sokal integrated autocorrelation times, and the level errors use across-pin scatter; the profile and junction errors additionally use the larger of propagated and across-level scatter. The finite-lattice results are

$$Q - Q(0) = (0.1440 \pm 0.0025)m, \qquad Q(0) = 0.0075,$$
$$\left\langle \frac{\Delta f_1 - \Delta f_1(0)}{Q - Q(0)} \right\rangle_{m>0} = 4.9965 \pm 0.5244,$$

and the normalized dressing-subtracted shell profile is

d	1	2	3	4	5	6	7	8
$\Delta f(d)/\Delta f(1)$	1.000	0.822	0.675	0.532	0.411	0.303	0.212	0.144.

The power-law fit gives -0.62 ± 0.11. The screening derivative is $+0.1062 \pm 0.0551$, statistically consistent with zero; the corresponding finite-range lower bound is $\xi > 4.6$ steps. The source fit is not perfectly linear: its maximum relative residual is 0.242, dominated by visible saturation at $m = 6$. The tail is also shallower than the ideal d^{-1}; the compact zero mode and finite slice radius flatten it, and the continuum scaling study must test convergence. Subject to those explicit diagnostics, the finite-lattice calculation demonstrates conservation, approximate source additivity, one junction ratio across the tested source values, power-law propagation, and no detected screening.

J.9 Capacity-decorated CDT and the continuum critical surface

These measurements establish the finite-lattice source-to-field behavior listed above. A microscopic CDT completion must still promote the capacity data to part of the joint transfer state and remove the regulator. For each triangulation T, the minimally sufficient configuration bundle is

$$\mathfrak{X}_{\mathrm{cap}}(T) = \{b_c,\ f_c,\ c_{A,c},\ \theta_A,\ H_{A,c;\ell}\}\,.$$

Here b_c is the existing closure state, f_c is transported free renewal bandwidth, and $c_{A,c}$ is the mass-equivalent committed allocation carrying the physical source label A. The variables θ_A and $H_{A,c;\ell}$ are respectively the source/worldline renewal phase and the internal history output at renewal depth ℓ. The index ℓ labels circuit records, not a fifth spacetime coordinate. The source-supported observables are

$$M^{\mathrm{lat}}_{c,A} = \sum_c c_{A,c},$$
$$\sigma_A = \frac{M^{\mathrm{lat}}_{c,A}}{M^{\mathrm{max}}_{c,A}},$$
$$\mathcal{C} = \text{single-stream integrability of the combined committed flow.}$$

There is no local per-channel "saturated" state and no microscopic coherence bit. After vacuum normalization the coarse availability observable is obtained from f_c, while σ_A is computed against the independently fixed per-source ceiling. The transfer kernel must route formation and release through the existing marked defect vertex, so $c_{A,c}$ is not an externally prescribed matter source. For a stable mark those stock-changing rates vanish; its ordinary closure dynamics instead determine the maintenance draw from f_c. Terminal exhaustion is omitted from this bulk transfer matrix because it belongs to the separate $q_{\mathrm{geo}} = 0$ boundary theory.

The ultraviolet configuration space is the disjoint union over causal triangulations,

$$\mathfrak{X}_{\mathrm{UV}} = \bigsqcup_{T \in \mathfrak{T}_{\mathrm{CDT}}} \mathfrak{X}_{\mathrm{cap}}(T),$$

with joint partition function

$$Z(\mathbf{g}) = \sum_T \frac{1}{C_T} e^{-S_{\mathrm{CDT}}[T;\kappa_0,\Delta,\kappa_4]} Z_{\mathrm{cap}}[T; \mathbf{g}_{\mathrm{cap}}].$$

The microscopic theory fixes $\mathbf{g}_{\mathrm{cap}}$; the physical capacity slice is therefore

$$\mathcal{P}_{\mathrm{cap}} = \{(\kappa_0, \Delta, \kappa_4; \mathbf{g}^*_{\mathrm{cap}})\}.$$

Integrating out the capacity variables gives $S_{\mathrm{eff}} = S_{\mathrm{CDT}} - \ln Z_{\mathrm{cap}}$, making their displacement of the host critical manifold a measurable output.

Increasing the number of simplices at a generic coupling point is not a continuum limit. First κ_4 must approach its critical value so the four-volume can diverge. A remaining relevant direction t must then approach a continuous geometric transition at which

$$\frac{\xi_{\mathrm{geom}}}{a} \sim |t|^{-\nu} \longrightarrow \infty.$$

Four-dimensional CDT contains candidate continuous transition lines that can support this construction [77, 78]. In the enlarged decorated theory define

$$\mathfrak{C}_* = \{\mathbf{g} : \kappa_4 = \kappa_4^c, \quad \xi_{\mathrm{geom}}/a = \infty\}.$$

The completion succeeds only if

$$\boxed{\mathcal{P}_{\mathrm{cap}} \cap \mathfrak{C}_* \neq \varnothing.}$$

If a capacity coupling is irrelevant, it shifts the location of the critical surface without adding a tuning. If it is relevant, its already-fixed microscopic value must lie on the surface. If the geometry and capacity sectors form a new joint fixed point, its exponents become predictions of the decorated theory. In every case, failure of the fixed capacity slice to intersect $\mathfrak{C}_*$ rejects this embedding.

The volume-operator normalization itself is already fixed kinematically. With Euclidean asymmetry α,

$$\mathcal{V}_4 = a^4 \left[\frac{\sqrt{8\alpha - 3}}{96} N_{41} + \frac{\sqrt{12\alpha - 7}}{96} N_{32} \right].$$

After the mixed non-volume directions have been projected out, the two independently read source coefficients must satisfy

$$\boxed{\frac{t_{41}}{a^4 v_{41}} = \frac{t_{32}}{a^4 v_{32}} = \rho_{\mathrm{vol,R}}.}$$

Equivalently, the target-blind double ratio

$$\boxed{\mathcal{R}_V(a) := \frac{t_{41}v_{32}}{t_{32}v_{41}} \longrightarrow 1}$$

must converge before either coefficient is converted to Λ_{R}. This equality uses two measurements and has no adjustable relative normalization. At fixed $\xi = N_{32}/N_{41}$ it reduces to $Z_V^{\mathrm{geom}} = (v_{41} + \xi v_{32})/(1+\xi)$. The scaling trajectory and its operator projection remain to be measured.

The required finite-size program is:

1. auto-tune κ_4 at each (κ_0, Δ), repeat the phase scan across increasing N_4, measure $\alpha(a)$ and $\xi(a)$, set a from a target-blind geometric observable, and verify the two-source equality $t_{41}/(a^4 v_{41}) = t_{32}/(a^4 v_{32})$ after critical-eigenoperator projection;
2. identify a continuous boundary using susceptibility peaks, Binder-cumulant crossings, histogram shape, transfer-matrix gaps, and pseudo-critical drift;
3. diagonalize the connected covariance matrix of the action-conjugate geometric and capacity operators,
$$\mathsf{C}_{ij} = \langle \mathcal{O}_i \mathcal{O}_j \rangle_c, \qquad \mathcal{O}_i = -\frac{\partial \ln w}{\partial g_i},$$
to identify the relevant mixed eigenoperator and the renormalized q observable;
4. verify the augmented-event incidence identity configuration by configuration and show that any screening mass vanishes with increasing volume;
5. measure D_q, χ_q, and α_{maint} separately, including their lattice-spacing factors, before forming $\chi_q \alpha_{\mathrm{maint}}/D_q$;
6. apply the electron-derived L_* and G_* only after the dimensionless continuum extrapolation and test the junction condition $\chi_q \alpha_{\mathrm{maint}}/D_q = 8\pi G_*/c^2$.

The ultraviolet and infrared limits do different work. The CDT critical surface removes the geometric cutoff. The incidence Ward identity protects the hydrodynamic zero mode of the proposed bandwidth carrier. Neither result supplies the other, and conservation alone does not establish the metric bridge, source purity, causal response, or junction normalization. At fixed lattice spacing, the larger host has the expected stronger extended-region indicators, and the transport calculation demonstrates conservation, approximate source additivity, a source-independent lattice junction ratio, power-law propagation, and no detected screening over the measured range. The visible boundary's order, the decorated critical trajectory, the target-blind operator and lattice-spacing matching, and the absolute junction to $8\pi G_*/c^2$ remain open. No continuum claim is assigned to the completed finite-volume results.

Appendix K: Numerical and Consistency Checks

Appendix K recomputes the numerical consequences of the stated derivations and collects the substrate-scale ledger. These checks support reproducibility but do not establish the ontology independently. Section 24 and Appendix J perform a separate task by coupling the same coefficient chain to a dynamical substrate. The complete self-contained reproduction code appears in terminal Appendix O.

The numbers recomputed here, and where each enters the main text, are: the admissibility entropy $g_{\mathrm{share,eff}}$ (Section 13, Appendices B and D.4); the substrate length L_* (Sections 2 and 13, Appendix D.4); the induced scale G_* (Sections 10 and 13, Appendices D.4 and K); the galactic scale a_0 (Section 15, Appendix C.7); the diamond-lattice Green constant $G_{\text{tet}}(0)$ (Appendix C.5); the edge-kernel chain $J_{\text{bare}} \to \gamma$ and the return sum Σ_{ret} (Appendices C.3–C.4); the charged-lepton ladder with its support-exponent cross-check (Section 13, Appendix I.1); and the primitive route, adjoint-channel, and causal-blocking identities of the conditional color branch (Appendix I.2).

K.1 Cross-sector numerical checks

The cross-check program includes:

- the one-bit fermionic defect check $\Delta S_f = \ln 2$;
- the rooted-shell convergence check $\sigma_{\text{ind}}^{(2)} \simeq \sigma_{\text{ind}}^{(3)}$;
- the UV closed-branch moments $\langle K^2 \rangle_{\eta_*}$, $\text{Var}_{\eta_*}(K^2)$, and a_{UV};
- the Gaussian closure-amplitude identity, nine-state response Gram matrix, unitary pair dilation, 21-block edge Hessian, and 56,800-state routing enumeration;
- the closed routing covariance, primitive route Gram spectrum, marked-fusion representation match, $t_8 = 13/14$ channel, and $2\sqrt{6}$ diamond blocking factor;
- cross-sector consistency among the electron anchor, the substrate length L_*, the induced scale G_*, the Green-matched Newton closure, and the galactic scale a_0.

These checks independently reproduce the coefficient chain in every sector where closure is claimed. The derivations remain the source of the coefficients.

K.2 Reproducibility ledger for the substrate scale

The substrate-length calculation is short enough to record as a numerical ledger. Enumerating the 1680 oriented injective tetrahedral states with labels $m = -3, \ldots, 3$, weighting them by $e^{-\eta K^2}$, and solving

$$\langle K^2 \rangle_\eta = \frac{3}{2\eta}$$

gives

$$\eta_* = 0.02986684439352237, \qquad g_{\text{share,eff}} = 7.419800023570903.$$

The one-pass seven-sector history support and its transverse export are then

$$e^{7g_{\text{share,eff}}} = 3.602860521062804 \times 10^{22},$$

$$\frac{2}{3} e^{7g_{\text{share,eff}}} = 2.401907014041869 \times 10^{22}.$$

Using $\lambda_e = \hbar/(m_e c) = 3.861592671986303 \times 10^{-13}$ m and $r = e^{-7g_{\text{share,eff}}}$ first gives the baseline

$$L_*^{(0)} = -\frac{3}{2} \lambda_e \ln(1 - r) = 1.607719470158885 \times 10^{-35} \text{ m},$$

while the marked vertex gives

$$\zeta_* = 0.005123584484947, \qquad Z_e = 1.005308283809514.$$

Therefore

$$L_* = Z_e L_*^{(0)} = 1.616253701392569 \times 10^{-35} \text{ m},$$

and

$$G_* = \frac{c^3 L_*^2}{\hbar} = 6.674289077220912 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}.$$

The source-side Green constant is obtained independently from the diamond-lattice integral in Appendix C.5. Uniform-grid quadrature with endpoint extrapolation gives

$$G_{\mathrm{tet}}(0) = 0.448220394388\ldots,$$

confirming the exact Joyce value

$$G_{\mathrm{tet}}(0) = \frac{3\Gamma(1/3)^6}{2^{14/3}\pi^4}.$$

This is the value used in the source theorem. These numbers are not additional inputs; they are the numerical evaluation of the finite spectrum, the seven-sector history support, and the graph Green function already defined in the derivation.

K.3 Conditional color and record-retention audits

The color audit first recomputes the weighted route covariance directly from the same 1680 states used for admissibility. It returns

$$C_R = 11.7667312558\, P_{\mathbf{1}} + 35.3002035759\, P_{\mathbf{2}},$$

and verifies

$$\lambda_{\mathbf{1}} = \frac{3}{4}\,\mathrm{Var}(K^2)$$

to numerical precision. The primitive recoupling matrix has spectrum

$$\mathrm{spec}\, G = \left\{\frac{9}{10}, \frac{21}{20}, \frac{21}{20}\right\}, \qquad \frac{\lambda_{\mathbf{2}}}{\lambda_{\mathbf{1}}} = \frac{7}{6}.$$

After maximum-throughput normalization, the direct and complementary branches satisfy

$$K = P_{\mathbf{2}} + \frac{6}{7}P_{\mathbf{1}}, \qquad L = \frac{\sqrt{13}}{7}P_{\mathbf{1}}, \qquad K^\dagger K + L^\dagger L = I.$$

The phase-invariant loop cost obeys the representation identity

$$\overline{\Delta_p} = \frac{3}{2}\left(1 - \frac{|\,\mathrm{Tr}_{\mathbf{3}}\, U_p|^2}{9}\right) = \frac{4}{3}\left(1 - \frac{1}{8}\,\mathrm{Tr}_{\mathbf{8}}\,\mathrm{Ad}\, U_p\right),$$

and is unchanged by multiplying any lift of U_p by a common phase. This checks that the derived action is well defined on the projective channel group without assuming the still-open fundamental endpoint lift. The Haar identities of Appendix I.2 then give a unital trace-preserving channel with singlet eigenvalue one and eight equal traceless eigenvalues 13/14. The heat-kernel coordinate and equal-transfer diamond blocking evaluate to

$$g^2_{\mathrm{HK,step}} = \frac{2}{3}\ln\frac{14}{13} = 0.04940531477,$$

$$2\sqrt{6} = 4.89897948557, \qquad g^2_{\mathrm{graph,HK}} = \frac{4\sqrt{6}}{3}\ln\frac{14}{13} = 0.24203562353.$$

These computations audit the conditional projective pure-gauge construction. They are not a comparison with a continuum QCD coupling.

The generation cutoff has also been checked without choosing a preferred excluded alphabet. Exhaustive enumeration over all reduced alphabets gives nonzero closure variance for every one-label exclusion (all seven cases, variance range 9.95556–16.17778) and every two-label exclusion (all 21 cases, range 1.24444–26.02667). Every three-label exclusion (all 35 cases) gives zero variance. The cutoff therefore follows from the remaining alphabet size and the permutation symmetry of K^2, not from a specially selected sequence of labels.

The record-retention formula has a separate normalization check. For any orthogonal set of fine records merged into $\bar{R}$, summing $p(h, R_i)$ over both h and i gives $p(\bar{R})$, and the conditional distribution in Section 22 therefore sums to one. Record erasure changes the accessible partition of the same joint measure; it adds no transition weight and cannot alter the unconditioned Born marginals.

Appendix L: Fork accounting for the scale chain

This appendix records the construction choices behind the induced value of G_* and quantifies the surrounding discrete-choice fork space. The comparison shows how much freedom was available in the scale chain.

Provenance. By the author's recollection, a target entropy near 7.42 was identified early in the framework's development by inverting the measured Newton constant, with the ensemble constructed to deliver it jointly with other structural constraints; the documentary record does not reach that genesis layer. The realized ensemble was fixed and motivated in print by information-independence arguments before the induced-G_* chain existed. Its original electron scale and the baseline charged-lepton ladder landed at discrepancies of roughly one percent and one-half percent. Those residuals were public parts of the construction before the decorated marked vertex was developed.

Why the residuals suggested one missing object. The amplitudes can be compared on the same scale. The original Newton value was low by about 1.05%, while the length entering it was low by half that amount because $G_* \propto L_*^2$. The three required multiplicative uplifts were

baseline quantity	required uplift	structural reading
$L_*^{(0)}$ inferred from G	0.5309%	electron feedback
$m_\mu^{(0)}/m_e$	0.5124%	universal marked response
$m_\tau^{(0)}/m_e$	0.6562%	universal response plus second shell

Dividing the tau uplift by the muon uplift leaves a further 0.1431% second-shell contribution. The pattern is not three identical residuals, and it was not used to impose a universal rescaling. It is more specific: two observables require nearly the same charged-transfer amplitude, while the third requires that common amplitude followed by one smaller shell passage. This was the motivation for seeking one marked response with different graph traces.

Why the repair had to be additive. By that stage the tetrahedral construction was already doing several independent jobs. Its label space and admissibility weight fixed the closure spectrum and sharing entropy; its edge projection fed the ordinary weak-field coefficient; its transverse export entered the scale, horizon normalization, and galactic branch; and its shell algebra produced the baseline charged-lepton ladder. Removing or retuning any of those ingredients to close one residual would propagate through the rest of the microscopic and macroscopic chain. A viable repair therefore had to preserve the unmarked tetrahedral ensemble and its ordinary transfer operator.

This requirement narrowed the search to a defect-bound additive sector: it must be invisible in the unmarked vacuum, reuse the closure response already present in the renewed cell, activate only on occupied charged-channel pairs, and route through the existing electron and shell graph. These conditions explain the form of the calculation in Appendix H. They do not by themselves prove its nine-state fiber, 21 pair records, Majorana determinant, or routing polynomials; those are the independent action-level results and audits. Showing this part of the development is appropriate because the residuals were already known. It makes clear which features were motivated by the discrepancy, which were protected by the existing theory, and which had to be derived rather than chosen.

The vertex in Appendix H was consequently found with the discrepancies known. It contains no continuously adjusted coefficient: $g_{\mathrm{share,eff}}$, η_*, 2/7, the nine-state trace, 21 pair records, and the three routing polynomials are fixed by the displayed action. That fact makes the result an action-level postdiction rather than a fit, but it does not make it a blind prediction. Its added evidential content is that one field space and one routing graph account for G, m_μ/m_e, and m_τ/m_e simultaneously, while the nearby shared-mode and off-diagonal alternatives fail. The cluster capacity bound, committed abundance, redshift direction of a_0, and sub-g_c rotation-curve regime remain post-fixation tests of the original ensemble; none entered either the early entropy calibration or the marked residual calculation.

The audit. The exact substrate length is proportional to $-Z_e \ln(1 - e^{-7g_{\mathrm{share,eff}}})$, with leading behavior $L_* \propto Z_e e^{-7g_{\mathrm{share,eff}}}$, so the calibration is meaningful only relative to the space of constructions that could have been written instead. The original grammar contains six label counts, five multiplicity rules, five closure coefficients, four support exponents, and five export prefactors, for 3000 candidate constructions. Those counts predate the marked vertex and must be recomputed with its $Z_e(g, \eta)$ dependence before they are used quantitatively.

The marked-sector audit separately enumerates 576 nearby formulas obtained by varying the response dimension, pair count, direction factor, determinant power, statistics, and overlap. Only the decorated-vertex formula lies within one muon standard deviation, and that exclusivity must be read carefully. The one-sigma muon window on ζ_* is about four parts per million relative, far finer than the spacing of any discrete formula menu, so at most one candidate could ever land inside it, and the failure of the other 575 carries little information on its own. What the menu records is qualitative brittleness: a shared edge register, directed complex determinant, stable bosonic determinant, or six-state symmetric response gives a visibly different result, and no probability measure is assigned to the menu. The quantitative audit is the one-integer scan of Section 13.5, which holds ζ_* fixed and asks the measured Newton constant to select the routing integer: $n = 7$ is the unique survivor over $|n| \le 200$, its neighbors fail at 2.3–2.4σ, and the tau's second-shell rational is supported but not uniquely resolved.

The galactic scale depends only linearly on $g_{\mathrm{share,eff}}$, so it tests the coupling form more strongly than the exact ensemble choice. The corrected G_* and charged-lepton ratios test the shared marked vertex through three different graph factors, with the postdiction status stated above.

Appendix M: Why the saturated CMB carrier must be committed capacity, not a lagged response

The microwave background requires a gravitating component that carries the growing mode of the baryon perturbations while rejecting their acoustic oscillation. This appendix records the quantitative exclusion of every relaxational realization of that component and the no-go that selects the constraint reading of Section 20.

Growth-limited kernels. For development dynamics $dW/dt = (W_{\max} - W)/\tau$ with $\tau = \beta\, t_{\mathrm{ff}}(\rho_{\mathrm{local}})$, the measured cluster radial decline of the source weight fixes $\beta \simeq 15.6$, while saturation of the cosmological bath by recombination requires $\beta \lesssim 0.56$: the window is empty by a factor of order thirty. The radial ratio between R_{500} and R_{200}, a β-independent prediction of the free-fall clock, agrees with the data at the several-percent level, so the exclusion is of the absolute clock, not of the radial structure.

Lagged response. A first-order lag transmits a fraction $T = [1+(\omega\tau)^2]^{-1/2}$ of an oscillation at frequency ω. The transmitted component acts as additional effective baryon loading $(1/\epsilon - 1)\,T$ in the acoustic driving; Einstein–Boltzmann computation shows a 0.3% temperature-spectrum tolerance bounds $T \leq 2\times 10^{-3}$, requiring $\beta \gtrsim 37$ against the development requirement $\beta \lesssim 0.56$: empty by a factor of order sixty-six. More generally, by recombination a third-peak mode has completed only a few oscillations, so no causal filter of any order achieves the required rejection.

Sound-speed classification. For $L = f(X)$, perturbations carry $c_s^2 = f_X/(f_X + 2Xf_{XX})$. The released branches give $c_s^2 = \frac{1}{2}$ (deep) and 1 (Newtonian): free-streaming. Plateau approaches $f = f_{\max} - A/X^n$ give $c_s^2 = -1/(2n+1)$: gradient-unstable. Both natural cap readings — stationarity of the canonical momentum and of the energy density — impose $f_X + 2Xf_{XX} = 0$, the pole of c_s^2: a rigid medium carrying no perturbation. The only locus with $c_s^2 = 0$ and healthy density response is the constraint surface of the mimetic class, X pinned with a multiplier, which is the reading of Section 20.

No-go for relaxation-plus-cap carriers. Within dynamics consisting of relaxation toward a demand together with a hard cap, with the cell weight as the only state variable, no coupling of the weight to the demand carries a secular component of the modulation while rejecting its oscillation: a pinned weight retains no memory of the modulation, and any instantaneous coupling of bounded periodic inputs is itself bounded and periodic. A secular component requires an additional conserved integrating variable. The conserved spatial density of committed cells is that variable, and it is native to the saturated phase rather than added to it.

Appendix N: Action Reconstruction, Parent-Theory Audit, and Transverse-Sector Target

This appendix asks whether the ordinary capacity functional is a second scalar action or the gravitational constraint action written in a different variable. The static longitudinal calculation answers that question; the transverse thermal calculation remains incomplete.

N.1 One parent metric, one matter coupling

The minimal covariant parent of the ordinary branch is

$$I_0[g,\psi] = I_{\mathrm{EH}}[g] + I_{\mathrm{GHY}}[g] + I_{\mathrm{matter}}[g,\psi],$$

$$I_{\mathrm{EH}} = \frac{c^3}{16\pi G}\int_{\mathcal{M}} d^4x\,\sqrt{-g}\,(R - 2\Lambda).$$

Here and throughout the covariant formulas, $x^0 = ct$. Thus $d^4x = dx^0 d^3x$ and the Einstein–Hilbert coefficient is $c^3/(16\pi G)$. After the split $dx^0 = c\,dt$, the ADM action written with t in seconds carries $c^4/(16\pi G)$. This convention makes the final Newton action an ordinary $\int dt\,L$ action. All matter, including radiation, couples to the one physical metric. Its stress tensor is

$$T_{\mu\nu} = -\frac{2c}{\sqrt{-g}}\frac{\delta I_{\mathrm{matter}}}{\delta g^{\mu\nu}},$$

and the equations are

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}.$$

The Bianchi identity and the matter equations give $\nabla_\mu T^{\mu\nu} = 0$. No explicit coupling $S_{\rm ent} T^\mu{}_\mu$ is present. Such a term would vanish as a source for classical radiation, would have a nontrivial metric variation through $\delta T^\mu{}_\mu/\delta g^{\alpha\beta}$, and would duplicate the matter response already carried by the metric equation.

The capacity variable enters after gauge choice and constraint reduction:

$$\delta S = \delta S[g,\psi]\big|_{\rm reduced}.$$

This is analogous to using the Newtonian potential as a reduced gravitational variable. A reduced variable can have a useful functional without being a new covariant matter field.

N.2 Direct reduction of the Einstein action

ADM form and boundary terms. With the GHY term included, the bulk gravitational action is

$$I_{\rm ADM} = \frac{c^4}{16\pi G}\int dt\, d^3x\, N\sqrt{h}\left({}^{(3)}R + K_{ij}K^{ij} - K^2 - 2\Lambda\right) + I_\infty.$$

This line uses $t = x^0/c$ and is therefore consistent with the covariant c^3 normalization above. For a static scalar perturbation in Newtonian gauge,

$$N = \sqrt{1+\frac{2\Phi}{c^2}} = 1 + \frac{\Phi}{c^2} + O(c^{-4}), \qquad N^i = 0, \qquad h_{ij} = \left(1 - \frac{2\Psi}{c^2}\right)\delta_{ij}, \qquad K_{ij} = 0.$$

The asymptotic term I_∞ subtracts the flat reference contribution and cancels the total divergence generated when the spatial curvature is integrated by parts.

Define $\phi = \Phi/c^2$, $\psi_N = \Psi/c^2$, and write $h_{ij} = e^{2\zeta}\delta_{ij}$, with

$$\zeta = \frac{1}{2}\ln(1-2\psi_N) = -\psi_N - \psi_N^2 + O(\psi_N^3).$$

In three dimensions,

$${}^{(3)}R = e^{-2\zeta}\left[-4\nabla^2\zeta - 2(\nabla\zeta)^2\right].$$

Before integration by parts, the expansion through second order is

$$N\sqrt{h}\,{}^{(3)}R = 4\nabla^2\psi_N + 4\phi\nabla^2\psi_N + 4\psi_N\nabla^2\psi_N + 6(\nabla\psi_N)^2 + O(3).$$

The first term is a boundary term. Integrating the next two Laplacians by parts and combining all surface pieces with I_∞ gives

$$N\sqrt{h}\,{}^{(3)}R = \frac{2}{c^4}\left[(\nabla\Psi)^2 - 2\nabla\Phi\cdot\nabla\Psi\right] + \nabla_i B^i + O(c^{-6}).$$

The boundary integral of B^i is removed by the reference-subtracted variational prescription. For nonrelativistic matter with ρ the rest-mass density, expansion of $-mc^2\int d\tau$ gives

$$I^{(1)}_{\rm matter} = -\int dt\, d^3x\, \rho\Phi.$$

Consequently,

$$\boxed{I^{(2)}_{0,\rm scal}[\Phi,\Psi] = \int dt\, d^3x\left\{\frac{(\nabla\Psi)^2 - 2\nabla\Phi\cdot\nabla\Psi}{8\pi G} - \rho\Phi\right\}.}$$

Constraint equations. Variation with respect to the lapse potential gives

$$\delta_\Phi I^{(2)}_{0,\mathrm{scal}} = 0 \quad \Longrightarrow \quad \nabla^2\Psi = 4\pi G\rho.$$

Variation with respect to the scalar part of the spatial metric gives

$$\delta_\Psi I^{(2)}_{0,\mathrm{scal}} = 0 \quad \Longrightarrow \quad \nabla^2(\Phi - \Psi) = 0.$$

The elimination order is essential: Φ and Ψ are varied independently, both constraint equations are obtained, and only then is their solution substituted back. Imposing $\Phi = \Psi$ in the two-potential action before variation would erase one constraint and is not an equivalent reduction. For asymptotically flat isolated sources, the harmonic difference vanishes:

$$\Phi = \Psi.$$

Substituting this solution back into the two-potential action yields

$$\boxed{I_{\mathrm{Newton}}[\Phi] = \int dt\, d^3x \left[-\frac{(\nabla\Phi)^2}{8\pi G} - \rho\Phi\right].}$$

The sign of the reduced kinetic term is not a ghost signal. Φ is a nondynamical constraint potential, not a propagating scalar oscillator. In the full covariant parent, the lapse and shift impose constraints and the propagating gravitational content is the two tensor polarizations.

N.3 Capacity field redefinition and coefficient identity

On the renormalized static branch, the capacity functional is

$$I^{\mathrm{static}}_{\mathrm{cap}} = \int dt\, d^3x \left[-\frac{\gamma}{2}(\nabla\delta S)^2 + \kappa\rho\,\delta S\right].$$

The most general linear local field redefinition compatible with a constant vacuum background is $\delta S = A\Phi$. Matching the signs of the source and the weak-field clock shift fixes $A < 0$. The bounded relation $q = 1 - \delta S/S_\infty = N^2$ at first order fixes

$$A = -\frac{2S_\infty}{c^2}.$$

Substitution gives

$$I^{\mathrm{static}}_{\mathrm{cap}} = \int dt\, d^3x \left[-\frac{2\gamma S_\infty^2}{c^4}(\nabla\Phi)^2 - \frac{2\kappa S_\infty}{c^2}\rho\Phi\right].$$

The equality

$$G = \frac{c^2\kappa}{8\pi\gamma S_\infty}$$

then makes the kinetic coefficient and source coefficient agree simultaneously:

$$\boxed{I^{\mathrm{static}}_{\mathrm{cap}} = \frac{2\kappa S_\infty}{c^2} I_{\mathrm{Newton}}.}$$

The off-shell identity proves that the two expressions are the same classical functional, up to a field-independent normalization, throughout the reduced static field space. Agreement of their Poisson equations alone would not establish this equivalence. The statement uses matched Dirichlet or asymptotically flat boundary conditions. With a finite boundary, equality also requires mapping the Newton boundary functional through the same linear field redefinition.

Normalization freedom. The observable static equation depends only on

$$\frac{\kappa}{\gamma S_\infty}.$$

A change of entropy units

$$\delta S \to a\,\delta S, \qquad S_\infty \to a S_\infty, \qquad \gamma \to \frac{\gamma}{a^2}, \qquad \kappa \to \frac{\kappa}{a}$$

leaves both the action and the observable bridge invariant. Multiplying the entire reduced action by a further constant also leaves its classical equation invariant. The UV calculation can fix these conventions for fluctuation normalization and correlation functions, but the classical Newton constant is fixed only once through the invariant combination. A convenient unit-normalized representative has $Z_S = 1$, equivalently

$$\kappa = \frac{c^2}{2S_\infty}, \qquad \gamma = \frac{c^4}{16\pi G S_\infty^2},$$

but the manuscript's canonical UV convention need not be changed to use the equivalence.

Why canonical scalar stress cannot do this job. If $\delta S = O(\rho)$ about a constant background, then

$$T_{\mu\nu}^{(\delta S)} \sim \partial_\mu \delta S\, \partial_\nu \delta S = O(\rho^2).$$

The bridged potential is $O(\rho)$. It therefore cannot be attributed to the canonical stress of a second scalar. In the reconstruction above, the linear response comes from the metric constraint and the apparent scalar functional is simply its reduced representation.

N.4 Parent-action options and degree-of-freedom audit

Route	**Variables / action**	**Modes**	**Audit result**
Metric-only / composite capacity	$I_0[g,\psi]$; δS defined after weak-field constraint reduction and $q_{\rm geo} = (\nabla R)^2$ in spherical symmetry	two tensors	Passes the weak-field action identity, couples radiation correctly, gives baseline no slip and GR PPN, and passes the spherical exterior test. This is the preferred ordinary branch. A generic local covariant definition of capacity outside the controlled reductions remains open.
Algebraic ADM multiplier	$I_0 + \int \sqrt{h}\,\lambda(N^2 - q)$	no new mode only because the construction is empty	δq gives $\lambda = 0$ when q has no other bulk term. The relation only renames a foliation-dependent lapse and does not yield the capacity source equation. Excluded as a parent completion.
Clock-constrained auxiliary capacity	$I_0 + \int \sqrt{-g}\,\Lambda(qX + 1)$, $X = g^{\mu\nu}\partial_\mu\tau\partial_\nu\tau$	trivial if q is purely algebraic; extra scalar or dust-like mode once dynamics is supplied	The q equation again drives Λ to zero unless additional q dependence is introduced. Kinetic or higher-derivative terms generically introduce a preferred foliation, mimetic-dust behavior, or a scalar stability problem. Retain only as a control model with a full Hamiltonian analysis.

Route	Variables / action	Modes	Audit result
Scalar–tensor / disformal	$I[g,q]+I_{\rm matter}[\widetilde{g},\psi]$, $\widetilde{g}_{\mu\nu}=C(q)g_{\mu\nu}+D(q)u_\mu u_\nu$	two tensors plus at least one scalar	Universal matter coupling can be arranged, but fifth forces, gravitational slip, PPN shifts, and radiative stability become new obligations. It is not needed for the ordinary branch and is not selected by the present evidence.

The metric-only route is minimal in a precise sense: it adds no propagating field, uses the full stress tensor, and reproduces both controlled reductions. This does not prove that every substrate sector is metric-only. It proves that the ordinary longitudinal capacity mode must not be counted again as an independent force.

N.5 Exact spherical reduction and the Misner–Sharp first integral

Let

$$ds^2 = h_{ab}(x)dx^a dx^b + R^2(x)d\Omega^2.$$

We use $x^0 = ct$, so $d^2x = dx^0 dr$. If the orbit-space integral is instead written as $dt\,dr$, its prefactor is $c^4/(4G)$. The curvature identity is

$${}^{(4)}R = {}^{(2)}R + \frac{2}{R^2}\left[1-(\nabla R)^2 - 2R\Box R\right].$$

After the angular integral,

$$I_{\rm EH} = \frac{c^3}{4G}\int d^2x\,\sqrt{-h}\,\Big[R^{2\,(2)}R + 2\{1-(\nabla R)^2\} - 4R\Box R\Big].$$

Integrating the last term by parts together with the reduced GHY term gives

$$\boxed{I_{\rm sph} = \frac{c^3}{4G}\int d^2x\,\sqrt{-h}\,\Big[R^{2\,(2)}R + 2(\nabla R)^2 + 2\Big] + I^{(2)}_{\rm matter} + I^{(2)}_{\partial}.}$$

The reduced boundary term is not left implicit. For a product boundary $\partial\mathcal{M}_4 = \partial\mathcal{M}_2 \times S^2$,

$$K^{(4)} = K^{(1)} + \frac{2}{R}n^a\nabla_a R.$$

The second term cancels the boundary contribution generated when $-4R\Box R$ is integrated by parts. With the standard orientation sign ε, the surviving term is

$$I^{(2)}_{\partial} = \varepsilon\,\frac{c^3}{2G}\int_{\partial\mathcal{M}_2} dy\,\sqrt{|\gamma_{(1)}|}\,R^2K^{(1)} + I_{\rm joint} + I_{\rm ref}.$$

The vacuum Euler–Lagrange equations are

$$R\,{}^{(2)}R - 2\Box R = 0,$$

$$2R\,(h_{ab}\Box R - \nabla_a\nabla_b R) + h_{ab}\left[(\nabla R)^2 - 1\right] = 0.$$

Define

$$q_{\rm geo} \equiv (\nabla R)^2, \qquad M_{\rm MS} \equiv \frac{c^2 R}{2G}(1-q_{\rm geo}).$$

Contracting and differentiating the metric equation, with the R equation used to remove the curvature term, gives

$$\nabla_a M_{\rm MS} = 0$$

in vacuum. Hence

$$q_{\text{geo}} = 1 - \frac{2GM}{c^2 R}.$$

In the static areal gauge $R = r$, asymptotic flatness fixes the remaining normalization and returns

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2,$$

$$q_{\text{geo}} = N^2.$$

Thus q is a composite geometric scalar in spherical symmetry, and its vacuum profile is a first integral. No auxiliary scalar is required. The result is nonperturbative but symmetry restricted.

N.6 Covariant capacity observable beyond the controlled reductions

The action reconstruction fixes two limits but does not yet provide one local scalar valid in every geometry. In a stationary spacetime with a timelike Killing field ξ^μ normalized so that $\xi^\mu \xi_\mu \to -c^2$ at the asymptotic reference boundary, the lapse has the invariant representative

$$q_K = -\frac{\xi^\mu \xi_\mu}{c^2} = 1 + \frac{2\Phi}{c^2} + O(c^{-4}).$$

In spherical symmetry the invariant representative is instead

$$q_{\text{MS}} = (\nabla R)^2 = 1 - \frac{2GM_{\text{MS}}}{c^2 R}.$$

They coincide in the static Schwarzschild exterior. Outside stationary or spherical sectors neither a preferred Killing field nor an areal-radius dilaton exists, so neither formula is a general definition.

This is not evidence that a new fundamental field is required. A Newtonian potential is not a local curvature scalar: it depends on relational boundary data and gauge choice, whereas local curvature invariants measure tidal derivatives. The natural general object is therefore allowed to be quasilocal and state dependent,

$$q_{\text{cap}}(x) = \mathcal{Q}[g_{\mu\nu}, \rho_{\text{q}}; \mathcal{D}_x, \mathcal{A}(\mathcal{D}_x)]\,,$$

where $\mathcal{D}_x$ is a relationally specified causal diamond or world tube, $\mathcal{A}(\mathcal{D}_x)$ is its observable algebra, and ρ_{q} is the quantum state relative to the chosen vacuum. This notation is a target class, not a proposed closed formula.

An informational realization could use a vacuum-relative entropy or modular-energy functional on $\mathcal{A}(\mathcal{D}_x)$ followed by a bounded constitutive map. Because regional relative entropy is unbounded while $q_{\text{cap}} \in [0,1]$, a global linear identification is impossible. Schematically one would need

$$q_{\text{cap}} = \mathfrak{f}\left(\frac{S_{\text{rel}}}{S_\infty}, \text{geometric data}\right), \qquad \mathfrak{f}(0,\ldots) = 1, \qquad 0 \le \mathfrak{f} \le 1,$$

with its first variation reproducing the weak-field bridge and its spherical restriction reproducing q_{MS}.

Bianconi's geometric-relative-entropy action is a published example showing that the local, volumetric part of this target is mathematically viable: a positive metric-response operator enters through a trace logarithm, its low-curvature expansion contains the Einstein–Hilbert action, and its Legendre transform gives the convex mismatch energy used in Section 25 [24]. It is not a solution for $\mathcal{Q}$. Her response operator is independently dynamical and contributes second-derivative terms to the field equations, whereas the present target must introduce no new Cauchy data in the ordinary branch. The comparison fixes a useful functional neighborhood near equilibrium; the bounded, composite, metric-only completion remains to be derived here.

A successful definition must satisfy all of the following:

1. it is diffeomorphism invariant once its region, state, and reference data are specified relationally;
2. it introduces no independent Cauchy data and hence no extra propagating gravitational degree of freedom;
3. it reduces to q_K in the stationary weak-field sector and to $q_{\rm MS}$ in spherical symmetry;
4. its response is causal or explicitly quasilocal, with no dependence on an arbitrary coordinate foliation;
5. it is compatible with the metric Ward identity and the full stress-tensor source;
6. it supplies a controlled bounded continuation near saturation rather than identifying bounded capacity globally with unbounded relative entropy.

Determining $\mathcal{Q}$ is the general constitutive problem left by the action reconstruction.

N.7 The $q_{\rm geo} = 0$ variational boundary

The exterior action should be defined first on a stretched timelike surface

$$\mathcal{B}_\epsilon = \{q_{\rm geo} = \epsilon\}, \qquad \epsilon > 0.$$

For Dirichlet metric data, include

$$I_{\rm GHY}[\mathcal{B}_\epsilon] = \varepsilon \frac{c^3}{8\pi G} \int_{\mathcal{B}_\epsilon} d^3y \sqrt{|\gamma|}\, K,$$

the reference term at infinity, and the appropriate joints. Only after the variation is defined should the null limit $\epsilon \to 0^+$ be taken, using the null-boundary form and a fixed normalization of the generators. This supplies a well-posed exterior Einstein variational principle.

Three logically different claims must not be conflated:

1. The reduced metric equations produce a marginal sphere where $q_{\rm geo} = 0$. This is derived.
2. The substrate capacity fraction equals $q_{\rm geo}$ up to that surface. This is a constitutive identification, strongly matched in static spherical vacuum.
3. The physical spacetime ends there and does not realize $q_{\rm geo} < 0$. This is the bounded-domain postulate and requires new boundary dynamics.

The standard gravitational boundary term fixes neither a channel Hamiltonian nor a reflectivity. The missing microscopic functional may be denoted

$$\Gamma_{\partial q}[\sigma_{AB}, \zeta_\partial; \rho_\partial],$$

where ζ_∂ are boundary channel variables and ρ_∂ is their state. It must determine:

- whether the GR ingoing condition $\mathcal{R}(\omega) = 0$ is recovered;
- whether a stretched layer $q_{\rm geo} = \epsilon$ is dynamically selected;
- the relaxation spectrum and any echo kernel;
- the shape variation and boundary stress balance;

- the microscopic channel-to-area count behind the 1/4 normalization identity.

Until this functional is derived, the Schwarzschild exterior is a result, while interior excision and boundary phenomenology are hypotheses.

N.8 Transverse effective Hessian and influence-functional target

The transverse extension has a controlled symmetry reduction and a conditional effective potential. Neither result supplies the metric response by itself. This subsection derives both pieces and then states the closed-time-path calculation still required.

Tetrahedral kinetic normalization. Let $q_i(x)$, $i = 1, \ldots, 4$, denote fluctuations on the four equivalent tetrahedral face directions $\mathbf{n}_i$, normalized by

$$\mathbf{n}_i \cdot \mathbf{n}_j = \begin{cases} 1, & i = j, \\ -1/3, & i \neq j, \end{cases} \qquad \sum_i \mathbf{n}_i = 0.$$

For the normalization argument use the Euclidean quadratic term

$$\Gamma_{E,\mathrm{kin}} = \frac{Z_0}{2} \int d^4x_E \sum_i \partial_\mu q_i \partial_\mu q_i.$$

The q_i are dimensionless occupation fluctuations, and Z_0 has units of action per length squared. Lorentzian response is obtained by analytic continuation to the signature of Appendix A. The four-dimensional permutation representation splits into a uniform singlet and a three-dimensional sum-zero sector. On the latter define

$$A^a = \frac{\sqrt{3}}{2} \sum_i n_i^{\,a} q_i, \qquad \sum_i q_i = 0.$$

Using $\mathbf{n}_i \cdot \mathbf{n}_j = 4\delta_{ij}/3 - 1/3$ gives the exact norm identity

$$\mathbf{A}^2 = \sum_i q_i^2, \qquad \Gamma_{E,\mathrm{kin}} = \frac{Z_0}{2} \int d^4x_E \, \partial_\mu \mathbf{A} \cdot \partial_\mu \mathbf{A}.$$

The same result follows from Schur's lemma: the tetrahedral three-dimensional representation is irreducible, so a symmetry-invariant quadratic operator is proportional to the identity on it. A slowly varying baryonic field selects $\mathbf{e}_L = \nabla\Phi_{\mathrm{bar}}/|\nabla\Phi_{\mathrm{bar}}|$ and permits

$$\mathbf{A} = X\mathbf{e}_L + \mathbf{Y}, \qquad \mathbf{e}_L \cdot \mathbf{Y} = 0.$$

For a locally fixed $\mathbf{e}_L$, the longitudinal mode and both transverse components inherit the same coefficient,

$$Z_L = Z_T = Z_0.$$

Writing $Y_1 = R\cos\vartheta$ and $Y_2 = R\sin\vartheta$ gives

$$(\partial Y_1)^2 + (\partial Y_2)^2 = (\partial R)^2 + R^2(\partial\vartheta)^2,$$

so the radial occupation mode that mixes with X also carries Z_0. For each stable Fourier mode before the radial reduction, (Y_i, Π_i) are two canonical oscillator pairs. Their action–angle variables obey $\theta_i \sim \theta_i + 2\pi$ and $d\Pi_1\, dY_1\, d\Pi_2\, dY_2 = dJ_1\, d\theta_1\, dJ_2\, d\theta_2$. Quantization spaces the actions by $\hbar$, giving one state per phase-space volume $(2\pi\hbar)^2$ and the angular Haar volume $(2\pi)^2$ for one action cell. This proves the phase measure for the specified doublet. It does not decide that the physical state loads one sharing entropy into that cell or that the cell couples reversibly to the horizon bath. Spatial variation of $\mathbf{e}_L$ produces derivative connection terms suppressed by the curvature scale of the source direction. Higher-derivative and background-dependent operators can split the coefficients; the equality is a leading long-wavelength result.

One-capacity-invariant potential. After the relative sector of Appendix H is gapped, define the longitudinal and transverse occupations $N_L = \sigma^\dagger P_L \sigma$ and $N_T = \sigma^\dagger P_T \sigma$. The minimal infrared premise is that matter sees one total-capacity invariant,

$$\Delta C = w_L N_L + w_T N_T - N_*.$$

This premise is stronger than tetrahedral symmetry. The general local quartic potential contains

$$V_{\text{gen}} = \mu_L N_L + \mu_T N_T + \frac{\lambda_L}{2} N_L^2 + \lambda_{LT} N_L N_T + \frac{\lambda_T}{2} N_T^2 + \cdots .$$

Its quartic coupling matrix has rank at most one when

$$\lambda_{LT}^2 = \lambda_L \lambda_T.$$

It has rank one only if the matrix is nonzero. A stable one-invariant potential further requires the matrix to be positive semidefinite, so $\lambda_L, \lambda_T \geq 0$ and not both vanish. Under those conditions $\lambda_L = u_C w_L^2$, $\lambda_T = u_C w_T^2$, and $\lambda_{LT} = u_C w_L w_T$ with $u_C > 0$ and suitable signed weights. The linear terms must align with the same weights. A thermodynamic capacity-pressure variable makes the stable sign explicit through

$$V_{\text{eff}}(\Delta C) = \sup_{\lambda_C} \left[\lambda_C \Delta C - \frac{\chi_C}{2} \lambda_C^2 \right].$$

The stationary pressure $\lambda_C = \Delta C / \chi_C$ gives $V_{\text{eff}} = (\Delta C)^2/(2\chi_C)$. A path-integral Hubbard–Stratonovich representation requires the contour and Lorentzian/Euclidean sign to be chosen consistently; the equation above is the thermodynamic Legendre form. More generally $V_{\text{eff}} = F(\Delta C)$ with $F'(0) = 0$ and $F''(0) > 0$.

Near a transverse background, $N_L = N_{L0} + \zeta_L X + \cdots$ and $N_T = R^2 = Y_1^2 + Y_2^2$, so

$$\Delta C = \alpha_C + g_C X + \beta_C R^2.$$

For fields $\varphi_i = (X, R)$ on the stationary surface $\Delta C = 0$,

$$H_{ij} = \left. \frac{\partial^2 V_{\text{eff}}}{\partial \varphi_i \partial \varphi_j} \right|_{\Delta C = 0} = F''(0)\, \partial_i \Delta C\, \partial_j \Delta C.$$

This outer product has rank one wherever $\nabla \Delta C \neq 0$; it has rank zero at a point where that gradient vanishes. Explicitly,

$$H = F''(0) \begin{pmatrix} g_C^2 & 2 g_C \beta_C R_0 \\ 2 g_C \beta_C R_0 & 4 \beta_C^2 R_0^2 \end{pmatrix}, \qquad C_\times^2 = A_L A_T.$$

At fixed δX, minimizing the quadratic form gives $\delta R_* = -C_\times \delta X / A_T$. Equal kinetic normalization also defines the clamped frequencies $\omega_L^2 = A_L / Z_0$ and $\omega_T^2 = A_T / Z_0$, so

$$\frac{C_\times}{A_T} = \sqrt{\frac{A_L}{A_T}} = \frac{\omega_L}{\omega_T}.$$

The equality concerns the static response and the clamped curvatures. The freely relaxing Hessian has one zero eigenvalue and one eigenvalue $A_L + A_T$; neither is ω_L or ω_T separately. The soft direction redistributes a fixed total capacity; the orthogonal direction changes the total occupancy. Positive gradient terms stabilize finite-wavelength fluctuations.

This result is protected at the effective level when all local corrections remain functions of the same invariant ΔC. To state the decoupling condition explicitly, define $D = w_T N_L - w_L N_T$ and

$$V(C, D) = F(C) + \frac{\Delta_D}{2} D^2 + \eta_D C D^2 + J_D D + \cdots , \qquad C \equiv \Delta C.$$

The stationary solution is $D = -J_D/(\Delta_D + 2\eta_D C) + \cdots$. For large Δ_D, integrating out D changes $F(C)$ only through constants and powers of Δ_D^{-1}. This is an effective decoupling statement, not proof that the bare vertex is rank one. A standalone $m_X^2 X^2/2$ outside $F(\Delta C)$ would raise the Hessian rank and generate a Yukawa deformation; the X^2 curvature tied to the entries of the rank-one outer product does not.

The susceptibility is not fixed by the normalized 1680-state internal distribution. It depends on fluctuations of total condensate occupation,

$$\chi_C = \langle \widehat{C}^2 \rangle - \langle \widehat{C} \rangle^2, \qquad \widehat{C} = w_L \widehat{N}_L + w_T \widehat{N}_T,$$

in a specified GFT state. In an energy-normalized convention the corresponding susceptibility includes $\beta_H \equiv 1/(k_B T_H)$. For a single mode $\widehat{C} = c_0 \widehat{N}$, a thermal state gives $\chi_C = c_0^2 \bar{N}(1 + \bar{N})$, while a coherent condensate gives $\chi_C = c_0^2 \bar{N}$. The closure distribution fixes neither state nor $\bar{N}$. The susceptibility controls the massive direction and nonlinear corrections but cancels from the rank-one ratio.

Thermal matching and its limit. An acceleration a carries the Unruh energy $E(a) = \hbar a/(2\pi c)$, while $k_B T_H = \hbar H_0/(2\pi)$, so

$$\frac{E(a)}{k_B T_H} = \frac{a}{c H_0}.$$

The proposed effective identification is

$$A_L = \frac{g_{\mathrm{bar}}}{c H_0}, \qquad A_T = \frac{a_0}{c H_0}$$

and therefore $C_\times / A_T = \sqrt{g_{\mathrm{bar}}/a_0}$ after choosing the positive relative orientation. This closes the algebra inside the leading effective model. If the source-driven energy is $E_\perp = \hbar\omega_L$, the desired thermal argument obeys

$$\frac{E_\perp}{k_B T_H} = \frac{C_\times}{A_T} \quad \Longleftrightarrow \quad \hbar\omega_T = k_B T_H.$$

Thus the curvature-to-energy step and the transverse thermal normalization are one anchoring condition within the clamped-oscillator reading. A microscopic calculation must still show that the retarded response uses the clamped curvatures rather than the adiabatic eigenmodes. The environmental constancy of $A_T = 4F''(0)\beta_C^2 R_0^2$, the one-entropy loading of the canonical phase cell, and its reversible horizon coupling remain matching conditions.

The metric response is an open-system problem. The natural object is the closed-time-path generating functional obtained by integrating out transverse substrate variables ξ^A in their physical state $\rho_\perp$:

$$e^{i\Gamma_\perp[g_+, g_-]/\hbar} = \mathrm{Tr}_\perp \left(U_\perp[g_+]\, \rho_\perp\, U_\perp^\dagger[g_-] \right).$$

The total effective functional is

$$\Gamma_{\mathrm{eff}} = I_0[g_+, \psi_+] - I_0[g_-, \psi_-] + \Gamma_\perp[g_+, g_-; \psi_+, \psi_-].$$

The physical metric equation is obtained by varying the difference field and then setting the two histories equal. Unitarity requires

$$\Gamma_\perp[g, g] = 0, \qquad \Gamma_\perp[g_+, g_-]^* = -\Gamma_\perp[g_-, g_+].$$

Diffeomorphism invariance of the influence functional must yield the Ward identity that makes the induced response covariantly conserved.

Quadratic data and response convention. Here g_+ and g_- label the two CTP branches. Let $h^{(1,2)}_{\mu\nu}$ be their metric perturbations and define the average and difference fields $h_c = (h^{(1)} + h^{(2)})/2$ and $h_\Delta = h^{(1)} - h^{(2)}$. With $x^0 = ct$, the linear metric coupling is

$$\delta I_\perp = \frac{1}{2c}\int d^4x\,\sqrt{-\bar{g}}\,h_{\mu\nu}\widehat{T}^{\mu\nu}_\perp.$$

Define the densitized source

$$J_{\mu\nu} \equiv \frac{\sqrt{-\bar{g}}}{2c}h_{\mu\nu}.$$

Let $J^c = (J^{(1)} + J^{(2)})/2$ and $J^\Delta = J^{(1)} - J^{(2)}$. Then the universal quadratic form in this source convention is

$$\Gamma^{(2)}_\perp = \int d^4x\,d^4y\,J^\Delta_{\mu\nu}(x)\widehat{\Pi}^{\mu\nu\alpha\beta}_R(x,y)J^c_{\alpha\beta}(y) + \frac{i}{2}\int d^4x\,d^4y\,J^\Delta_{\mu\nu}(x)\widehat{\mathcal{N}}^{\mu\nu\alpha\beta}(x,y)J^\Delta_{\alpha\beta}(y).$$

The retarded kernel is

$$\boxed{\widehat{\Pi}^{\mu\nu\alpha\beta}_R(x,y) = +\frac{i}{\hbar}\theta(x^0 - y^0)\left\langle\left[\widehat{T}^{\mu\nu}_\perp(x),\widehat{T}^{\alpha\beta}_\perp(y)\right]\right\rangle_{\rho_\perp} + \widehat{\Pi}^{\mu\nu\alpha\beta}_{\text{local}}(x,y).}$$

The sign follows from the stated $+\frac{1}{2}h_{\mu\nu}T^{\mu\nu}$ action coupling and the Kubo susceptibility convention. The local or *seagull* term comes from the explicit metric variation of $\widehat{T}^{\mu\nu}_\perp$ and of the covariant measure. It is required, together with the commutator, for the diffeomorphism Ward identity. Define $\Delta\widehat{T}^{\mu\nu}_\perp = \widehat{T}^{\mu\nu}_\perp - \langle\widehat{T}^{\mu\nu}_\perp\rangle_{\rho_\perp}$. The noise kernel is the correspondingly normalized symmetrized correlator,

$$\widehat{\mathcal{N}}^{\mu\nu\alpha\beta}(x,y) = \frac{1}{2\hbar}\left\langle\left\{\Delta\widehat{T}^{\mu\nu}_\perp(x),\Delta\widehat{T}^{\alpha\beta}_\perp(y)\right\}\right\rangle_{\rho_\perp}.$$

A microscopic calculation begins with the transverse Hessian and metric vertex,

$$\mathcal{K}_{AB}(x,y) = \frac{\delta^2 I_\perp}{\delta\xi^A(x)\delta\xi^B(y)}, \qquad V^{\mu\nu}_A(x;y) = \frac{\delta^2 I}{\delta\xi^A(x)\delta g_{\mu\nu}(y)}.$$

In the physical state,

$$G^{AB}_R(x,y) = +\frac{i}{\hbar}\theta(x^0 - y^0)\,\mathrm{Tr}\big(\rho_\perp[\xi^A(x),\xi^B(y)]\big),$$

and schematically

$$\widehat{\Pi}^{\mu\nu\alpha\beta}_R = V^{\mu\nu}_A G^{AB}_R V^{\alpha\beta}_B + \widehat{\Pi}^{\mu\nu\alpha\beta}_{\text{contact}}.$$

This is the minimal correlator/Hessian calculation required before an action claim can be made.

Static and lensing targets. In the static scalar sector the kernel produces a two-component response,

$$\begin{pmatrix}\Delta\Phi_\perp(\mathbf{k})\\ \Delta\Psi_\perp(\mathbf{k})\end{pmatrix} = \mathbf{K}_\perp(0,\mathbf{k};\rho_\perp)\,\rho(\mathbf{k}) + \text{nonlinear terms}.$$

The time-time component must reproduce, for spherical baryonic sources,

$$g_{\text{obs}} = \frac{g_{\text{bar}}}{1 - \exp[-\sqrt{g_{\text{bar}}/a_0}]}.$$

The light-deflection potential is proportional to $\Phi + \Psi$, so the spatial components of the same kernel must determine whether

$$\Delta\Phi_\perp = \Delta\Psi_\perp.$$

The exact Bose factor $1 + n_B$, the argument $\sqrt{g_{\text{bar}}/a_0}$, and the absence of an additional spectral form factor require nonlinear vertices or state dependence beyond the quadratic Hessian. They must emerge from the same calculation; they cannot be imposed on the temporal equation while the spatial response is chosen separately.

Acceptance tests. A transverse completion is acceptable only if:

1. matter couples to one physical metric and the induced response satisfies its Ward identity;
2. the retarded kernel yields the RAR without a galaxy-by-galaxy parameter or an extra kernel $F(x)$;
3. the spatial response predicts lensing and is compatible with the observed dynamical-to-lensing relation;
4. the high-acceleration limit leaves Solar-System PPN values and tensor-wave propagation unchanged;
5. the retarded poles have no ghosts, tachyons, or gradient instabilities, and the noise kernel is positive;
6. any dissipative part obeys the state-appropriate fluctuation–dissipation relation and does not violate causal support;
7. the same calculation explains how momentum transfer is converted into acceleration and how total energy–momentum is conserved.

N.9 Action-level outcome

The action problem now divides into four parts:

$$\boxed{\begin{aligned} \text{ordinary longitudinal capacity} &= \text{reduced Einstein constraint action},\\ \text{spherical nonlinear capacity} &= \text{composite Misner–Sharp invariant},\\ \text{galactic transverse response} &= \text{conditional rank-one EFT},\\ &\qquad\text{with an open metric influence functional},\\ \text{capacity-exhaustion boundary} &= \text{open microscopic boundary functional}. \end{aligned}}$$

This division solves the original double-counting problem and closes the ordinary weak-field parent action. The transverse EFT now supplies a precise rank-one target. A specified simplicial GFT action must reproduce its projector, one-invariant potential, compact phase structure, and thermal matching; the state and metric vertex must then determine the full $\widehat{\Pi}_R^{\mu\nu\alpha\beta}$, including its local Ward-identity term. The boundary-channel functional at $q_{\rm geo} = 0$ remains separate work.

Appendix O: Reproduction Code

This terminal appendix contains the executable reproduction block after all conceptual, provenance, and action-level appendices. It recomputes the manuscript's numerical spine from the definitions stated in the paper; it does not add a new derivation.

The script below recomputes the numerical consequences of the stated definitions: the 1680-state spectrum, closure solution, sharing entropy, marked vertex, dressed scale, Joyce constant, edge-kernel chain, corrected charged-lepton ratios, slot mutual information, closed routing covariance, primitive route spectrum, adjoint channel, causal color-blocking factor, and alphabet-independent generation cutoff. It also verifies the algebraic normalization identity $n_{\rm hor} S_\infty^{\rm cell} = 1/4$. CODATA constants are comparison inputs; no observational target is fitted inside the script. It runs under Python 3 with `numpy` and `scipy`. The complete Hessian and graph enumeration are supplied in the accompanying audit script.

```
# ============================================================
# Reproduction script for the Entropic Scalar EFT manuscript.
# Recomputes the full numerical spine from stated definitions:
```

```
# the 1680-state K^2 spectrum, eta*, g_share,eff, marked
# vertex, dressed L* and G*, the
# Joyce constant, the 1/4 horizon identity, the edge-kernel
# chain, epsilon, sigma*, the lepton ladder, slot MI, and
# the conditional color-route algebra.
# Dependencies: Python 3, numpy, scipy.
# Every printed value is compared against the manuscript.
# ==============================================================
import itertools, math
from math import log, exp, pi, gamma, sqrt
from collections import Counter
import numpy as np
from scipy.optimize import brentq

# 1. Enumerate 1680 states: injective 4-of-7 labels m in -3..3, x2 parity
labels = range(-3,4)
states = list(itertools.permutations(labels,4))
print("P(7,4) =", len(states), " total with parity:", 2*len(states))

def K2(ms):
    S = sum(ms); Sig2 = sum(m*m for m in ms)
    return 48 - (S*S - Sig2)/3

spec = Counter()
for st in states:
    spec[round(K2(st)*3)] += 2   # parity doubling; key = 3*K2 to keep exact
print("Distinct K2 values and multiplicities (K2 as fraction /3):")
for k in sorted(spec): print(f"  K2={k}/3 = {k/3:.4f}  mult={spec[k]}")
print("Total multiplicity:", sum(spec.values()))

# 2. Solve closure condition <K2> = 3/(2 eta)
Ks = np.array([k/3 for k in sorted(spec)])
ns = np.array([spec[k] for k in sorted(spec)], dtype=float)
def avgK2(eta):
    w = ns*np.exp(-eta*Ks)
    return (w*Ks).sum()/w.sum()
f = lambda eta: avgK2(eta) - 3/(2*eta)
eta_star = brentq(f, 1e-4, 1.0, xtol=1e-16)
print("\neta* =", eta_star, " (paper: 0.02986684439352237)")

w = ns*np.exp(-eta_star*Ks); Z = w.sum(); p_class = w/Z
p_micro = np.exp(-eta_star*Ks)/Z
H = -(ns*p_micro*np.log(p_micro)).sum()
print("g_share,eff =", H, " (paper: 7.419800023570903)")
avg = avgK2(eta_star)
var = (p_class*(Ks-avg)**2).sum()
print("<K2> =", avg, " (paper 50.2229154254)   3/(2eta*) =", 3/(2*eta_star))
print("Var(K2) =", var, " (paper 15.6889750078)   a_UV = 1/Var =", 1/var)
print("C_cl = eta* <K2> =", eta_star*avg)

# 3. Decorated marked vertex, L*, G*
hbar=1.054571817e-34; c=299792458.0; me=9.1093837139e-31  # CODATA 2022
lam_e = hbar/(me*c)
print("\nlambda_e =", lam_e, " (paper 3.861592671986303e-13)")
r = exp(-7*H)
L0 = -1.5*lam_e*math.log1p(-r)
q = 2/7
zeta = 9*exp(-H)*(1-eta_star*8/49)**(21/2)
Zmu = 1+zeta
Ze = (1+zeta)*(1+7*zeta*zeta)
Ztau2 = 1+q*zeta
Lstar = Ze*L0
print("e^{7g} =", exp(7*H), " (paper 3.602860521062804e22)")
```

```
print("(2/3)e^{7g} =", (2/3)*exp(7*H), " (paper 2.401907014041869e22)")
print("zeta* =", zeta, " (paper 0.005123584484947)")
print("Z_mu, Z_e, Z_tau2 =", Zmu, Ze, Ztau2)
print("L0 =", L0, " (paper 1.607719470158885e-35)")
print("L* =", Lstar, " (paper 1.616253701392569e-35)")
Gstar = c**3*Lstar**2/hbar
print("G* =", Gstar, " (paper 6.674289077220912e-11)")
G_codata = 6.67430e-11
LP = sqrt(hbar*G_codata/c**3)
print("L* vs Planck length:", (LP-Lstar)/LP*100, "%")
print("G* vs CODATA:", (G_codata-Gstar)/G_codata*100, "%")

# 4. Joyce constant
Gtet = 3*gamma(1/3)**6/(2**(14/3)*pi**4)
print("\nG_tet(0) =", Gtet, " (paper 0.4482203943883814)")
print("ln(7/6)/(pi*Gtet) =", log(7/6)/(pi*Gtet), " (paper 0.109472228)")
print("S_inf_cell = 3ln2/(32 pi Gtet) =", 3*log(2)/(32*pi*Gtet), " (paper 0.0461482516)")
print("n_hor*S_inf_cell =", (8*pi*Gtet/(3*log(2))) * (3*log(2)/(32*pi*Gtet)))

# 5. Edge kernel chain
Jbare = 2/3*eta_star
Jtree = Jbare/3
Sig = 7+2/9
cloop = 1/(1+Jtree*Sig)
Jren = Jtree*cloop
print("\nJ_bare =", Jbare, " (paper 0.0199112296)")
print("J_tree =", Jtree, " (paper 0.0066370765)")
print("c_loop =", cloop, " (paper 0.95426)")
print("J_ren =", Jren, " (paper 0.00633348)")
print("gamma coeff 4 J_ren/(3 pi^2) =", 4*Jren/(3*pi**2), " (paper 8.556e-4)")

# 6. epsilon and cluster numbers
epsv = H/(4*pi**2)
print("\nepsilon = g/(4pi^2) =", epsv, " 1-eps =", 1-epsv, " ceiling =", 2-epsv)
print("sigma* = pi/g =", pi/H, " (paper 0.42340665)")

# 7. Corrected charged-lepton ladder
mmu0 = 720*q; mtau0 = 720**2*q**4
mmu = mmu0*Zmu; mtau = mtau0*Zmu*Ztau2
print("\nbaseline m_mu/m_e =", mmu0, " baseline m_tau/m_e =", mtau0)
print("m_mu/m_e =", mmu, " (paper 206.768280237)")
print("m_tau/m_e =", mtau, " (paper 3477.343310)")
print("alpha_mu =", log(206.7683)/log(mmu), " alpha_tau =", log(3477.23)/log(mtau))

# 8. mixing-time ratio
tau_star = Lstar/c; tau_e = hbar/(me*c**2)
print("\ntau* =", tau_star, " tau_e =", tau_e, " ratio =", tau_e/tau_star)

# 9. slot mutual information (marginal of slot under admissibility)
joint = Counter()
for st in states:
    wgt = exp(-eta_star*K2(st))
    joint[(st[0],st[1])] += 2*wgt
tot = sum(joint.values())
pj = {k:v/tot for k,v in joint.items()}
p0 = Counter(); p1 = Counter()
for (a,b),v in pj.items(): p0[a]+=v; p1[b]+=v
I = sum(v*log(v/(p0[a]*p1[b])) for (a,b),v in pj.items())
H0 = -sum(v*log(v) for v in p0.values())
print("\nslot MI =", I, " (paper 0.1545)   H(slot) =", H0, " I/H =", I/H0, " (paper 0.079)")

# 10. Conditional color-route checks
route_rows = []
```

```
route_weights = []
for st in states:
    m1,m2,m3,m4 = st
    route_rows.append([
        m1*m2 + m3*m4,
        m1*m3 + m2*m4,
        m1*m4 + m2*m3,
    ])
    route_weights.append(2*exp(-eta_star*K2(st)))  # parity doubling
route_rows = np.asarray(route_rows, dtype=float)
route_weights = np.asarray(route_weights, dtype=float)
route_weights /= route_weights.sum()
route_mean = (route_weights[:,None]*route_rows).sum(axis=0)
centered = route_rows-route_mean
Croute = (route_weights[:,None,None]
          * centered[:,:,None]*centered[:,None,:]).sum(axis=0)
P1 = np.ones((3,3))/3
P2 = np.eye(3)-P1
lam1 = np.trace(P1@Croute)
lam2 = np.trace(P2@Croute)/2
print("\nroute covariance =\n", Croute)
print("route singlet/doublet eigenvalues =", lam1, lam2)
print("(3/4) Var(K2) =", 0.75*var)
assert np.allclose(Croute, lam1*P1+lam2*P2, atol=1e-10)
assert abs(lam1-0.75*var) < 1e-10

Groute = np.eye(3)- (np.ones((3,3))-np.eye(3))/20
Graw = np.linalg.eigvalsh(Groute)
Kroute = (20/21)*Groute
Lroute = sqrt(13)/7*P1
print("primitive route Gram eigenvalues =", Graw)
print("lambda2/lambda1 =", Graw[-1]/Graw[0], " (paper 7/6)")
print("dilation residual =",
      np.linalg.norm(Kroute.T@Kroute + Lroute.T@Lroute-np.eye(3)))

Utest = np.diag(np.exp(1j*np.array([0.2, -0.5, 0.3])))
trU = np.trace(Utest)
return_projective = 1.5*(1-abs(trU)**2/9)
return_adjoint = (4/3)*(1-(abs(trU)**2-1)/8)
phase = np.exp(0.37j)
return_rephased = 1.5*(1-abs(np.trace(phase*Utest))**2/9)
print("projective/adjoint return costs =",
      return_projective, return_adjoint, return_rephased)
assert abs(return_projective-return_adjoint) < 1e-14
assert abs(return_projective-return_rephased) < 1e-14

t8 = 13/14
g_step = (2/3)*log(14/13)
block = 2*sqrt(6)
g_graph = block*g_step
print("t8 =", t8, " -ln(t8) =", -log(t8))
print("g_HK,step^2 =", g_step)
print("diamond blocking =", block, " g_graph,HK^2 =", g_graph)

# 11. Alphabet-independent generation cutoff
for excluded_count in (1, 2, 3):
    exclusion_variances = []
    for excluded in itertools.combinations(labels, excluded_count):
        reduced = [m for m in labels if m not in excluded]
        reduced_K2 = [K2(st) for st in itertools.permutations(reduced, 4)]
        exclusion_variances.append(float(np.var(reduced_K2)))
    nonzero = sum(value > 1e-12 for value in exclusion_variances)
    zero = sum(abs(value) <= 1e-12 for value in exclusion_variances)
```

```
print(
    f"excluded labels = {excluded_count}: cases = {len(exclusion_variances)}, "
    f"nonzero = {nonzero}, zero = {zero}, "
    f"variance range = [{min(exclusion_variances)}, {max(exclusion_variances)}]"
)
if excluded_count < 3:
    assert nonzero == len(exclusion_variances)
else:
    assert zero == len(exclusion_variances)
```

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